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A NEW INTUITIONISTIC FUZZY MODAL OPERATOR

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In memory of one of the pioneers
of the intuitionistic fuzzy sets theory
Prof. Dogan Coker (1951 - 2003)

1. Introduction

The modal operators, defined over the Intuitionistic Fuzzy Sets (IFSs; see [1]) and in the frameworks of the Intuitionistic Fuzzy Modal Logic (IFML; see [1, 2]) are modicications and extensions of the ordinary modal logic operators (see, e.g., [3]). Now, we shall construct new modification of these intuitionistic fuzzy operators.

Initially, we shall start with short remarks on IFML and after this will introduce the new operator.

2. Short remarks on IFML

To each proposition p in the intuitionistic fuzzy predicative logic (see [1, 4]) we can assign a "truth degree" $\mu(p) \in [0, 1]$ and a "falsity degree" $\nu(p) \in [0, 1]$, such that

$$\mu(p) + \nu(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

When the values $V(p)$ and $V(q)$ of the propositions p and q are known, we can define the operations "negation", "conjunction", "disjunction" and two versions of implications "(max, min)-implication" and "sg-implication", as follows:

$$\neg V(p) = V(\neg p) = \langle \nu(p), \mu(p) \rangle,$$

$$V(p) \& V(q) = V(p \& q) = \langle min(\mu(p), \mu(q)), max(\nu(p), \nu(q)) \rangle,$$

$$V(p) \vee V(q) = V(p \vee q) = \langle max(\mu(p), \mu(q)), min(\nu(p), \nu(q)) \rangle.$$

$$V(p) \rightarrow V(q) = V(p \rightarrow q) = \langle max(\nu(p), \mu(q)), min(\mu(p), \nu(q)) \rangle,$$

$$V(p) \supset V(q) = V(p \supset q)$$

$$= \langle 1-(1-\mu(q)).sg(\mu(p)-\mu(q)), \nu(q).sg(\mu(p)-\mu(q)).sg(\nu(q)-\nu(p))\rangle,$$

where:

$$sg(x) = \begin{cases} 1 & \text{if } x > 0 \\ \\ 0 & \text{if } x \leq 0 \end{cases}$$

The proposition p has the geometrical interpretation From Fig. 1, where S is a set of propositions ($p \in S$).

Following [1, 2] we shall define also the operators:

$$V(\Box p) = \langle \mu(p), 1-\mu(p)\rangle,$$

$$V(\Diamond p) = \langle 1-\nu(p), \nu(p)\rangle$$

which are analogous of the modal operators "necessity" and "possibility".

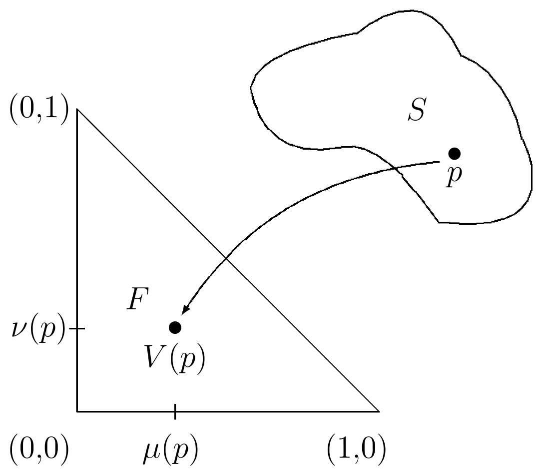

Fig. 1.

They have the geometrical interpretations from Fig. 2 and 3.

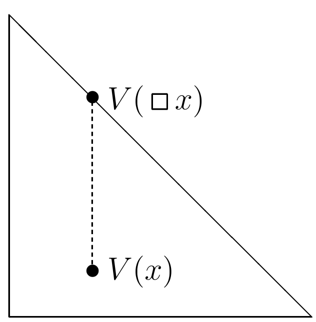

Fig 2.

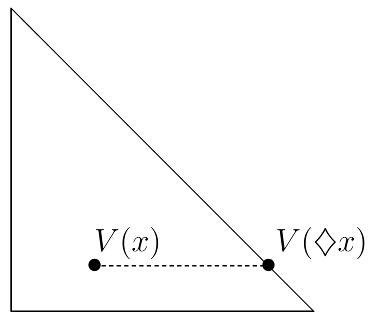

Fig 3.

For the needs of the discussion below we shall define the notions of "a tautology" and of "an Intuitionistic Fuzzy Tautology" (IFT) through (see [1, 4]):

"p is a tautology" **iff** $V(p) = \langle 1, 0 \rangle$,
"p is an IFT" **iff** if $V(p) = \langle \mu(p), \nu(p) \rangle$, then $\mu(p) \geq \nu(p)$.

If p and q are two propositions, then $V(p) \geq V(q)$ **iff**

$$\mu(p) \geq \mu(q) \quad \text{and} \quad \nu(p) \leq \nu(q).$$

and $V(p) > V(q)$ **iff**

$$\mu(p) \geq \mu(q) \quad \text{and} \quad \nu(p) \leq \nu(q)$$

where at least one of the inequalities is strong.

3. A new intuitionistic fuzzy modal operator

Now we shall introduce a new intuitionistic fuzzy modal operator having the form

$$V(\bigcirc p) = \langle \frac{\mu(p)}{\mu(p) + \nu(p)}, \frac{\nu(p)}{\mu(p) + \nu(p)} \rangle,$$

(see Fig. 4).

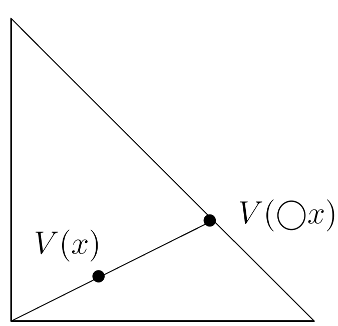

Fig 4.

The following assertions are valid.

THEOREM 1: For every proposition p:

(a) $V(\bigcirc\bigcirc p) = V(\bigcirc p)$,

(b) $V(\bigcirc\Box p) = V(\Box p)$,

(c) $V(\Box\bigcirc p) = V(\bigcirc p)$,

(d) $V(\bigcirc\Diamond p) = V(\Diamond p)$,

(e) $V(\Diamond\bigcirc p) = V(\bigcirc p)$.

THEOREM 2: For every two propositions p and q:

$$V(\bigcirc\neg p) = V(\neg\bigcirc p).$$

Unfortunately, the other equalities, that are valid for operators $\Box$ and $\Diamond$, are not valid here.

THEOREM 3: For every proposition p:

(a) p is an IFT **iff** $\bigcirc p$ is an IFT,

(b) $V(p \to \bigcirc p)$ is an IFT,

(b) $V(\bigcirc p \to p)$ is an IFT,

(d) $V(p \supset \bigcirc p)$ is a tautology,

(e) $V(\bigcirc p \supset p) = V(p)$.

Following [1, 2] we can note that:

(a) if $\Box p$ is an IFT, then p is an IFT, but the opposite case is not valid;

(b) if p is an IFT, then $\Diamond p$ is an IFT, but the opposite case is not valid.

For operator $\bigcirc$, when $V(p) > \langle 0, 0\rangle$ is valid

THEOREM 4: p is an IFT **iff** $\bigcirc p$ is an IFT.

Proof: Let p be an IFT. Therefore $\mu(p) \geq \nu(p)$. Hence,

$$\frac{\mu(p)}{\mu(p)+\nu(p)} \geq \frac{\nu(p)}{\mu(p)+\nu(p)},$$

i.e., $\bigcirc p$ is an IFT. The opposite direction is checked by analogy.

When p has no proper intuitionistic fuzzy values, i.e., when

$$V(p) = \langle \mu(p), 1-\mu(p)\rangle,$$

then

$$V(\bigcirc p) = V(p).$$

Therefore, in this case

$$V(\Box p) = V(p) = V(\bigcirc p) = V(\Diamond p),$$

while for a p with proper intuitionistic fuzzy values the following assertion is valid.

THEOREM 5: $V(\Box p) \leq V(\bigcirc p) \leq V(\Diamond p)$.

References

[1] K. Atanassov, Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[2] K. Atanassov, Two variants of intuitionistic fuzzy modal logic. Preprint IM-MFAIS-3-89, Sofia, 1989.

[3] R. Feys, Modal logics, Gauthier-Villars, Paris, 1965.

[4] K. Atanassov, Two variants of intuitonistc fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia, 1988.