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ON SOME INTUITIONISTIC FUZZY NEGATIONS

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Abstract

Fifteen intutionistic fuzzy implications are given and the negations that they generate are studied. The relations between the negations and the Law for Excluded Middle is described. Relations between the intuitionistic fuzzy negations are shown.

1 Introduction: On some previous results

Variants of intuitionistic fuzzy implications are discussed in [3, 4, 5, 7, 8]. In [6] they are the basis for obtaining of intuitionistic fuzzy negations. Here we shall study some properties of these negations and will show that they satisfy the properties of the intuitionistic negation.

Let x be a variable. Then its intuitionistic fuzzy truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x. Any other formula is estimated by analogy.

Obviously, when V is ordinary fuzzy truth-value estimation, for it

$$b = 1 - a.$$

Everywhere below we shall assume that for the three variables x, y and z equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle, V(z) = \langle e, f \rangle$ $(a, b, c, d, e, f, a + b, c + d, e + f \in [0, 1])$ hold.

For the needs of the discussion below we shall define the notion of Intuitionistic Fuzzy Tautology (IFT, see, [1, 2]) by:

$$x \text{ is an IFT if and only if } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$.

In some definitions we shall use functions sg and $\overline{sg}$:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

$$\overline{sg}(x) = \begin{cases} 0 & \text{if } x > 0 \\ \\ 1 & \text{if } x \leq 0 \end{cases}$$

In ordinary intuitionistic fuzzy logic (see [1, 2]) the negation of variable x is $N(x)$ such that

$$V(N(x)) = \langle b, a \rangle.$$

For two variables x and y operations "conjunction" (&) and "disjunction" ($\vee$) are defined by:

$$V(x \& y) = \langle \min(a, c), \max(b, d) \rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d) \rangle.$$

Following [5] (that is based on [9]), we shall mention that we can define at least 15 different implications - see Table 1.

TABLE 1: List of intuitionistic fuzzy implications

Notation	Name	Form of implication
$\rightarrow_1$	Zadeh	$\langle \max(b, \min(a, c)), \min(a, d) \rangle$
$\rightarrow_2$	Gaines-Rescher	$\langle 1 - \mathrm{sg}(a - c), d.\mathrm{sg}(a - c) \rangle$
$\rightarrow_3$	Gödel	$\langle 1 - (1 - c).\mathrm{sg}(a - c), d.\mathrm{sg}(a - c) \rangle$
$\rightarrow_4$	Kleene-Dienes	$\langle \max(b, c), \min(a, d) \rangle$
$\rightarrow_5$	Lukasiewicz	$\langle \min(1, b + c), \max(0, a + d - 1) \rangle$
$\rightarrow_6$	Reichenbach	$\langle b + ac, ad \rangle$
$\rightarrow_7$	Willmott	$\langle \min(\max(b, c), \max(a, b), \max(c, d)),$ $\max(\min(a, d), \min(a, b), \min(c, d)) \rangle$
$\rightarrow_8$	Wu	$\langle 1 - (1 - \min(b, c)).\mathrm{sg}(a - c),$ $\max(a, d).\mathrm{sg}(a - c).\mathrm{sg}(d - b) \rangle$
$\rightarrow_9$	Klir and Yuan 1	$\langle b + a^2 c, ab + a^2 d \rangle$
$\rightarrow_{10}$	Klir and Yuan 2	$\langle c.\overline{sg}(1 - a) + \mathrm{sg}(1 - a).(\overline{sg}(1 - c) + b.\mathrm{sg}(1 - c)),$ $d.\overline{sg}(1 - a) + a.\mathrm{sg}(1 - a).\mathrm{sg}(1 - c) \rangle$
$\rightarrow_{11}$	Atanassov 1	$\langle 1 - (1 - c).\mathrm{sg}(a - c), d.\mathrm{sg}(a - c).\mathrm{sg}(d - b) \rangle$
$\rightarrow_{12}$	Atanassov 2	$\langle \max(b, c), 1 - \max(b, c) \rangle$
$\rightarrow_{13}$	Atanassov and Kolev	$\langle b + c - b.c, a.d \rangle$
$\rightarrow_{14}$	Atanassov and Trifonov	$\langle 1 - (1 - c).\mathrm{sg}(a - c) - d.\overline{sg}(a - c).\mathrm{sg}(d - b),$ $d.\mathrm{sg}(d - b) \rangle$
$\rightarrow_{15}$	Atanassov 3 (see below)	$\langle 1 - (1 - \min(b, c)).\mathrm{sg}(\mathrm{sg}(a - c) + \mathrm{sg}(d - b))$ $- \min(b, c).\mathrm{sg}(a - c).\mathrm{sg}(d - b),$ $1 - (1 - \max(a, d)).\mathrm{sg}(\overline{sg}(a - c) + \overline{sg}(d - b))$ $- \max(a, d).\overline{sg}(a - c).\overline{sg}(d - b) \rangle$

For some of them in [6] we constructed respective negations. Below we shall study these and new negations and will discuss their properties.

2 Main results

First, following [6] and using as a basis equality

$$\neg x = x \to 0$$

or

$$\neg\langle a, b\rangle = \langle a, b\rangle \to \langle 0, 1\rangle$$

we shall construct negations, corresponding to each of the above implications. These negations are introduced in Table 2. The table is extension of Table 2 from [6] with the negations, generated by the author's implications.

TABLE 2: List of intuitionistic fuzzy negations

Name	Form of negation
Zadeh	$\langle b, a\rangle$
Gaines-Rescher	$\langle 1 - \mathrm{sg}(a), \mathrm{sg}(a)\rangle$
Gödel	$\langle 1 - \mathrm{sg}(a), \mathrm{sg}(a)\rangle$
Kleene-Dienes	$\langle b, a\rangle$
Lukasiewicz	$\langle b, a\rangle$
Reichenbach	$\langle b, a\rangle$
Willmott	$\langle b, a\rangle$
Wu	$\langle 1 - \mathrm{sg}(a), \mathrm{sg}(a).\mathrm{sg}(1 - b)\rangle$
Klir and Yuan 1	$\langle b, a.b + a^2\rangle$
Klir and Yuan 2	$\langle b, a\rangle$
Atanassov 1	$\langle 1 - \mathrm{sg}(a), \mathrm{sg}(a).\mathrm{sg}(1 - b)\rangle$
Atanassov 2	$\langle b, 1 - b\rangle$
Atanassov and Kolev	$\langle b, a\rangle$
Atanassov and Trifonov	$\langle 1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1 - b), \mathrm{sg}(1 - b)\rangle$
Atanassov 3	$\langle 1 - \mathrm{sg}(\mathrm{sg}(a) + \mathrm{sg}(1 - b)), 1 - \overline{sg}(a).\overline{sg}(1 - b)\rangle$

First, we shall note that some negations coincide. Obviously, Zadeh, Kleene-Dienes, Lukasiewicz, Reichenbach, Willmott and second Klir and Yuan's negations coincide. Analogically, it can be seen that negations of Gaines-Rescher and Gödel coincide and that negation introduced by me in [1] coincides with Wu's negations. It can be easily seen that the latter four mentioned negations coincide.

Really, their first components coincide, while for the expression

$$X \equiv \mathrm{sg}(a).\mathrm{sg}(1 - b) - \mathrm{sg}(a)$$

holds:
if $a = 0$: $X = 0.\mathrm{sg}(1 - b) - 0 = 0$
if $a > 0$ and therefore, $1 - b \geq a > 0$: $X = 1.1 - 1 = 0$,
i.e., always

$$\mathrm{sg}(a).\mathrm{sg}(1 - b) - \mathrm{sg}(a).$$

The proof that Atanassov and Trifonov's and Third Atanassov's negations coincide is analogical. Therefore, we can also write it in the following modified form

$$\langle 1 - \mathrm{sg}(\mathrm{sg}(a) + \mathrm{sg}(1 - b)), \mathrm{sg}(1 - b)\rangle.$$

For the first 10 of the above 15 implications the following three properties are checked in [4]:
Propertiy P1: $A \to \neg\neg A$,
Propertiy P2: $\neg\neg A \to A$,
Propertiy P3: $\neg\neg\neg A = \neg A$.

Now, we shall formulate similar assertions as in [4], but for all negations.

Theorem 1: Each of the negations from Table 2 satisfies Property 1.

Theorem 2: Negations of Zadeh, Kleene-Dienes, Lukasiewicz, Reichenbach, Willmott, Klir and Yuan 2 and Atanassov and Kolev satisfy Property 2, while negations of Gaines-Rescher, Gödel, Wu, Klir and Yuan 1, the three Atanassov's, Atanassov and Trifonov's implications do not satisfy it.

Theorem 3: Each of the negations from Table 2 satisfies Property 3.

We shall prove Theorem 3 for the case of Atanassov and Trifonov's negation.

$$\neg\neg\neg\langle a, b\rangle = \neg\neg\langle 1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b), \mathrm{sg}(1-b)\rangle$$

$$= \neg\langle 1 - \mathrm{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b)) - \overline{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b)).\mathrm{sg}(1 - \mathrm{sg}(1-b)),$$
$$\mathrm{sg}(1 - \mathrm{sg}(1-b))\rangle$$

$$= \langle 1 - \mathrm{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b))$$
$$- \overline{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b)).\mathrm{sg}(1 - \mathrm{sg}(1-b)))$$
$$- \overline{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b)) - \overline{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b))$$
$$.\mathrm{sg}(1 - \mathrm{sg}(1-b))).\mathrm{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(1-b))), \mathrm{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(1-b)))\rangle$$

Let

$$X \equiv 1 - \mathrm{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b))$$
$$- \overline{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b)).\mathrm{sg}(1 - \mathrm{sg}(1-b)))$$
$$- \overline{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b)) - \overline{sg}(1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b))$$
$$.\mathrm{sg}(1 - \mathrm{sg}(1-b))).\mathrm{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(1-b))) - (1 - \mathrm{sg}(a) - \overline{sg}(a).\mathrm{sg}(1-b))$$

Let $a = 0$. Then $\mathrm{sg}(a) = 0$, $\overline{sg}(a) = 1$ and

$$X = 1 - \mathrm{sg}(1 - \mathrm{sg}(1 - \mathrm{sg}(1-b)) - \overline{sg}(1 - \mathrm{sg}(1-b)).\mathrm{sg}(1 - \mathrm{sg}(1-b)))$$

$$-\overline{sg}(1-\mathrm{sg}(1-\mathrm{sg}(1-b))-\overline{sg}(1-\mathrm{sg}(1-b)).\mathrm{sg}(1-\mathrm{sg}(1-b))).\mathrm{sg}(1-\mathrm{sg}(1-\mathrm{sg}(1-b)))-(1-\mathrm{sg}(1-b))$$

If $b = 1$, then $\mathrm{sg}(1-b) = 0$ and

$$X = 1 - \mathrm{sg}(1 - \mathrm{sg}(1) - \overline{sg}(1).\mathrm{sg}(1)) - \overline{sg}(1 - \mathrm{sg}(1) - \overline{sg}(1).\mathrm{sg}(1)).\mathrm{sg}(1 - \mathrm{sg}(1)) - 1$$

$$= 1 - \mathrm{sg}(1-1) - \overline{sg}(1-1).\mathrm{sg}(0) - 1 = 1 - 1 = 0.$$

If $b < 1$, then $\mathrm{sg}(1-b) = 1$ and

$$X = 1 - \mathrm{sg}(1 - \mathrm{sg}(1-1) - \overline{sg}(1-1).\mathrm{sg}(1-1)) - \overline{sg}(1 - \mathrm{sg}(1-1) - \overline{sg}(1-1).\mathrm{sg}(1-1))$$

$$.\mathrm{sg}(1 - \mathrm{sg}(1-1)) - (1-1)$$
$$= 1 - \mathrm{sg}(1) - \overline{sg}(1).\mathrm{sg}(1) = 1 - 1 = 0.$$

Let $a > 0$. Then $\text{sg}(a) = 1$, $\overline{sg}(a) = 0$, $\text{sg}(1-b) = 1$ and

$$X = 1 - \text{sg}(1 - \text{sg}(1-1) - \overline{sg}(1-1).\text{sg}(1-1)) - \overline{sg}(1 - \text{sg}(1-1) - \overline{sg}(1-1)$$

$$.\text{sg}(1-1)).\text{sg}(1 - \text{sg}(1-1)) - (1-1)$$
$$= 1 - \text{sg}(1) - \overline{sg}(1).\text{sg}(1) = 1 - 1 = 0.$$

Therefore Property 3 is valid for Atanassov and Trifonov's and for the Third Atanassov's negations.

The comparison is also interesting

Second, we can number the different negations as it is shown on Table 3.

Table 3: List of the different intuitionistic fuzzy negations

Notation	Form of negation
$\neg_1$	$\langle b, a\rangle$
$\neg_2$	$\langle 1 - \text{sg}(a), \text{sg}(a)\rangle$
$\neg_3$	$\langle b, a.b + a^2\rangle$
$\neg_4$	$\langle b, 1 - b\rangle$
$\neg_5$	$\langle 1 - \text{sg}(\text{sg}(a) + \text{sg}(1-b)), \text{sg}(1-b)\rangle$

We shall study the validity of the Law for Excluded Middle (LEM) in the forms:

$$\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle 1, 0\rangle$$

(tautology-form) and

$$\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle p, q\rangle,$$

and a Modified LEM in the forms:

$$\neg\neg\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle 1, 0\rangle$$

(tautology-form) and

$$\neg\neg\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle p, q\rangle,$$

(IFT-form), where $1 \geq p \geq q \geq 0$ and $i = 1, 2, ..., 5$.

Theorem 4: No one negation satisfies the LEM in the tautological form.

Theorem 5: Negations $\neg_1, \neg_3$ and $\neg_4$ satisfy the LEM in the IFT-form.

Theorem 6: Only $\neg_2$ and $\neg_5$ satisfy the Modified LEM in the tautological form.

Theorem 7: All negations satisfy the Modified LEM in the IFT-form.

As illustration we shall prove Theorems 6 and 7 for the case of implication $\neg_5$.

$$\neg_5\neg_5\langle a, b\rangle \vee \neg_5\langle a, b\rangle$$

$$= \langle 1 - \text{sg}(a) - \overline{sg}(a).\text{sg}(1-b), \text{sg}(1-b)\rangle \vee \langle 1 - \text{sg}(1 - \text{sg}(a) - \overline{sg}(a).\text{sg}(1-b))$$
$$-\overline{sg}(1 - \text{sg}(a) - \overline{sg}(a).\text{sg}(1-b)).\text{sg}(1 - \text{sg}(1-b)), \text{sg}(1 - \text{sg}(1-b))\rangle$$
$$= \langle \max(1 - \text{sg}(a) - \overline{sg}(a).\text{sg}(1-b), 1 - \text{sg}(1 - \text{sg}(a) - \overline{sg}(a).\text{sg}(1-b))$$
$$-\overline{sg}(1 - \text{sg}(a) - \overline{sg}(a).\text{sg}(1-b)).\text{sg}(1 - \text{sg}(1-b))), \min(\text{sg}(1-b), \text{sg}(1 - \text{sg}(1-b)))\rangle$$

Let

$$X \equiv \max(1-\mathrm{sg}(a)-\overline{sg}(a).\mathrm{sg}(1-b), 1-\mathrm{sg}(1-\mathrm{sg}(a)-\overline{sg}(a).\mathrm{sg}(1-b))$$
$$-\overline{sg}(1-\mathrm{sg}(a)-\overline{sg}(a).\mathrm{sg}(1-b)).\mathrm{sg}(1-\mathrm{sg}(1-b)))$$
$$-\min(\mathrm{sg}(1-b), \mathrm{sg}(1-\mathrm{sg}(1-b)))$$

Let $a = 0$. Then $\mathrm{sg}(a) = 0$, $\overline{sg}(a) = 1$ and

$$X = \max(1-\mathrm{sg}(1-b), 1-\mathrm{sg}(1-\mathrm{sg}(1-b))-\overline{sg}(1-\mathrm{sg}(1-b)).\mathrm{sg}(1-\mathrm{sg}(1-b)))$$
$$-\min(\mathrm{sg}(1-b), \mathrm{sg}(1-\mathrm{sg}(1-b))).$$

If $b = 1$, then $\mathrm{sg}(1-b) = 0$ and

$$X = \max(1, 1-\mathrm{sg}(1)-\overline{sg}(1).\mathrm{sg}(1)) - \min(0, \mathrm{sg}(1)) = \max(1,0) - \min(0,1) = 1.$$

If $b < 1$, then $\mathrm{sg}(1-b) = 1$ and

$$X = \max(1-1, 1-\mathrm{sg}(1-1)-\overline{sg}(1-1).\mathrm{sg}(1-1)) - \min(1, \mathrm{sg}(1-1)) = \max(0,1) - \min(1,0) = 1.$$

Let $a > 0$. Then $\mathrm{sg}(a) = 1$, $\overline{sg}(a) = 0$, $\mathrm{sg}(1-b) = 1$ and

$$X \equiv \max(1-1, 1-\mathrm{sg}(1-1)-\overline{sg}(1-1).\mathrm{sg}(1-1)) - \min(1, \mathrm{sg}(1-1))$$
$$= \max(0,1) - \min(1,0) = 1.$$

Therefore, negation $\neg_5$ satisfies the Modified LEM in the IFT-form. On the other hand, in all cases the evaluation of the expression is equal to $\langle 1, 0\rangle$, i.e., this negation satisfies the Modified LEM in the tautological form.

Third, we shall study the relations between the different negations. By direct checks we can see the validity of the following Table 4.

The lack of relation between two implications is noted in Table 4 by "$*$".

Table 4: List of the relations between the different intuitionistic fuzzy negations

	$\neg_1$	$\neg_2$	$\neg_3$	$\neg_4$	$\neg_5$
$\neg_1$	$=$	$*$	$\leq$	$\geq$	$\geq$
$\neg_2$	$*$	$=$	$*$	$*$	$\geq$
$\neg_3$	$\geq$	$*$	$=$	$\geq$	$\geq$
$\neg_4$	$\leq$	$*$	$\leq$	$=$	$\geq$
$\neg_5$	$\leq$	$\leq$	$*$	$\leq$	$=$

The values from Table 5 are also interesting.

Table 5: List of the values of some special constants for the different intuitionistic fuzzy negations

$V(x)$	$\neg_1 V(x)$	$\neg_2 V(x)$	$\neg_3 V(x)$	$\neg_4 V(x)$	$\neg_5 V(x)$
$\langle 1, 0\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
$\langle 0, 1\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$
$\langle 0, 0\rangle$	$\langle 0, 0\rangle$	$\langle 1, 0\rangle$	$\langle 0, 0\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$

3 Conclusion: a new argument that the intuitionistic fuzzy sets have intuitionistic nature

The above assertions shown that all negations without the first one satisfy conditions of an intuitionistic logic, but not of a classical logic. These five negations were generated by implications, that were generated by fuzzy implications. Now, let us return from the intuitionistic fuzzy negations to ordinary fuzzy negations. The result is shown on Table 5, where $b = 1 - a$.

Table 5: List of the fuzzy negations, generated by intuitionistic fuzzy negations

Notation	Form of the intuitionistic fuzzy negation	Form of the fuzzy negation
$\neg_1$	$\langle b, a\rangle$	b
$\neg_2$	$\langle 1 - \mathrm{sg}(a), \mathrm{sg}(a)\rangle$	$1 - \mathrm{sg}(a)$
$\neg_3$	$\langle b, a.b + a^2\rangle$	b
$\neg_4$	$\langle b, 1 - b\rangle$	b
$\neg_5$	$\langle 1 - \mathrm{sg}(\mathrm{sg}(a) + \mathrm{sg}(1 - b)), \mathrm{sg}(1 - b)\rangle$	$1 - \mathrm{sg}(a)$

Really, having in mind the above mentioned equality $b = 1 - a$ for the fuzzy case, we can see directly that

$$1 - \mathrm{sg}(\mathrm{sg}(a) + \mathrm{sg}(1 - b)) = 1 - \mathrm{sg}(\mathrm{sg}(a) + \mathrm{sg}(a)) = 1 - \mathrm{sg}(\mathrm{sg}(a)) = 1 - \mathrm{sg}(a).$$

Therefore, from the intuitionistic fuzzy negations we can generate fuzzy negations, so that two of them ($\neg_3$ and $\neg_4$) coincide with the standard fuzzy negation ($\neg_1$). Therefore, there are intuitionistic fuzzy negations that loss their properties when they are restricted to ordinary fuzzy case. With other words, the construction of the intuitionistic fuzzy estimation

$$\langle \text{degree of membership/validity}, \text{degree of non-membership/non-validity}\rangle$$

that is specific for the intuitionistic fuzzy sets, is the reason for the intuitionistic behaviour of these sets. Over them we can define as intuitionistic, as well as classical negations. The other two negations ($\neg_2$ and $\neg_5$) also coincide and this fuzzy negation satisfies Properties 1 and 3 and does not satisfy Property 2, i.e., it has intuitionistic character.

Finally, we must note that as the latter fuzzy negations, as well as the five intuitionistic fuzzy negations are very simple. They can be extended essentially, if we use extended intuitionistic fuzzy modal operators and this will be a theme for a next reseach.

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