First Int. Conf. on IFS, Sofia, 18-19 Oct. 1997
NIFS 3 (1997) 4, 14-20

ON INTUITIONISTIC FUZZY SETS

Supriya Kumar De
Ranjit Biswas*
Akhil Ranjan Roy
Department of Mathematics,
Indian Institute of Technology,
Kharagpur - 721302, West Bengal,
INDIA. [1]

Abstract

In this paper we define α-cut of an IFS, nearest ordinary set of an IFS, distance between two IFSs, index of intuitionistic fuzziness and study their properties with examples.

Keywords : Fuzzy set, intuitionistic fuzzy set (IFS), α-cut, nearest ordinary set, index of intuitionistic fuzziness.

1 INTRODUCTION

Fuzzy sets have been generalised in many ways for various purposes, out of which [1], [7], [8], [12], [13] [14], [15], [16] are interesting. One of the existing higher order fuzzy sets is the notion of intuitionistic fuzzy sets (IFSs) introduced by Atanassov [1]. Where the fuzzy sets can be viewed as intuitionistic fuzzy sets, the converse is not necessarily true. In the present paper we study intuitionistic fuzzy sets to define α-cut of intuitionistic fuzzy sets, nearest ordinary sets of intuitionistic fuzzy sets, distance between two intuitionistic fuzzy sets and prove some propositions.

2 PRELIMINARIES

We present here some basic preliminaries for the progress of our works.

Definition 2.1

Let E be any set, a mapping μ_A : E$\rightarrow$ $[0, 1]$ is called a fuzzy subset of E.

Definition 2.2

Let A be a fuzzy subset of a set E. The complement of A is A^c with membership function μ_{A^c} defined by

[1]Correspondence to : Dr. Ranjit Biswas, Department of Mathematics, Indian Institute of Technology, Kharagpur-721302, W.B., INDIA.
(E-mail : ranjit@maths.iitkgp.ernet.in)

$$\mu_{A^c} = 1 - \mu_A(x) \ , \forall \ x \in E.$$

Definition 2.3

Let a set E is fixed. An intuitionistic fuzzy set or IFS A in E is an object having the form

$$A = \{<x, \mu_A(x), \nu_A(x) > | \, x \in E \}$$

where the function $\mu_A : E \rightarrow [0,1]$ and $\nu_A : E \rightarrow [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Definition 2.4

If A and B are two IFSs of the set E, then

$A \subset B$ iff $\forall x \in E$, $[\ \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)\ ]$

$A \subset B$ iff $B \supset A$

$A = B$ iff $\forall\ x \in E$, $[\ \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)\ ]$

$\bar{A} = \{<x, \nu_A(x), \mu_A(x) > | \, x \in E \}$

$A \cap B = \{<x \ , \min(\mu_A(x) \ , \mu_B(x)) \ , \max(\nu_A(x), \nu_B(x)) > | \, x \in E \}$

$A \cup B = \{<x \ , \max(\mu_A(x) \ , \mu_B(x)) \ , \min(\nu_A(x), \nu_B(x)) > | \, x \in E \}$

$A + B = \{<x \ , \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x) \ , \nu_A(x).\nu_B(x) > | \, x \in E \}$

$A \ . \ B = \{<x \ , \mu_A(x).\mu_B(x) \ , \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x) > | \, x \in E \}$

$\Box \ A = \{<x, \mu_A(x), 1 - \mu_A(x) > | \, x \in E \}$

$\Diamond \ A = \{<x, 1 - \nu_A(x), \nu_A(x) > | \, x \in E \}$

$C(A) = \{<x, K, L> | \, x \in E \}$

where $K = \max_{x \in E} \mu_A(x)$,

$L = \min_{x \in E} \nu_A(x)$

$I(A) = \{<x, k, l> | \, x \in E \}$

where $k = \min_{x \in E} \mu_A(x)$,

$l = \max_{x \in E} \nu_A(x)$

Obviously every fuzzy set has the form $\{<x \ , \mu_A(x) \ , \mu_{A^c}(x) > | \, x \in E \}$.

In [1], Atanassov presented an example of an IFS which is not a fuzzy set.

3 SOME CHARACTERIZATION

We start this section with some examples of IFS.

Example 3.1

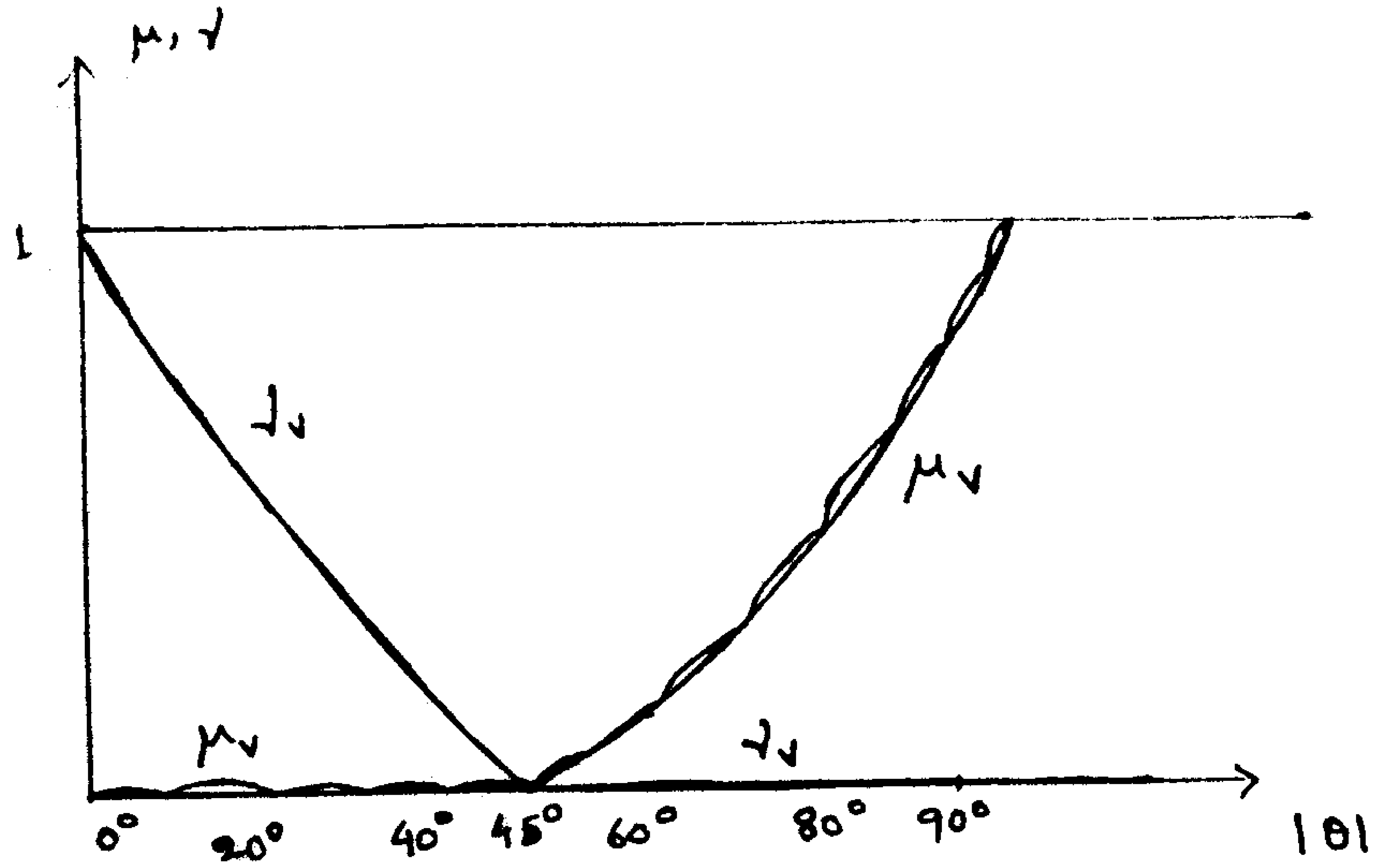
The intuitionistic fuzzy set of straight lines labelled "Vertical" is given by the membership function

$$\mu_V(x) = \begin{cases} 1 - \left| \frac{1}{m_x} \right|^{F_e}, & \text{if } | m_x | > 1 \\ 0, & \text{otherwise} \end{cases}$$

and the non-membership function

$$\nu_V(x) = \begin{cases} 1 - \mid m_x \mid^{F_e}, & \text{if } \mid m_x \mid < 1 \\ 0, & \text{otherwise} \end{cases}$$

where $m_x = \tan\theta$ is the slope of the line and F_e is some positive integer. The graphical representation of this IFS is shown below:



Example 3.2

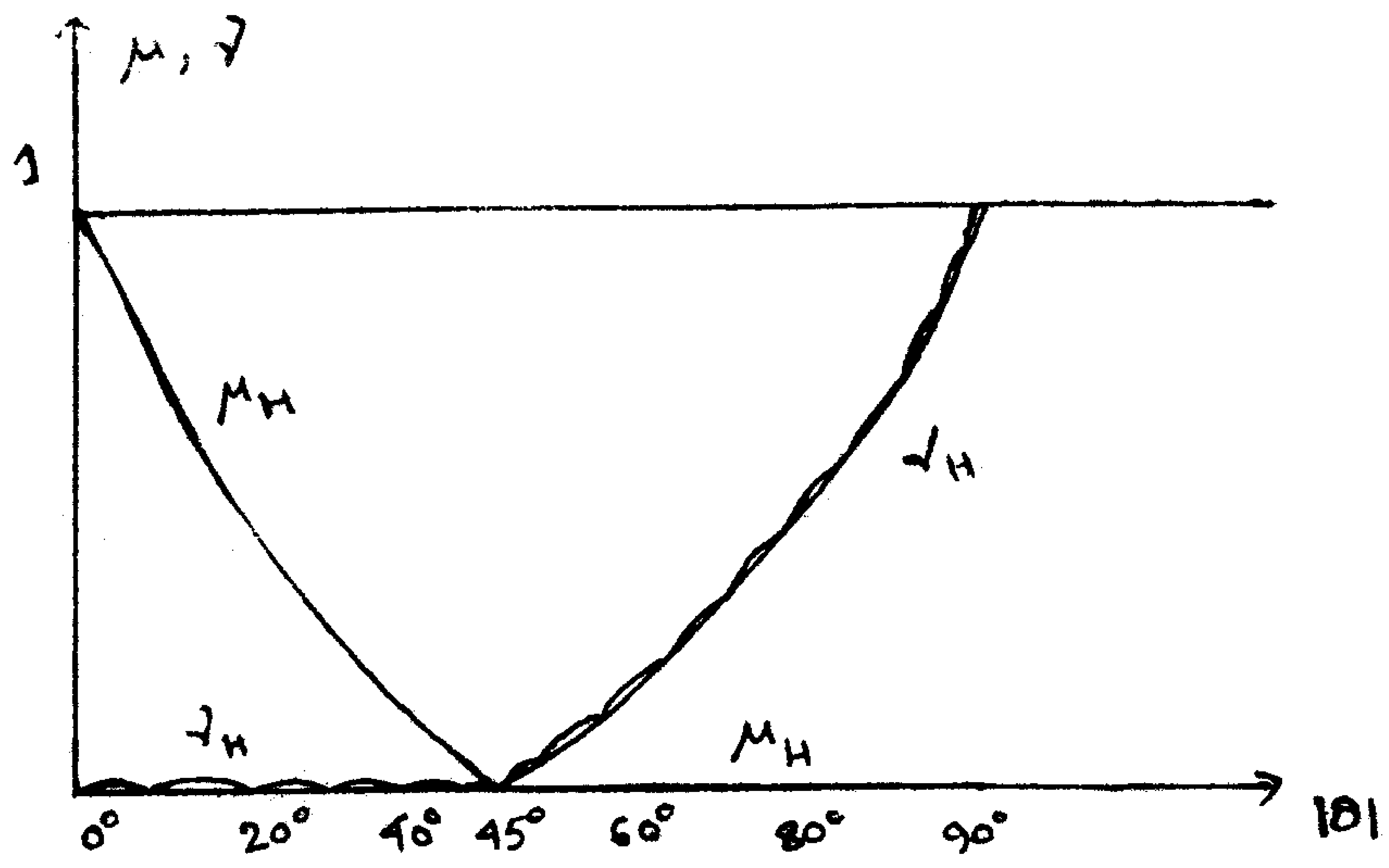
The IFS of a straight line labelled "Horozontal" is given by the membership function

$$\mu_H(x) = \begin{cases} 1 - \mid m_x \mid^{F_e}, & \text{if } \mid m_x \mid < 1 \\ 0, & \text{otherwise} \end{cases}$$

and the non-membership function

$$\nu_V(x) = \begin{cases} 1 - \mid \frac{1}{m_x} \mid^{F_e}, & \text{if } \mid m_x \mid > 1 \\ 0, & \text{otherwise} \end{cases}$$

where $m_x = \tan\theta$ be the slope of the line and F_e is some positive integer. The graphical representation of this IFS is shown below:



Note: Let E be a set. We can view E itself as an intuitionistic fuzzy set with membership value given by $\mu_E(x) = 1 \; \forall$ x $\in$ E and the non-membership values given by $\nu_E(x) = 0 \; \forall$ x $\in$ E. Similarly, a null set ϕ also can be viewed as $\mu_\phi(x) = 0 \; \forall$ x $\in$ E and $\nu_\phi(x) = 1 \; \forall$ x $\in$ E. We call it null intuitionistic fuzzy set or null IFS.

Nearest ordinary sets of fuzzy sets have got applications in many areas because the index of fuzziness of a fuzzy set is defined with the help of nearest ordinary sets. A similar study can be made in case of IFSs too. First of all let us defined α-cut of intuitionistic fuzzy sets.

Definition 3.1

Let A be an IFS of the set E. For $\alpha \in$ [0,1], the α-cut of A is the crisp set A_α defined by $A_\alpha = \{$ x : x $\in$ E , either $\mu_A(x) \geq \alpha$ or $\nu_A(x) \leq 1 - \alpha$ $\}$.

Clearly A_0 = E.

It may be noted that the condition $\mu_A(x) \geq \alpha$ ensures $\nu_A(x) \leq 1 - \alpha$ but not conversely.

Example 3.3

Let E = $\{ e_1 , e_2 , e_3 , e_4 \}$ be a set and

A = $\{ (.5 , .4)/ e_1 , (.4 , .5)/ e_2 , (.2 , .8)/ e_3 , (.8 , .1)/ e_4 \}$ be an IFS of E .

Then, the .3-cut of the IFS A is given by

$A_{.3} = \{ e_1 , e_2 , e_4 \}$

The .5-cut of the IFS A is given by

$A_{.5} = \{ e_1 , e_2 , e_4 \}$ and

the .8-cut of the IFS A is given by

$A_{.8} = \{ e_4 \}$.

Definition 3.2

The nearest ordinary set of an IFS A of the set E is denoted by A_{ne} and is defined by the characterstic function $C_{A_{ne}}$ given by

$$C_A(ne) = \begin{cases} 1, & \text{if either } \mu_A(x) > 0.5 \text{ or } \nu_A(x) < 0.5 \\ 1 \text{ or } 0 & , \text{if } \mu_A(x) = 0.5 \text{ and } \nu_A(x) = 0.5 \\ 0, & \text{otherwise .} \end{cases}$$

Example 3.4

Consider the IFS A as chosen in the previous example (example 3.3). We can see that

$A_{ne} = \{ e_1 , e_4 \}$

The following proposition is straightforward

Proposition 3.1

If α_1 , $\alpha_2 \in [0,1]$, then

(i) $\alpha_2 \geq \alpha_1 \Rightarrow A_{\alpha_2} \subseteq A_{\alpha_1}$

(ii) for $\alpha_2 \geq \alpha_1$, if $A_{\alpha_2} = A_{\alpha_1}$, then $\forall\ \alpha \in [\ \alpha_1 , \alpha_2\ ]$, A_α is fixed.

Proposition 3.2

If A and B are intuitionistic fuzzy sets of E , then $\forall\ \alpha \in [0,1]$

(i) $(A \cup B)_\alpha = A_\alpha \cup B_\alpha$

(ii) $(A \cap B)_\alpha = A_\alpha \cap B_\alpha$

Proof (i) Suppose x $\in (A \cup B)_\alpha$

$\Rightarrow \mu_{A \cup B}(x) \geq \alpha$ or $\nu_{A \cup B} \leq 1 - \alpha$

Case (i) : If $\mu_{A \cup B}(x) \geq \alpha$, then max $\{ \mu_A(x) , \mu_B(x) \} \geq \alpha$

$\Rightarrow$ either $\mu_A(x) \geq \alpha$ or $\mu_B(x) \geq \alpha$

or both the cases hold.

Case (ii) If $\nu_{A \cup B}(x) \leq 1 - \alpha$

then min $\{ \nu_A(x) , \nu_B(x) \} \leq 1 - \alpha$

$\Rightarrow$ either $\nu_A(x) \leq 1 - \alpha$ or $\nu_B(x) \leq 1 - \alpha$ or both the cases hold.

Case (i) and case (ii) reveals that x $\in A_\alpha \cup B_\alpha$. Similarly, we can prove that

if x $\in A_\alpha \cup B_\alpha$ then x $\in (A \cup B)_\alpha$

$\Rightarrow (A \cup B)_\alpha = A_\alpha \cup B_\alpha$.

(ii) This proof is similar to (i).

The following propositions is now obvious.

Proposition 3.3

If A and B are intuitionistic fuzzy sets of E, then

(i) $(A \cap B)_{ne} = A_{ne} \cap B_{ne}$.

(ii) $(A \cup B)_{ne} = A_{ne} \cup B_{ne}$.

Definition 3.3

If A and B are two IFSs of a finite set E, then the Hamming distance between A and B is given by

$$d(A,B) = \sum_{i=1}^{n} min\{d_i, r_i\}$$

where $d_i = \mid \mu_A(x_i) - \mu_B(x_i) \mid$, and
$r_i = \mid \nu_A(x_i) - \nu_B(x_i) \mid$

and the Euclidean distance between A and B is given by

$$e(A,B) = [\sum_{i=1}^{n} min({d_i}^2, {r_i}^2)]^{\frac{1}{2}}$$

where $d_i = \mu_A(x_i) - \mu_B(x_i)$, and
$r_i = \nu_A(x_i) - \nu_B(x_i)$

where n is the cardinality of E.
The following results are obvious.

Proposition 3.4

If A, B, C are intuitionistic fuzzy sets of E, then

(i) d(A,B) $\geq$ 0
(ii) d(A,B) =0 if A = B, but the converse is not necessarily true.
(iii) d(A,B) = d(B,A)
(iv) The inequality d(A,B) $\leq$ d(A,C) + d(C,B) is not true in general.

Definition 3.5

The index of intuitionistic fuzziness of an IFS A of E with n supporting points is given by

i(A) = $\frac{2}{n^k}$ d (A,A_{ne})

where d(A,A_{ne}) denotes the distance (Hamming or Eucledian) between IFS A and the ordinary set A_{ne} (Viewing as an IFS).

Example 3.5

Consider the IFS A as chosen in example 3.3. in A the index of intuitionistic fuzziness is given by

i(A) = $\frac{2}{n}$ d(A,A_{ne}) = .55

where the distance is Hamming distance.

References

[1] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems. 20 (1986) 87-96.

[2] Atanassov, K., Two operators on intuitionistic fuzzy sets, Comptes Rendus de l'Academic Bulgare des Sciences 41(5) (1988) 35-38.

[3] Atanassov, K., More on intuitionistic fuzzy sets, Fuzzy sets and Systems. 33 (1989) 37-46.

[4] Atanassov, K., A universal operator over intuitionistic fuzzy sets, Comptes Rendus de l'Academic Bulgare des Sciences. 46 (1) 1993 13-15.

[5] Atanassov, K., New operations defined over intuitionistic fuzzy sets, Fuzzy Sets and Systems, 61 (1994) 137-142.

[6] Atanassov, K. and Georgeiv, C., Intuitionistic fuzzy prolog, Fuzzy Sets and Systems. 53 (1993) 121-128.

[7] Bassu, K., Deb, R. and Pattanaik, P.K., Soft sets: An ordinal formulation of vagueness with some applications to the theory of choice, 45 (1992) 45-88.

[8] Biswas, R., Square zero (2): reducing vagueness in zero (0), to appear in Bull. Pour. Sous. Ens. Flous. Appl. (BUSEFAL).

[9] Biswas, R., Similarity measurements in IFSs, Notes on IFSs, 2(3) (1996) 5-14.

[10] Biswas, R., Intuitionistic fuzzy relations, in Bull. Sous. Ens. Flous. Appl. (BUSEFAL) 70 (1997).

[11] Burillo, P. and Bustinee, H., Construction theorams for intuitionistic fuzzy sets, Fuzzy Sets and Systems, 84 (1996) 271-281.

[12] Dubois, D. and Prade, H., Twofold fuzzy sets and rough sets : some issues in knowledge representation, Fuzzy Sets and Systems 23 (1987) 3-18.

[13] Dubois, D. and Prade, H., Toll sets and toll logic in fuzzy logic : State of the Art, R. Lowen and M. Roubens eds., Dordrecht: Kluwer Aca. Publisher (to appear).

[14] Goguen, J.A., L-fuzzy sets, Jou. Maths. Anal. Appl. 18 (1967) 145-174.

[15] Hirota, K., Concepts of probabilistic sets, Fuzzy Sets and Systems 5(1) (1981) 31-46.

[16] Mizumto, M. and Tanaka, K., Some properties of fuzzy sets of type 2., Info. and Control. 31 (1976) 312-340.

[17] Zadeh, L.A., Fuzzy sets, Information and Control. 8 (1965) 338-353.