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GENERALIZED NETS HAVING TRANSITIONS WITH LIMITED TIME FOR ACTIVE STATUS

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In a series of papers [1,2], a variety of different types of Generalized Nets (GN) have been defined, and each of them has been proven to be a conservative extension of the ordinary GNs.

In this paper, we will introduce yet another extension and will prove that it is conservative, too.

We shall use the following notations throughout:

• $\mathcal{N} = \{0, 1, 2, \ldots\} \cup \{\infty\}$;

• $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in \mathcal{N}, n \geq 1$, and $1 \leq i \leq n$. More generally, for a given n-dimensional set X $(n \geq 2)$

$$pr_{i_1,i_2,\ldots i_k} X = \prod_{j=1}^{k} pr_{i_j} X$$

where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$.

• $card(X)$ is the cardinality of set X.

All other notations related to GNs are given in [1,2].

The formal definition of the new type of extension coincide with this of the ordinary GN. Every transition is described by a eight-tuple (see Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r, M, \Box, t_3 \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 1 these are

$$L' = \{l'_1, l'_2, \ldots, l'_m\}$$

and

$$L'' = \{l''_1, l''_2, \ldots, l''_n\};$$

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will transfer from the transition's inputs to its outputs. Parameter r has the form of an Index Matrix (IM, see [1,2]):

$$r = \begin{array}{c|c} & l''_1 \ \dots \ l''_j \ \dots \ l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \quad - \ \text{predicate}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} \quad ;$$

where $r_{i,j}$ is the predicate which expresses the condition for transfer from the i-th input place to the j-th output place. When $r_{i,j}$ has truth-value "*true*", then a token from the i-th input place can be transferred to the j-th output place; otherwise, this is impossible;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ \dots \ l''_j \ \dots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \ge 0 - \ \text{natural number or } \infty) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} \quad ;$$

(f) $\square$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places, and it is an expression constructed of variables and the Boolean connectives $\wedge$ and $\vee$ determining the following conditions:

$\wedge(l_{i_1}, l_{i_2}, \dots, l_{i_u})$	$-$	every place $l_{i_1}, l_{i_2}, \dots, l_{i_u}$ must contain at least one token,
$\vee(l_{i_1}, l_{i_2}, \dots, l_{i_u})$	$-$	there must be at least one token in the set of places $l_{i_1}, l_{i_2}, \dots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \dots, l_{i_u}\} \subset L'$;

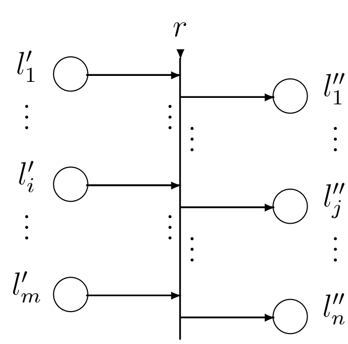

Fig. 1.

Whenever the value of a type (calculated as a Boolean expression) is "*true*", the transition can become active, otherwise it cannot;

(g) t_3 is a constant corresponding to the limited duration of the active status of the transition.

The ordered quadruple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle$$

will be called a *Generalized Net having Transitions with Limited Time for Active Status (GN-TLTAS)* if: **(a)** A is a set of transitions (see above);

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \to \mathcal{N}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to \mathcal{N}$, where

$$L = pr_1 A \cup pr_2 A$$

and obviously, L is the set of all GN-places;

(d) c is a function giving the capacities of the places, i.e., $c : L \to \mathcal{N}$;

(e) f is a function that calculates the truth values of the predicates of the transition's conditions; for the (ordinary) GNs, described in this section, function f obtain values "$false$" or "$true$", or values from set $\{0, 1\}$; if $\mathcal{P}$ is the set of the predicates used in a given model, then we can define f as $f : \mathcal{P} \to \{0, 1\}$;

(f) θ_1 is a function giving the next time-moment, for which a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$; the value of this function is calculated at the moment when the transition terminates its functioning;

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i.e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$; the value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \to \mathcal{N}$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the GN starts functioning; this moment is determined with respect to a fixed (global) time-scale;

(l) t^0 is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the GN functioning;

(n) in all publications on GNs (see, e.g., [1]), X is defined to be the set of all initial characteristics that the tokens can receive when they enter the net; in [2], for a first time another interpretation of X was introduced. Namely, X is a function which assigns initial characteristics to every token when it enters input place of the net; if $\alpha \in K$, then it enters the GN with initial characteristic x_0^α;

(o) Φ is a characteristic function that assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition; if $\alpha \in K$, then it, entering an output place of some GN-transition and having as current characteristic x_{cu}^α, obtains the next characteristic x_{cu+1}^α;

(p) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \to \mathcal{N}$.

In the ordinary case, the constant t_3, associated to a given transition, is equal to ∞. In this case, the GN-TLTAS is an ordinary GN. Therefore, a GN-TLTAS is an extension of a GN.

Theorem: The functioning and the results of the work of each GN-TLTAS can be represent by an ordinary GN.
Proof. Let E be a given GN-TLTAS. We construct the ordinary GN F with the form

$$F = \langle\langle A^*, \pi_A, \pi_L^*, c, f, \theta_1, \theta_2\rangle, \langle K^*, \pi_K^*, \theta_K^*\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b\rangle\rangle,$$

where A^* is the set of the F-transitions. Let transition Z^* of F, corresponding to transition Z of E, have the form

$$Z^* = \langle L'^*, L''^*, t_1, t_2, r^*, M^*, \square^*\rangle$$

(see Fig. 2) where t_1 and t_2 are as above and

$$L'^* = L' \cup \{l_Z\},$$

$$L''^* = L'' \cup \{l_Z\},$$

$$\square^* = \wedge(\square, l_Z)$$

and if

$$r = pr_5 Z = [L', L'', \{r_{l_i,l_j}\}]$$

has the form of an IM, then

$$r^* = pr_5 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r^*_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(r^*_{l_i,l_j} = r_{l_i,l_j} \& \text{“}x_{cu}^{\alpha_Z} < t_3\text{”}),$$

where $x_{cu}^{\alpha_Z}$ denotes the current characteristic of token α_Z that stays permanently in place l_Z.

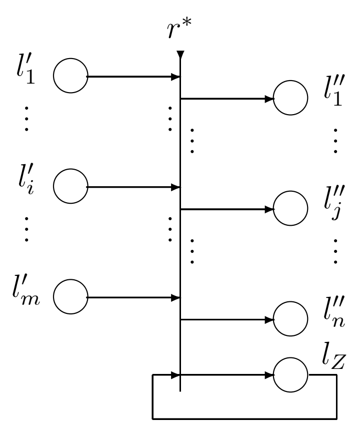

Fig. 2.

If

$$M = pr_6 Z = [L', L'', \{m_{l_i,l_j}\}]$$

has the form of an IM, then

$$M^* = pr_6 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m^*_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(m^*_{l_i,l_j} = m_{l_i,l_j}),$$
$$(\forall l_i \in L')(\forall l_j \in L'')(m^*_{l_i,l_Z} = m_{l_Z,l_j} = 0),$$
$$m^*_{l_Z,l_Z} = 1.$$

$$\pi^*_L = \pi_L \cup \pi_{\{l_Z|Z \in A\}},$$

where function $\pi_{\{l_Z|Z\in A\}}$ determines the priorities of the new places, that are elements of set $\{l_Z|Z \in A\}$ and the priorities of l_Z-places for every transition $Z \in A$ are the minimum among the place priorities of this transition Z.

$$c^*_L = c \cup c_{\{l_Z|Z\in A\}},$$

where function $c_{\{l_Z|Z\in A\}}$ satisfy equality

$$c_{\{l_Z|Z\in A\}}(l_Z) = 1$$

for all place l_Z.

The set of all tokens of GN F is

$$K^* = K \cup \{\alpha_Z | Z \in A\};$$

$$\theta^*_K = \theta_K \cup \theta_{\{l_Z|Z\in A\}},$$

where function $\theta_{\{l_Z|Z\in A\}}$ determines that each α_Z-token will stays in the initial time-moment T in its place;

$$\pi^*_K = \pi_K \cup \pi_{\{l_Z|Z\in A\}},$$

where function $\pi_{\{l_Z|Z\in A\}}$ determines that each α_Z-token has the minimal priority;

$$X^* = X \cup \{x_0^{\alpha_Z} | Z \in A\},$$

where $x_0^{\alpha_Z}$ is the initial α_Z-token characteristic and it is “0”;

$$\Phi^* = \Psi \cup \Psi_{\{l_Z|Z\in A\}},$$

where function $\Psi_{\{l_Z|Z\in A\}}$ determines the characteristics of the α_Z-tokens in the form

$$\Psi_{\{l_Z|Z\in A\}}(\alpha_Z) = \text{“}x_{cu}^{\alpha_Z} + t^o\text{”}.$$

We shall now prove that both GNs E and F function in the same way. To this end, we compare the functioning of one arbitrary transition Z of GN E and its respective transition Z^* from F. Obviously, these transitions start functioning at some time-moment and have equal duration of functioning for each of their activations. When the limited time of functioning of transition Z finishes, it stops function, but the transition Z^* has the same behavior, because of the form of its predicates. Therefore, both transitions will have the same way of functioning. The coincidence of the parameters of the rest transition components is obvious.

To conclude, we shall mention that in future research we intend to study the optimal way of functioning of the transitions with limited time for active status. It will be important to construct special algorithms determining the moments of transition activations and their durations.

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References

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