

Multiplicatively equivalent intuitionistic fuzzy sets

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Abstract: In this paper, we propose the concept of multiplicative equivalence of two intuitionistic fuzzy sets. We also consider some other possible equivalence relations.

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1 Introduction

Intuitionistic fuzzy sets (shortly IFSs) were introduced by K. Atanassov in 1983 [1]. Their definition is as follows:

Definition 1 (cf. [3]). Let X be a universe set. Let A be a set. Then an intuitionistic fuzzy set A^* over X is an object of the form

$$A^* = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$$

where the mappings $\mu_A : X \rightarrow [0, 1]$, $\nu_A : X \rightarrow [0, 1]$ are such that $\forall x \in X :$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

The mapping μ_A is called the membership function and ν_A is called the non-membership function (of the element x to the set A). Then, the mapping π_A defined by:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

for all $x \in X$, is called the hesitancy degree or indeterminacy degree of the element x .

In what follows we shall present a new concept of equivalence (in terms of equivalence classes) between different IFSs.

2 Multiplicative equivalence and other types of equivalence between intuitionistic fuzzy sets

Further we investigate the idea of multiplicative equivalence of two intuitionistic fuzzy sets A and B , and define a more general notion of equivalence between IFSs, and we finish with an Open Problem for further research. As was done for another notion of equivalence between IFSs, introduced in [6], we first start by the necessary definitions.

Definition 2. Let A, B be IFSs over the same universe set X . Then we shall call A and B , *fully multiplicatively equivalent* if for all $x \in X$:

$$\mu_A(x)\nu_A(x)\pi_A(x) = \mu_B(x)\nu_B(x)\pi_B(x). \quad (1)$$

And we shall call A, B *partially multiplicatively equivalent* if and only if

$$\mu_A(x)\nu_A(x) = \mu_B(x)\nu_B(x). \quad (2)$$

Bearing in mind the fact that

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = \mu_B(x) + \nu_B(x) + \pi_B(x) = 1,$$

we can, circumventing the slight technical detail that not all values need be positive, view the above definition as the statement that the geometric and the arithmetic means of the membership, non-membership and indeterminacy degrees of the two intuitionistic fuzzy sets must coincide.

It is easy to show that the class of *fully multiplicatively equivalent* IFSs to a given set is non-empty, since any permutation of the degrees fulfills the required condition. In the case when one of the degrees is zero, any triple of degrees with at least one zero is *multiplicatively equivalent* to it.

Definition 3. Let A, B be IFSs over the same universe set X . Then we shall call A and B , *dual multiplicatively equivalent* if for all $x \in X$:

$$\mu_A(x)\nu_B(x) = \mu_B(x)\nu_A(x). \quad (3)$$

Proposition 1. Let A be an IFSs. Then the following statements are true:

- A is partially multiplicatively equivalent to $\neg A$.
- $\Box A$ is partially multiplicatively equivalent to $\Diamond(\neg A)$.
- $\Diamond A$ is partially multiplicatively equivalent to $\Box(\neg A)$.
- Let $B \in \text{IFS}(X, \tilde{\pi})$ (in the sense of [5]) with $\max_{x \in X} \pi_B(x) > 0$, then A is partially multiplicatively equivalent to B if and only if A is fully multiplicatively equivalent to B .

- Let $A, B \in \text{IFS}(X)$ be such that for all $x \in X$ we have

$$\nu_A(x)\nu_B(x) > 0.$$

Then A and B are dual multiplicatively equivalent if and only if for all $\gamma \geq 0$, we have

$$N_\gamma(A) = N_\gamma(B),$$

where $N_\gamma(A)$ is defined as in [4].

We omit the proof, since the statements are directly checked.

By analogy with Definition 2 we may introduce the following one:

Definition 4. Let A, B be IFSs over the same universe set X . Then we shall call A and B , *r-mean equivalent* if $\forall x \in X$:

$$\mu_A(x)^r + \nu_A(x)^r + \pi_A(x)^r = \mu_B(x)^r + \nu_B(x)^r + \pi_B(x)^r,$$

where $r \in \mathbb{N}$, and under $r = 0$ we understand the condition (1) in Definition 2.

Note that similar arguments apply for the fact that the class of *r-mean equivalent* IFSs is non-empty.

Remark. The fact that Definition 4 defines an equivalence relation is obvious. In particular, any two IFSs are always *1-mean equivalent*.

We have not considered here the case when r may be a negative integer (similarly to the Harmonic mean, etc.) since the possibility that one of the degrees may be zero presents technical difficulties. Such investigation is planned for the near future.

Open Problem. Is it possible for two IFSs A, B to be $0, 1, 2, \dots, k$ -mean equivalent simultaneously and under what conditions? In other words, under what conditions it possible that the following system has a solution:

$$\begin{cases} \mu_A(x)\nu_A(x)\pi_A(x) = \mu_B(x)\nu_B(x)\pi_B(x) \\ \mu_A(x)^2 + \nu_A(x)^2 + \pi_A(x)^2 = \mu_B(x)^2 + \nu_B(x)^2 + \pi_B(x)^2 \\ \vdots \\ \mu_A(x)^k + \nu_A(x)^k + \pi_A(x)^k = \mu_B(x)^k + \nu_B(x)^k + \pi_B(x)^k \end{cases}$$

3 Conclusion

We have introduced a new concept of equivalence between IFSs. This opens the path to investigation of transformations, operations, etc., which preserve such equivalence classes. We have also posed the question of the conditions that need to be satisfied for simultaneous equivalence to be present, i.e., that the considered IFSs fall in all consecutive equivalence classes. In future work we plan to investigate the proposed and related concepts in more detail.

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