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TEMPORAL INTUITIONISTIC FUZZY GRAPHS

Krassimir T. Atanassov
Centre for Biomedical Engineering - Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, BULGARIA
e-mail: krat@bgcict.acad.bg

Here we will consider an application of the Intuitionistic Fuzzy Sets (IFSs; [1]), Temporal IFSs (TIFSs; [2]), Intuitionistic Fuzzy Graphs (IFGs; [3]), Intuitionistic Fuzzy Relations (IFRs; [4,5]) and Index Matrices (IMs; [6]) to graph theory. Following [1-5], the concept of *Temporal IFG (TIFG)* will be introduced.

Let E_1 and E_2 be two sets; let everywhere below $x \in E_1$ and $y \in E_2$ and let operation $\times$ denote the standard Cartesian product operation. Therefore $\langle x, y\rangle \in E_1 \times E_2$. Let the operation $o \in \{\times_1, \times_2, \ldots, \times_5\}$, where operations $\times_1, \times_2, \ldots, \times_5$, are Cartesian products over the IFSs (see [4,5]). Let T is a fixed set which we shall call "time-scale" and let it be strictly oriented by the relation "$<$").

Let us use below the notations

$$T' = \{t' \mid t' \in T \& t' < t\} \quad \text{and} \quad T'' = \{t'' \mid t'' \in T \& t'' > t\}.$$

The set

$$G(t) = \{\langle\langle x, y\rangle, \mu_G(x, y, t), \nu_G(x, y, t)\rangle \mid x \in E_1, y \in E_2\}$$

for $t \in T$ is called an o-TIFG (or briefly, a TIFG) if the functions $\mu_G : E_1 \times E_2 \times T \to [0, 1]$ and $\nu_G : E_1 \times E_2 \times T \to [0, 1]$ define the degree of membership and the degree of non-membership, respectively, of the element $\langle x, y\rangle \in E_1 \times E_2$ to the set $G \subset E_1 \times E_2$ at a time-moment t (these functions have the forms of the corresponding components of the o-Cartesian product over IFSs) and for all $\langle x, y, t\rangle \in E_1 \times E_2 \times T : 0 \leq \mu_G(x, y, t) + \nu_G(x, y, t) \leq 1$.

Let the oriented graph $G = (V, A)$ be given, where $V = \{v_1, v_2, ..., v_n\}$ is a set of vertices and A is a set of arcs. Every graph arc connects two graph vertices. In [5] it is shown that $A \subset V \times V$ can be described as a $(1, 0)$-IM. If the graph is fuzzy, the IM has elements from the set $[0, 1]$; if the graph is an IFG, the IM has elements from the set $[0, 1] \times [0, 1]$. In the present case the graph will have the form $G(t) = (V, A(t))$, where $t \in T$ and T is a fixed time-scale and the degrees of every arc are functions of t.

The graph $G(t)$ has the following IM:

$$A(t) = \begin{array}{c|cccc} & v_1 & v_2 & \ldots & v_n \\ \hline v_1 & a_{1,1} & a_{1,2} & \ldots & a_{1,n} \\ v_2 & a_{2,1} & a_{2,2} & \ldots & a_{2,n} \\ \vdots & \ldots & \ldots & \ldots & \ldots \\ v_n & a_{n,1} & a_{n,2} & \ldots & a_{n,n} \end{array}$$

where $a_{i,j} =< \mu_G(v_i, v_j, t), \nu_G(v_i, v_j, t) >\in [0, 1] \times [0, 1]$ where $0 \leq \mu_G(v_i, v_j, t) + \nu_G(v_i, v_j, t) \leq 1$ $(1 \leq i, j \leq n)$. We can write briefly:

$$G(t) = [V, V, A(t)].$$

It can be easily seen that the above IM can be modified to the following form:

$$G(t) = [V_I \cup \overline{V}, \overline{V} \cup V_O, A(t)],$$

where V_I, V_O and $\overline{V}$ are respectively the sets of the graph input, output and internal vertices. At least one arc leaves every vertex of the first type, but none enters; at least one arc enters each vertex of the second type but none leaves it; every vertex of the third type has at least one arc ending in it and at least one arc starting from it.

Obviously, first, the graph matrix (in the sense of IM) now will be of a smaller dimension than the ordinary graph matrix, and second, it can be nonsquare, unlike the ordinary graph matrices.

As in the ordinary case, the vertex $v_p \in \overline{V}$ has a loop iff $a_{p,p} = \langle \mu_G(v_p, v_p, t), \nu_G(v_p, v_p), t) \rangle$ for the vertex v_p and $\mu_G(v_p, v_p, t) > 0$ and $\nu_G(v_p, v_p, t) < 1$.

Let graphs $G_1(t)$ and $G_2(t)$ be given and let $G_s(t) = [V_s', V_s'', A(t)]$, where $s = 1, 2$ and V_s' and V_s'' are the sets of the graph vertices.

Then, using the apparatus of the IMs, we can construct the graph which is an union of the graphs $G_1(t)$ and $G_2(t)$. The new graph has the description

$$G(t) = G_1(t) \cup G_2(t) = [V_1' \cup V_2', V_1'' \cup V_2'', A(t)],$$

where $A(t)$ is determined by the above IM-formulas, using min-max operations between its elements, for the case of operation "+" between IMs.

Analogously, we can construct a graph which is the intersection of the two given graphs $G_1(t)$ and $G_2(t)$. It would have the form

$$G(t) = G_1(t) \cap G_2(t) = [V_1' \cap V_2', V_1'' \cap V_2'', A(t)],$$

where $A(t)$ is determined by the above IM-formulas, using min-max operations between its elements, for the case of operation "." between IMs.

Obviously, we can describe every TIFG $G(V, t)$ in the equivalent form

$$G(V, t) = \{\langle \langle x, y, t \rangle, \mu_G(x, y, t), \nu_G(x, y, t) \rangle \mid x \in E_1, y \in E_2\},$$

which contains as a parameter the time-moment, too. From the point of a view of a separate graph, this addition is not essential, but it is important for the next constructions.

For a given (standard) IFG $G(V)$ with fixed set of vertices V and for a given time-scale T we can construct the set

$$G^*(V, T) = \{G(V, t) \mid t \in T\}$$

of all TIFGs $G(V, t)$, describing in the last form on the basis of the given IFG.

We shall call this set *a* (V, T)*-atlas,* by analogy with the same concept in the differential geometry (see, e.g., [7,8]). For it we can define also the sets

$$\underline{Closure}(G^*(V, T)) = \bigcup_{t \in T} C(V, t) \quad \text{and} \quad \underline{Interior}(G^*(V, T)) = \bigcup_{t \in T} C(V, t).$$

Now, for the TIFGs $G(V, t'), G(V, t'') \in G^*(V, T)$ we can obtain the graphs $G(V, t') \cup G(V, t'')$ and $G(V, t') \cap G(V, t'')$, having equal set of vertices.

Following [2], here we shall define a Temporal IFS (TIFS) as the following object

$$A(T) = \{\langle x, \mu_A(x, t), \nu_A(x, t) \rangle | \langle x, t \rangle \in E \times T\},$$

where:

(a) $A \subset E$ is a fixed set,

(b) $\mu_A(x,t) + \nu_A(x,t) \leq 1$ for every $\langle x,t \rangle \in E \times T$,

(c) $\mu_A(x,t)$ and $\nu_A(x,t)$ are the degrees of membership and non-membership, respectively, of the element $x \in E$ at the time-moment $t \in T$.

The specific operators over TIFSs are (see [2])

$$C^*(A(T)) = \{\langle x, \max_{t \in T} \mu_{A(T)}(x,t), \min_{t \in T} \nu_{A(T)}(x,t)\rangle | x \in E\},$$

$$I^*(A(T)) = \{\langle x, \min_{t \in T} \mu_{A(T)}(x,t), \max_{t \in T} \nu_{A(T)}(x,t)\rangle | x \in E\}.$$

Immediatelly we can seen that $C^*(G^*(V,T))$ and $I^*(G^*(V,T))$ are IFGs, which are not TIFGs. These IFGs are special ones. They have the forms

$$C^*(G^*(V,T)) = \{\langle x, \max_{t \in T} \mu_G(A(T)(x,t), \min_{t \in T} \nu_{A(T)}(x,t)\rangle | x \in E\},$$

and they correspond to the *closure* and *interior* of the (V,T)-atlas.

Let for the IFS A (see [1]):

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\} \text{ and } \Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.$$

We can prove for the TIFGs, which, obviously, are TIFSs, the following

THEOREM: For every (V,T)-atlas $G(V,T)$:

(a) $\overline{C^*(\overline{G(V,T)})} = I^*(G(V,T))$;

(b) $C^*(\Box G(V,T)) = \Box C^*(G(V,T))$,

(c) $C^*(\Diamond G(V,T)) = \Diamond C^*(G(V,T))$,

(d) $I^*(\Box G(V,T)) = \Box I^*(G(V,T))$,

(e) $I^*(\Diamond G(V,T)) = \Diamond I^*(G(V,T))$.

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