

Intuitionistic Fuzzy Filters of Ordered Semigroups

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ABSTRACT. The notion of intuitionistic fuzzy filters in ordered semigroups is introduced and relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals is investigated. The notion of intuitionistic fuzzy bi-ideal subsets and intuitionistic fuzzy bi-filters are provided and relation between intuitionistic fuzzy bi-filters and intuitionistic fuzzy prime bi-ideal subsets is established. The concept of intuitionistic fuzzy right filters(left filters) is given and their relation with intuitionistic fuzzy prime left ideals is discussed.

1. Introduction

The theory of fuzzy sets proposed by Zadeh [27], has achieved a great success in various fields. Out of several higher order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [2], have been found to be highly useful to deal with vagueness. Gau and Buehrer [12] presented the concept of vague sets. But, Burillo and Bustince showed that the notion of vague sets coincides with that of intuitionistic fuzzy sets. Szmidt and Kacprzyk [25] proposed a non-probabilistic-type entropy measure for intuitionistic fuzzy sets. De et al. [9] studied the Sanchez's approach for medical diagnosis and extended this concept with the notion of intuitionistic fuzzy set theory. There are many applications of intuitionistic fuzzy set theory in mathematics. Davvaz et al. [8] applied the concept of this notion to H_v -modules. They introduced the concept of intuitionistic fuzzy H_v -submodules of an H_v -module. Dudek et al. [10] considered the intuitionistic fuzzification of the concept of sub-hyperquasigroups of a hyperquasigroup. Several authors studied intuitionistic fuzzification of subrings/ideals of a ring [1],[5]. In particular, Jun used intuitionistic fuzzification of bi-ideals in ordered semigroups and introduced several basic properties of this notion with many other characterizations of ordered semigroups [13].

The concept of fuzzy sets in ordered semigroups was first given by Kehayopulu and Tsingelis [18]. In [20], they studied several fuzzy analogous results for ordered semigroup. In [18], the concept of fuzzy sets in ordered groupoids/ordered semigroups is given, and it is shown that a fuzzy set f of an ordered groupoid/ordered semigroup S is a fuzzy filter of S if and only if the complement f' of f is a prime fuzzy ideal of S .

Key words and phrases. Intuitionistic fuzzy sets; Ordered semigroups; Intuitionistic fuzzy filters; Intuitionistic fuzzy ideals; Intuitionistic fuzzy bi-ideal subsets; Intuitionistic fuzzy bi-filters; Intuitionistic fuzzy left ideals; Intuitionistic fuzzy left filters.

In this paper we follow the ideas of fuzzy filters given in [18], [24] for ordered semigroups and define the intuitionistic fuzzy analogous results for ordered semigroups. We establish a relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals of ordered semigroups. We study the relation between the intuitionistic fuzzy bi-filters and intuitionistic fuzzy prime bi-ideal subsets of ordered semigroups. We provide an intuitionistic fuzzification link between one-sided filters and one-sided prime ideals of ordered semigroups.

2. Basic Concepts in Ordered Semigroups

An ordered semigroup is an ordered set S at the same time a semigroup such that

$$a, b \in S, a \leq b \implies xa \leq xb \text{ and } ax \leq bx \text{ for all } x \in S.$$

Let (S, \cdot, \leq) be an ordered semigroup. For $A \subseteq S$, we denote

$$(A) := \{t \in S \mid t \leq h \text{ for some } h \in A\}.$$

For $A, B \subseteq S$, we denote,

$$AB := \{ab \mid a \in A, b \in B\}.$$

Let (S, \cdot, \leq) be an ordered semigroup, $\emptyset \neq A \subseteq S$, A is called a *subsemigroup* of S if $A^2 \subseteq A$.

DEFINITION 1. [15]. Let (S, \cdot, \leq) be an ordered semigroup. $\emptyset \neq A \subseteq S$ is called a *right (resp. left) ideal* of S if:

- (1) $AS \subseteq A$ (resp. $SA \subseteq A$) and
- (2) If $a \in A$ and $S \ni b \leq a$, then $b \in A$.

If A is both a right and a left ideal of S , then it is called an *ideal* of S .

DEFINITION 2. [7]. Let (S, \cdot, \leq) be an ordered semigroup. A non-empty subset B of S is called a *bi-ideal subset* of S if:

- (1) $a \in B, x \in S$ implies $axa \in B$;
- (2) If $a \in B, S \ni b \leq a$, then $b \in B$.

DEFINITION 3. [16]. A subsemigroup F of an ordered semigroup S is called a *filter* of S if

- (1) $a, b \in S, ab \in F$ implies $a, b \in F$;
- (2) $a \in F, S \ni b \geq a$ implies $b \in F$.

DEFINITION 4. [7]. Let (S, \cdot, \leq) be an ordered semigroup. A subsemigroup F of S is called a *left (resp. right) filter* of S if:

- (1) $a, b \in S, ab \in F$ implies $b \in F$ (resp. $a \in F$);
- (2) $a \in F, S \ni b \geq a$ implies $b \in F$.

F is called a *bi-filter* of S if F satisfies condition (2) of definition 4 and

- (1') $a, b \in S, aba \in F$ implies $a \in F$.

3. Basic Definition on (Intuitionistic) Fuzzy Ideals

Let (S, \leq) be an ordered semigroup. By a fuzzy subset f of S , we mean a mapping $f : S \rightarrow [0, 1]$.

If (S, \leq) is an ordered semigroup and $A \subseteq S$, the characteristic function f_A of A is a fuzzy subset of S defined as follows:

$$f_A : S \rightarrow [0, 1] | x \mapsto f_A(x) := \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

DEFINITION 5. [20]. Let S be an ordered semigroup. A fuzzy subset f of S is called a fuzzy subsemigroup of S if:

$$f(xy) \geq \min\{f(x), f(y)\} \text{ for all } x, y \in S.$$

DEFINITION 6. [18]. Let S be an ordered semigroup. A fuzzy subset f of S is called a fuzzy left (resp. right) ideal of S if:

- (1) $x \leq y \implies f(x) \geq f(y)$;
- (2) $f(xy) \geq f(y)$ (resp. $f(xy) \geq f(x)$) for all $x, y \in S$.

If f is both a fuzzy left ideal and a fuzzy right ideal of S , then f is called a fuzzy ideal of S or a fuzzy two-sided ideal of S .

Equivalently, f is called a fuzzy ideal of S if:

- (1) $x \leq y \implies f(x) \geq f(y)$;
- (2) $f(xy) \geq \max\{f(x), f(y)\}$ for all $x, y \in S$.

DEFINITION 7. [18]. Let (S, \leq) be an ordered semigroup. A fuzzy subset f of S is called a fuzzy filter of S if:

- (1) $x \leq y \implies f(x) \leq f(y)$;
- (2) $f(xy) = \min\{f(x), f(y)\}$ for all $x, y \in S$.

DEFINITION 8. [24]. Let (S, \leq) be an ordered semigroup. A fuzzy subset f of S is called a fuzzy left (resp. right) filter of S if:

- (1) $x \leq y \implies f(x) \leq f(y)$;
- (2) $f(xy) \geq \min\{f(x), f(y)\}$ for all $x, y \in S$;
- (3) $f(xy) \leq f(y)$ (resp. $f(xy) \leq f(x)$) for all $x, y \in S$.

DEFINITION 9. [24]. Let (S, \leq) be an ordered semigroup. A fuzzy subset f of S called a fuzzy bi-filter of S if:

- (1) $x \leq y \implies f(x) \leq f(y)$;
- (2) $f(xy) \geq \min\{f(x), f(y)\}$;
- (2) $f(xyx) \leq f(x)$ for all $x, y \in S$.

DEFINITION 10. [24]. Let S be an ordered semigroup. A fuzzy subset f of S is called a fuzzy bi-ideal subset of S if:

- (1) $x \leq y \implies f(x) \geq f(y)$;
- (2) $f(xy) \geq f(x)$ for all $x, y \in S$.

Let (S, \cdot, \leq) be an ordered semigroup and f a fuzzy subset of S . Then the function defined by

$$f' : S \rightarrow [0, 1] | a \longrightarrow f'(a) = 1 - f(a),$$

is a fuzzy subset of S , called the complement of f in S .

As an important generalization of the notion of fuzzy sets in S , Atanassov [2] introduced the concept of an intuitionistic fuzzy set (*IFS* for short) defined on a non-empty set S as objects having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle | x \in S\},$$

where the functions $\mu_A : S \rightarrow [0, 1]$ and $\gamma_A : S \rightarrow [0, 1]$ denote the *degree of membership* (namely $\mu_A(x)$) and the *degree of nonmembership* (namely $\gamma_A(x)$) of each element $x \in S$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in S$ [12].

For any two IFSs A and B of an ordered semigroup S we define:

- (1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in S$,
- (2) $A^c = \{\langle x, \gamma_A(x), \mu_A(x) \rangle | x \in S\}$,
- (3) $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\} \rangle | x \in S\}$,
- (4) $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\} \rangle | x \in S\}$.

The intuitionistic fuzzy sets are studied by many authors (see for example three journals: 1) *Information Sciences* 2) *Fuzzy Sets and Systems* and 3) *Notes on Intuitionistic Fuzzy Sets*) and have many interesting applications in mathematics (see for example Kim, Dudek and Jun in [21]). Also in [22], [23], Kim and Jun introduced the concept of intuitionistic fuzzy (interior) ideals of semigroups.

Let (S, \cdot, \leq) be an ordered semigroup and $A \subseteq S$, the intuitionistic characteristic function χ_A of A is defined as follows:

$$\chi_A := \{\langle x, \mu_{\chi_A}, \gamma_{\chi_A} \rangle | x \in S\},$$

where μ_{χ_A} and γ_{χ_A} are fuzzy subsets defined as follows:

$$\begin{aligned} \mu_{\chi_A} : S \rightarrow [0, 1] | x \longrightarrow \mu_{\chi_A}(x) &:= \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A, \end{cases} \\ \text{and } \gamma_{\chi_A} : S \rightarrow [0, 1] | x \longrightarrow \gamma_{\chi_A}(x) &:= \begin{cases} 0 & \text{if } x \in A, \\ 1 & \text{if } x \notin A. \end{cases} \end{aligned}$$

DEFINITION 11. [13]. Let (S, \cdot, \leq) be an ordered semigroup, an IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy subsemigroup of S if:

- (1) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,
- (2) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.

DEFINITION 12. Let (S, \cdot, \leq) be an ordered. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy left ideal of S if:

- (1) $x \leq y \implies \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
- (2) $\mu_A(xy) \geq \mu_A(y)$ for all $x, y \in S$,
- (3) $\gamma_A(xy) \leq \gamma_A(y)$ for all $x, y \in S$.

DEFINITION 13. Let (S, \cdot, \leq) be an ordered. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy right ideal of S if:

- (1) $x \leq y \implies \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
- (2) $\mu_A(xy) \geq \mu_A(x)$ for all $x, y \in S$,
- (3) $\gamma_A(xy) \leq \gamma_A(x)$ for all $x, y \in S$.

An IFS $A = \langle \mu_A, \gamma_A \rangle$ of an ordered semigroup S is called a two-sided intuitionistic fuzzy ideal of S , if it is both a left and a right intuitionistic fuzzy ideal of S . Equivalent definition:

DEFINITION 14. Let (S, \cdot, \leq) be an ordered. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy ideal of S if:

- (1) $x \leq y \implies \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
- (2) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
- (3) $\gamma_A(xy) \leq \min\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$.

4. Relation Between Intuitionistic Fuzzy Filters and Intuitionistic Fuzzy Prime ideals of Ordered Semigroups

It is well known that a fuzzy subset f of an ordered semigroup S is a fuzzy filter of S if and only if the complement f' of f is a fuzzy prime ideal of S [18].

In this paragraph we give a relation between intuitionistic fuzzy filters and intuitionistic fuzzy prime ideals. We show that an IFS $A = \langle \mu_A, \gamma_A \rangle$ in an ordered semigroup S is an intuitionistic fuzzy filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime ideal of S .

DEFINITION 15. Let (S, \cdot, \leq) be an ordered. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy filter of S if:

- (1) $x \leq y \implies \mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$,
- (2) $\mu_A(xy) = \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
- (3) $\gamma_A(xy) = \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$.

4.1. Lemma. [27]. Let S be a semigroup and A a non-empty subset of S . Then

- (1) A is a subsemigroup of S if and only if the intuitionistic characteristic function $\chi_A = \langle \mu_{\chi_A}, \gamma_{\chi_A} \rangle$ of A is an intuitionistic fuzzy subsemigroup of S .
- (2) A is a left(right, two-sided) ideal of S if and only if $\chi_A = \langle \mu_{\chi_A}, \gamma_{\chi_A} \rangle$ of A is an intuitionistic fuzzy left(right, two-sided) ideal of S .

4.2. Proposition. Let S be an ordered semigroup, $\emptyset \neq F \subseteq S$. Then F is a filter of S if and only if the intuitionistic characteristic function $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ of F is an intuitionistic fuzzy filter of S .

PROOF. \implies . Let S be an ordered semigroup, F a filter of S and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ the intuitionistic characteristic function of F . Then χ_F is an intuitionistic fuzzy filter of S . Indeed: By Lemma 4.1, χ_F is an intuitionistic fuzzy subsemigroup of S . Let $x, y \in S$, $x \leq y$. If $x \notin F$ then $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$. Since $\mu_{\chi_F}(y) \geq 0$ and $\gamma_{\chi_F}(y) \leq 1$ for all $x, y \in S$. Then $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Let $x \in F$ then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $y \geq x \in F$ and F is a filter of S we have $y \in F$, and hence $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Again $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Hence condition (1) of definition 15, is satisfied.

Let $x, y \in S$, if $xy \notin F$. Then $\mu_{\chi_F}(xy) = 0$ and $\gamma_{\chi_F}(xy) = 1$. Since F is a filter of S and $xy \notin F \implies x \notin F$ or $y \notin F$. Hence

$$\begin{aligned}\mu_{\chi_F}(x) &= 0 \text{ and } \gamma_{\chi_F}(x) = 1, \\ \mu_{\chi_F}(y) &= 0 \text{ and } \gamma_{\chi_F}(y) = 1.\end{aligned}$$

Thus

$$\begin{aligned}\min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= 0 = \mu_{\chi_F}(xy) \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= 1 = \gamma_{\chi_F}(xy).\end{aligned}$$

Let $xy \in F$. Then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. $xy \in F \implies x \in F$ and $y \in F$. Then

$$\mu_{\chi_F}(x) = 1 \text{ and } \gamma_{\chi_F}(x) = 0$$

and

$$\mu_{\chi_F}(y) = 1 \text{ and } \gamma_{\chi_F}(y) = 0.$$

Thus

$$\min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} = 1 = \mu_{\chi_F}(xy)$$

and

$$\max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} = 0 = \gamma_{\chi_F}(xy),$$

hence conditions (2) and (3) of definition 15, are satisfied.

\Leftarrow . Let $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ be an intuitionistic fuzzy filter of S . By Lemma 4.1, F is a subsemigroup of S .

Let $x, y \in S$. If $xy \in F$ then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. Since χ_F is an intuitionistic fuzzy filter of S . By conditions (2) and (3) of definition 15, we have

$$\begin{aligned}\min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= \mu_{\chi_F}(xy), \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= \gamma_{\chi_F}(xy).\end{aligned}$$

Hence

$$\begin{aligned}\min\{\mu_{\chi_F}(x), \mu_{\chi_F}(y)\} &= 1, \\ \text{and } \max\{\gamma_{\chi_F}(x), \gamma_{\chi_F}(y)\} &= 0,\end{aligned}$$

and we have

$$\begin{aligned}\mu_{\chi_F}(x) &= 1 \text{ and } \gamma_{\chi_F}(y) = 0, \\ \mu_{\chi_F}(y) &= 1 \text{ and } \gamma_{\chi_F}(x) = 0.\end{aligned}$$

Thus we have $x \in F$ and $y \in F$.

Let $x, y \in S$, $x \leq y$, if $x \in F$. Then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $x \leq y$ and χ_F is an intuitionistic fuzzy filter of S , we have $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Therefore $y \in F$. \square

DEFINITION 16. Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S . Then A is called an intuitionistic fuzzy prime if:

- (1) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
- (2) $\gamma_A(xy) \geq \min\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$.

If $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy ideal of S , then A is called an intuitionistic fuzzy prime ideal of S .

4.3. Proposition. Let $(S, ., \leq)$ be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S . Then A is an intuitionistic fuzzy filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime ideal of S .

PROOF. \implies . Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy filter of an ordered semigroup S . Then $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy ideal of S . In fact: Let $x, y \in S$. Then

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y),$$

and

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y).$$

Let $x, y \in S$. Since A is an intuitionistic fuzzy filter of S . Then by condition (2) and (3) of definition 15, we have

$$\gamma_A(xy) = \max\{\gamma_A(x), \gamma_A(y)\},$$

and

$$\mu_A(xy) = \min\{\mu_A(x), \mu_A(y)\},$$

by condition (1) and (2) of definition 16, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime ideal of S .

\Leftarrow . Let $A^c = \langle \gamma_A, \mu_A \rangle$ be an intuitionistic fuzzy prime ideal of S . Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy filter of S . Indeed: Let $x, y \in S$, $x \leq y$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy ideal of S , we have

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y),$$

and

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y).$$

Let $x, y \in S$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy ideal of S . By condition (2) and (3) of definition 14, we have

$$\gamma_A(xy) \geq \max\{\gamma_A(x), \gamma_A(y)\} \quad (*)$$

$$\text{and } \mu_A(xy) \leq \min\{\mu_A(x), \mu_A(y)\} \quad (**)$$

Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime ideal of S , by condition (1) and (2) of definition 16, we have

$$\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\} \quad (***)$$

$$\text{and } \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \quad (****),$$

By (*), (**), (***) and (****), we have

$$\begin{aligned} \mu_A(xy) &= \min\{\mu_A(x), \mu_A(y)\} \\ \text{and } \gamma_A(xy) &= \max\{\gamma_A(x), \gamma_A(y)\}. \end{aligned}$$

Thus $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy filter of S . \square

5. Relation Between Intuitionistic Fuzzy Bi-filters and Intuitionistic Fuzzy Prime Bi-ideal Subsets of Ordered Semigroups

In [24], Shabir et al. has shown that a fuzzy subset f of an ordered semigroup S is a fuzzy bi-filter of S if and only if the complement f' of f is a prime fuzzy bi-ideal subset of S .

In this paragraph, we show that an IFS $A = \langle \mu_A, \gamma_A \rangle$ in S is a fuzzy bi-filter of S if and only if the complement A^c of A is an intuitionistic fuzzy prime bi-ideal subset of S .

In definition 16, of intuitionistic fuzzy prime subsets of S , if $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S , then A is called an intuitionistic fuzzy prime bi-ideal subset of S .

DEFINITION 17. Let (S, \cdot, \leq) be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy bi-ideal subset of S if:

- (1) $x \leq y \implies \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
- (2) $\mu_A(xyx) \geq \mu_A(x)$ for all $x, y \in S$,
- (3) $\gamma_A(xyx) \leq \gamma_A(x)$ for all $x, y \in S$.

5.1. Proposition. Let S be an ordered semigroup, $\emptyset \neq B \subseteq S$. Then B is a bi-ideal subset of S if and only if the intuitionistic characteristic function $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ of B is an intuitionistic fuzzy bi-ideal subset of S .

PROOF. \implies . Let B be a bi-ideal subset of an ordered semigroup S and $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ the intuitionistic characteristic function of B . Then χ_B is an intuitionistic fuzzy bi-ideal subset of S . In fact: Let $x, y \in S$, $x \leq y$. If $y \notin B$, then $\mu_{\chi_B}(y) = 0$ and $\gamma_{\chi_B}(y) = 1$. Since $\mu_{\chi_B}(x) \geq 0$ and $\gamma_{\chi_B}(x) \leq 1$ for all $x, y \in S$. We have $\mu_{\chi_B}(x) \geq \mu_{\chi_B}(y)$ and $\gamma_{\chi_B}(x) \leq \gamma_{\chi_B}(y)$. Let $y \in B$ then $\mu_{\chi_B}(y) = 1$ and $\gamma_{\chi_B}(y) = 0$. Since B is a bi-ideal subset of S and $x \leq y \in B$, then $x \in B$. Hence $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$. Again $\mu_{\chi_B}(x) \geq \mu_{\chi_B}(y)$ and $\gamma_{\chi_B}(x) \leq \gamma_{\chi_B}(y)$.

Let $x, y \in S$. If $x \notin B$, then $\mu_{\chi_B}(x) = 0$ and $\gamma_{\chi_B}(x) = 1$. Hence $\mu_{\chi_B}(xyx) \geq \mu_{\chi_B}(x)$ and $\gamma_{\chi_B}(xyx) \leq \gamma_{\chi_B}(x)$. Let $x \in B$, then $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$. Since B is a bi-ideal subset of S and $x \in B$, we have $xyx \in B$. Then $\mu_{\chi_B}(xyx) = 1$ and $\gamma_{\chi_B}(xyx) = 0$. Hence $\mu_{\chi_B}(xyx) \geq \mu_{\chi_B}(x)$ and $\gamma_{\chi_B}(xyx) \leq \gamma_{\chi_B}(x)$. Thus χ_B is an intuitionistic fuzzy bi-ideal subset of S .

\impliedby . Let $\chi_B = \langle \mu_{\chi_B}, \gamma_{\chi_B} \rangle$ be an intuitionistic fuzzy bi-ideal subset of S . Then B is a bi-ideal subset of S . In fact: Let $x, y \in S$. If $x \in B$. Then $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$. Since χ_B is an intuitionistic fuzzy bi-ideal subset of S , we have $\mu_{\chi_B}(xyx) \geq \mu_{\chi_B}(x)$ and $\gamma_{\chi_B}(xyx) \leq \gamma_{\chi_B}(x)$. Then $\mu_{\chi_B}(xyx) = 1$ and $\gamma_{\chi_B}(xyx) = 0$, and hence $xyx \in B$.

Let $x, y \in S$, $x \leq y$. If $y \in B$. Then $\mu_{\chi_B}(y) = 1$ and $\gamma_{\chi_B}(y) = 0$. Since $x \leq y$, we have $\mu_{\chi_B}(x) \geq \mu_{\chi_B}(y)$ and $\gamma_{\chi_B}(x) \leq \gamma_{\chi_B}(y)$. Then $\mu_{\chi_B}(x) = 1$ and $\gamma_{\chi_B}(x) = 0$. Thus $x \in B$. \square

DEFINITION 18. Let (S, \cdot, \leq) be an ordered semigroup. An IFS $A = \langle \mu_A, \gamma_A \rangle$ of S is called an intuitionistic fuzzy bi-filter of S if:

- (1) $x \leq y \implies \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$,
- (2) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
- (3) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$,
- (4) $\mu_A(xyx) \geq \mu_A(x)$ for all $x, y \in S$,
- (5) $\gamma_A(xyx) \leq \gamma_A(x)$ for all $x, y \in S$.

5.2. Proposition. Let (S, \leq) be an ordered semigroup and $\emptyset \neq F \subseteq S$. Then F is a bi-filter of S if and only if $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$ is an intuitionistic fuzzy bi-filter of S .

PROOF. \implies . Let F be a bi-filter of an ordered semigroup S and $\chi_F = \langle \mu_{\chi_F}, \gamma_{\chi_F} \rangle$, the intuitionistic characteristic function of F . Then χ_F is an intuitionistic fuzzy bi-filter of S . Indeed: By Lemma 4.1, χ_F is an intuitionistic fuzzy subsemigroup of S . Let $x, y \in S$, $x \leq y$. If $x \notin F$, then $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$. Since $\mu_{\chi_F}(y) \geq 0$ and $\gamma_{\chi_F}(y) \leq 1$ for all $x, y \in S$. Thus $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Let $x \in F$ then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $y \geq x \in F$ and F is a bi-filter of S , we have $y \in F$, and hence $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Again $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Hence condition (1) of definition 18, is satisfied.

Let $x, y \in S$, if $xyx \notin F$. Then $\mu_{\chi_F}(xyx) = 0$ and $\gamma_{\chi_F}(xyx) = 1$. Since F is a bi-filter of S and $xyx \notin F \implies x \notin F$. Hence

$$\mu_{\chi_F}(x) = 0 \text{ and } \gamma_{\chi_F}(x) = 1,$$

Thus $\mu_{\chi_F}(xyx) \geq \mu_{\chi_F}(x)$ and $\gamma_{\chi_F}(xyx) \leq \gamma_{\chi_F}(x)$.

\impliedby . Let χ_F be an intuitionistic fuzzy bi-filter of S . By Lemma 4.1, F is a subsemigroup of S .

Let $x, y \in S$. If $xyx \in F$ then $\mu_{\chi_F}(xyx) = 1$ and $\gamma_{\chi_F}(xyx) = 0$. Since χ_F is an intuitionistic fuzzy bi-filter of S we have $\mu_{\chi_F}(xyx) \geq \mu_{\chi_F}(x)$ and $\gamma_{\chi_F}(xyx) \leq \gamma_{\chi_F}(x)$. Thus $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$, and so $x \in F$.

Let $x, y \in S$, $x \leq y$, if $x \in F$. Then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $x \leq y$ and χ_F is an intuitionistic fuzzy bi-filter of S , we have $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Therefore $y \in F$.

□

In definition 16, of intuitionistic fuzzy prime subsets of an ordered semigroup S if we consider the *IFS* $A = \langle \mu_A, \gamma_A \rangle$ in S as an intuitionistic fuzzy bi-ideal subset of S . Then A is called an intuitionistic fuzzy prime bi-ideal subset of S .

5.3. Proposition. Let (S, \leq) be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an *IFS* of S . Then A is an intuitionistic fuzzy bi-filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime bi-ideal subset of S .

PROOF. \implies . Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy bi-filter of an ordered semigroup S . We show that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime bi-ideal subset of S . Since A is an intuitionistic fuzzy bi-filter of S then A^c is an intuitionistic fuzzy bi-ideal subset of S . In fact: Let $x, y \in S$. Then

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y),$$

and

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y).$$

Let $x, y \in S$. Since A is an intuitionistic fuzzy bi-filter of S . Then by condition (4) and (5) of definition 18, we have

$$\mu_A(xyx) \geq \mu_A(x)$$

and

$$\gamma_A(xyx) \leq \gamma_A(x),$$

by conditions (2) and (3) of definition 18, it follows that A^c is an intuitionistic fuzzy bi-ideal subset of S .

Since A is an intuitionistic fuzzy bi-filter of S , by condition (2) and (3) of definition 18, we have

$$\begin{aligned} \gamma_A(xy) &\leq \max\{\gamma_A(x), \gamma_A(y)\}, \\ \text{and } \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\}, \end{aligned}$$

by condition (1) and (2) of definition 16, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime ideal of S .

\Leftarrow . Let $A^c = \langle \gamma_A, \mu_A \rangle$ be an intuitionistic fuzzy prime bi-ideal subset of S . Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-filter of S . Indeed: Let $x, y \in S$, $x \leq y$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S , we have

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y),$$

and

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A^c(x).$$

Let $x, y \in S$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy bi-ideal subset of S . By condition (2) and (3) of definition 18, we have

$$\begin{aligned} \gamma_A(xyx) &\geq \gamma_A(x) \\ \text{and } \mu_A(xyx) &\leq \mu_A(x) \end{aligned}$$

Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime ideal of S , by condition (1) and (2) of definition 16, we have

$$\begin{aligned} \gamma_A(xy) &\leq \max\{\gamma_A(x), \gamma_A(y)\} \\ \text{and } \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

Thus $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi-filter of S . \square

6. Relation Between Intuitionistic Fuzzy left (resp. right) Filters and Intuitionistic Fuzzy Prime left (resp. right) ideal of Ordered Semigroups

It is well known that a fuzzy subset f of an ordered semigroup S is a left filter of S if and only if the complement f' of f is a fuzzy prime left ideal of S [24].

In this paragraph, we show that an IFS, $A = \langle \mu_A, \gamma_A \rangle$ of an ordered semigroup S is a left (resp. right) filter of S if and only if the complement A^c of A is an intuitionistic fuzzy prime left (resp. right) ideal of S .

DEFINITION 19. Let (S, \cdot, \leq) be an ordered semigroup, $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S . Then A is called an intuitionistic fuzzy left filter of S if:

- (1) $x \leq y \implies \mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$,
- (2) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
- (3) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$,
- (4) $\mu_A(xy) \geq \mu_A(y)$ for all $x, y \in S$,
- (5) $\gamma_A(xy) \leq \gamma_A(y)$ for all $x, y \in S$.

DEFINITION 20. Let (S, \cdot, \leq) be an ordered semigroup, $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S . Then A is called an intuitionistic fuzzy right filter of S if:

- (1) $x \leq y \implies \mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$,
- (2) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in S$,
- (3) $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in S$,
- (4) $\mu_A(xy) \geq \mu_A(x)$ for all $x, y \in S$,
- (5) $\gamma_A(xy) \leq \gamma_A(x)$ for all $x, y \in S$.

6.1. Proposition. Let (S, \cdot, \leq) be an ordered semigroup and $\emptyset \neq F \subseteq S$. Then F is a left filter of S if and only if $\chi_F = \langle \mu_F, \gamma_F \rangle$ is an intuitionistic fuzzy left filter of S .

PROOF. \implies . Let F be a left filter of an ordered semigroup S and $\chi_F = \langle \mu_F, \gamma_F \rangle$, the intuitionistic characteristic function of F . Then χ_F is an intuitionistic fuzzy left filter of S . Indeed: By Lemma 4.1, χ_F is an intuitionistic fuzzy subsemigroup of S . Let $x, y \in S$, $x \leq y$. If $x \notin F$, then $\mu_{\chi_F}(x) = 0$ and $\gamma_{\chi_F}(x) = 1$. Since $\mu_{\chi_F}(y) \geq 0$ and $\gamma_{\chi_F}(y) \leq 1$ for all $x, y \in S$, we have $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Let $x \in F$ then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $y \geq x \in F$ and F is a left filter of S , we have $y \in F$, and hence $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Again $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Hence condition (1) of definition 19, is satisfied.

Let $x, y \in S$, if $xy \in F$. Then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. Since F is a left filter of S and $xy \in F \implies y \in F$. Hence

$$\mu_{\chi_F}(y) = 1 \text{ and } \gamma_{\chi_F}(y) = 0,$$

Thus $\mu_{\chi_F}(xy) \geq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(xy) \leq \gamma_{\chi_F}(y)$.

\impliedby . Let χ_F be an intuitionistic fuzzy left filter of S . By Lemma 4.1, F is a subsemigroup of S .

Let $x, y \in S$. If $xy \in F$ then $\mu_{\chi_F}(xy) = 1$ and $\gamma_{\chi_F}(xy) = 0$. Since χ_F is an intuitionistic fuzzy left filter of S we have $\mu_{\chi_F}(xy) \geq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(xy) \leq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$, and so $y \in F$.

Let $x, y \in S$, $x \leq y$, if $x \in F$. Then $\mu_{\chi_F}(x) = 1$ and $\gamma_{\chi_F}(x) = 0$. Since $x \leq y$ and χ_F is an intuitionistic fuzzy left filter of S , we have $\mu_{\chi_F}(x) \leq \mu_{\chi_F}(y)$ and $\gamma_{\chi_F}(x) \geq \gamma_{\chi_F}(y)$. Thus $\mu_{\chi_F}(y) = 1$ and $\gamma_{\chi_F}(y) = 0$. Therefore $y \in F$. \square

In a similar way we can prove the following:

6.2. Proposition. Let (S, \cdot, \leq) be an ordered semigroup and $\emptyset \neq F \subseteq S$. Then F is a right filter of S if and only if χ_F is an intuitionistic fuzzy right filter of S .

In definition 16, of an intuitionistic fuzzy prime subsets of an ordered semigroup S , if $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left (resp. right) ideal of S . Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy prime left (resp. right) ideal of S .

6.3. Proposition. Let (S, \cdot, \leq) be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S . Then A is an intuitionistic fuzzy left filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime left ideal of S .

PROOF. \Rightarrow . Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left filter of an ordered semigroup S . Then $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime left ideal of S . We first show that A^c is an intuitionistic fuzzy left ideal of S . Let $x, y \in S$, $x \leq y$. Then

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y),$$

and

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y).$$

Let $x, y \in S$. Since A is an intuitionistic fuzzy left filter of S . Then by condition (4) and (5) of definition 19, we have

$$\mu_A(xy) \geq \mu_A(y)$$

and

$$\gamma_A(xy) \leq \gamma_A(y)$$

by condition (2) and (3) of definition 12, it follows that $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S . Since A is an intuitionistic fuzzy left filter of S . By conditions (2) and (3) of definition 19, we have,

$$\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\},$$

and

$$\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\},$$

by conditions (1) and (2) of definition 16, it follows that A^c is an intuitionistic fuzzy prime left ideal of S .

\Leftarrow . Let $A^c = \langle \gamma_A, \mu_A \rangle$ be an intuitionistic fuzzy prime left ideal of S . Then A^c is an intuitionistic fuzzy left ideal of S . We show that $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left filter of S . Indeed: Let $x, y \in S$, $x \leq y$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S , we have

$$\mu_A(x) = \gamma_A^c(x) = 1 - \gamma_A(x) \leq 1 - \gamma_A(y) = \gamma_A^c(y) = \mu_A(y),$$

and

$$\gamma_A(x) = \mu_A^c(x) = 1 - \mu_A(x) \geq 1 - \mu_A(y) = \mu_A^c(y) = \gamma_A(y).$$

Let $x, y \in S$. Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy prime left ideal of S . We have,

$$\begin{aligned} \gamma_A(xy) &\geq \max\{\gamma_A(x), \gamma_A(y)\} \\ \text{and } \mu_A(xy) &\leq \min\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

By condition (1) and (2) of definition 11, it follows that A is an intuitionistic fuzzy subsemigroup of S .

Since $A^c = \langle \gamma_A, \mu_A \rangle$ is an intuitionistic fuzzy left ideal of S , by condition (1) and (2) of definition 12, we have

$$\begin{aligned} \gamma_A(xy) &\geq \gamma_A(y) \\ \text{and } \mu_A(xy) &\leq \mu_A(y). \end{aligned}$$

Thus A is an intuitionistic fuzzy left filter of S . This completes the proof. \square

Similarly we can prove the following:

6.4. Proposition. Let (S, \leq) be an ordered semigroup and $A = \langle \mu_A, \gamma_A \rangle$ an IFS of S . Then A is an intuitionistic fuzzy right filter of S if and only if the complement $A^c = \langle \gamma_A, \mu_A \rangle$ of A is an intuitionistic fuzzy prime right ideal of S .

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