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Data-based approximation of intuitionistic fuzzy target sets

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Abstract: Approximation is an important process for one to know or recognize a crisp target set. Data reasoning based on an information system is a common source for such approximation. I will introduce how to characterize an intuitionisitc fuzzy target set via a classifier induced by intuitionistic fuzzy sets based on an information system, in particular, a messy information system. I will also show how to restore the data of a messy information system.
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1 Introduction

To handle the uncertainty and vagueness when forming a set, in 1965, Lotfi A. Zadeh introduced fuzzy sets based on the concept of membership. However, it is not always the case that one can decide whether a property is owned by an object or not. Hence, in 1986, Krassimir Atanassov introduced intuitionistic fuzzy set to capture the concept of membership, non-membership and indeterminacy.

In 1982, Zdzislaw Pawlak introduced a different setting for uncertain reasoning by the concept of rough sets. In particular, he further applied his approximation approach to information systems. Data in an information system is used very often in characterization of a crisp target set. For example, a universe U consists of only three elements u and v and w and it is equipped with an information system which contains only one attribute Temperature. The values of Temperature for u is hot, for v is cold and for w is hot. Then he forms the granule knowledge of hot

$\{e \in U : e \text{ is hot}\} = \{u, w\}$. It shows there is no way to tell the difference between element u and w based on the data provided.

As the target becomes more complicated, an approximation approach for a crisp target set is not enough. In this paper, I will consider the target set to be an intuitionistic fuzzy set (or intuitionistic fuzzy target set). The source is still an information system, but the reasoning is different. I will use intuitionistic fuzzy sets to represent a granule knowledge. For example, suppose a universe $U = \{u_1, u_2, u_3\}$ is equipped with an information system which contains two attributes Height and Distance. The values of Height for u_1 is 2, for u_2 is 1 and for u_3 is also 1. The values of Distance for u_1 is 20 km, for u_2 is 30 km and for u_3 the data is missing. Now one forms the granule knowledge via a classifier: Height=1 and Distance=20, by an intuitionistic fuzzy set $((\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, 0))$ (see Definition 1),where the first component $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is the membership function and $(\frac{1}{2}, \frac{1}{2}, 0))$ is the non-membership function formed from the data in the information system. In Section 3, I will how to characterize an intuitionistic fuzzy target set based on a classifier induced by an information system.

In the real world, the data in an information system might not be so perfect, for example, some of the data might be missing or too hard to collect or some data might be fiddled or wrongly typed or some data might be dubious. The concept of an intuitionistic fuzzy set is very useful dealing with such information systems. In Section 2, I will show how to extract information from problematic data of a messy information system through reasoning on intuitionistic fuzzy sets. In Section 4, I show a way to recover the missing, corrupted or contaminated data.

2 Fuzzy classifier: Data-driven fuzzy sets

A messy information system is an information system which contains incomplete, wrongly-typed, missing, misplaced, abnormal, corrupted or any dubious data. Before one proceeds this kind of information system, he needs to filter out the problematic data. Assume that we associate a bijective function to fix a finite universe U to specify its elements. We use $|v|$ to denote either the size of the set v or the absolute value of the real number v.

Example 1. Let us show an example of a messy information system defined over a Universe $U = \{u_1, u_2, u_3, u_4\}$ and a set of attributes $\{a_1, a_2, ..., a_{10}\}$ as follows. The data in the cell (u_1, a_4) is incomplete or missing, the data in (u_2, a_1) is wrongly-typed or dubious, the data in (u_3, a_8) is misplaced and the data in (u_4, a_2) is corrupted or abnormal.

Universe	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
u_1	low	short	1	?	hard	fly	hot	new	yes	0
u_2	high,low	long	0	red	soft	walk	warm	old	undecided	3
u_3	medium	long	1	black	soft	fly	warm	no	yes	4
u_4	medium	~~short~~	1	green	hard	swim	cold	new	no	0

Table 1: Messy information system

Now one turns the messy information system into a filtered information system based on his personal judgment or some mechanical processes.

Example 2. Let us continue Table 1 in Example 1. The data in doubt is filtered and disposed, and the cells are left empty. We use BLANK to denote such cells.

Universe	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
u_1	low	short	1	BLANK	hard	fly	hot	new	yes	0
u_2	BLANK	long	0	red	soft	walk	warm	old	undecided	3
u_3	medium	long	1	black	soft	fly	warm	BLANK	yes	4
u_4	medium	BLANK	1	green	hard	swim	cold	new	no	0

Table 2: Filtered information system $\mathcal{I}$

Now we want to show how one forms an intuitionistic fuzzy granule knowledge based on a filtered information system. We say $P \in [a_1]' \times [a_2]' ... \times [a_n]'$ is a knowledge specification, where $[a_i]'$ denotes the filtered domain of the attribute a_i, for example, $[a_7]' = \{\text{hot}, \text{warm}, \text{cold}\}$ in $\mathcal{I}$. A knowledge specification can be chosen mechanically or manually. It represents the concepts that the user or designer is interested in or is capable of accessing. Though the filtered information is presented, one needs a knowledge specification to convert it into his knowledge. Through a knowledge specification, one (a user) starts to form some knowledge of the universe. Then he uses this knowledge to classify or characterize other new target sets. This way of processing information, forming knowledge, and characterizing target sets has a wide range of applications in all fields.

Let U be a universe. Let $\mathfrak{A} = \{a_1, a_2, ..., a_n\}$ be a set of attributes and $\mathbb{A} = \{\mathcal{B}_1, \mathcal{B}_2, ..., \mathcal{B}_n\}$ be a set of domains of attribute values (numerical or non-numerical) of $\mathfrak{A}$. Let $\mathcal{D} = \{1, 2, ..., |\mathfrak{A}|\}$. Let

$$\mathcal{I} : U \to \prod^{i \in \mathcal{D}} \mathcal{B}_i$$

be an information system which assigns the attribute values to each element $u \in U$. First of all, I show how one converts the information from the filtered information system into the knowledge of membership and non-membership functions. Let $[0,1]^U$ denote the set of all the functions from U to $[0,1]$. Let $\rho^{(n)}$ be a projection function which projects the n-th component of an ordered set.

Definition 1 (Intuitionistic Fuzzy Sets)**.** Let $P \in [a_1]' \times [a_2]' ... \times [a_n]'$ be arbitrary. Define $\mathcal{MN}^{\mathcal{I}}_P : U \to [0,1] \times [0,1]$ by

$$\mathcal{MN}^{\mathcal{I}}_P(u) := (\mathcal{M}^{\mathcal{I}}_P(u), \mathcal{N}^{\mathcal{I}}_P(u)),$$

where

$$\mathcal{M}^{\mathcal{I}}_P(u) := \frac{|\{j \in \mathcal{D} : \rho^{(j)}(P) = \rho^{(j)}(\mathcal{I}(u))\}|}{|\mathfrak{A}|}$$

and

$$\mathcal{N}^{\mathcal{I}}_P(u) := \frac{|\{j \in \mathcal{D} : \rho^{(j)}(P) \neq \rho^{(j)}(\mathcal{I}(u))\}|}{|\mathfrak{A}|}.$$

We use $\mathcal{MN}_P^{\mathcal{I}}$ to denote $(\mathcal{M}_P^{\mathcal{I}}, \mathcal{N}_P^{\mathcal{I}})$. The first component denotes the membership function and the second one denotes the non-membership function. Observe that each $\mathcal{MN}_P^{\mathcal{I}}$ depends on a filtered information system $\mathcal{I}$ and a knowledge specification P.

This definition is a simplified one as it makes no differences regarding the individual data or knowledge specifications. One can put some weight on his measurement according to his needs or the reality of the situations. In addition, every knowledge specification P in $[a_1]' \times [a_2]' ... \times [a_n]$ is assumed. If one can only access to part of the information system or his access is limited, then P can be chosen only partially, for example, $P \in [a_2]' \times [a_4]' \times [a_7]'$. If this is the case, then one has to change the part $|\mathfrak{A}|$ in the definition to the size of P.

Example 3. Let us continue Example 2. Let the knowledge specification $P = (\text{medium}, \text{short}, 1, \text{green}, \text{soft}, \text{swim}, \text{hot}, \text{new}, \text{yes}, 0)$. Then the granule knowledge in the form of an intuitionistic fuzzy set can be represented by

$$\mathcal{MN}_P^{\mathcal{I}}(u_1) = (\frac{|\{\text{short}, 1, \text{hot}, \text{new}, \text{yes}, 0\}|}{10}, \frac{|\{\text{low}, \text{hard}, \text{fly}\}|}{10}) = (\frac{6}{10}, \frac{3}{10});$$

$$\mathcal{MN}_P^{\mathcal{I}}(u_2) = (\frac{|\{\text{short}\}|}{10}, \frac{|\{\text{long}, 0, \text{red}, \text{walk}, \text{warm}, \text{old}, \text{undecided}, 3\}|}{10}) = (\frac{1}{10}, \frac{8}{10});$$

$$\mathcal{MN}_P^{\mathcal{I}}(u_3) = (\frac{|\{\text{medium}, 1, \text{soft}, \text{yes}\}|}{10}, \frac{|\{\text{long}, \text{black}, \text{fly}, \text{warm}, 4\}|}{10}) = (\frac{4}{10}, \frac{5}{10});$$

$$\mathcal{MN}_P^{\mathcal{I}}(u_4) = (\frac{|\{\text{medium}, 1, \text{green}, \text{swim}, \text{new}, 0\}|}{10}, \frac{|\{\text{hard}, \text{cold}, \text{no}\}|}{10}) = (\frac{6}{10}, \frac{3}{10}).$$

These intuitionistic fuzzy sets induced by an information system $\mathcal{I}$ and a knowledge specification P can then be concisely represented by

$$\mathcal{MN}_P^{\mathcal{I}} = (\mathcal{M}_P^{\mathcal{I}}, \mathcal{N}_P^{\mathcal{I}}) = ((\frac{6}{10}, \frac{1}{10}, \frac{4}{10}, \frac{6}{10}), (\frac{3}{10}, \frac{8}{10}, \frac{5}{10}, \frac{3}{10})).$$

Now we know how to extract information from a filtered information system to form our knowledge via an intuitionistic fuzzy set. Next, we will start to consider the characterization of an intuitionistic fuzzy target set.

3 Characterizations

Let $\mathbb{P} = \{P_1, P_2, ..., P_n\}$ be a set of knowledge specifications. Let the set of all the intuitionistic fuzzy sets induced by $\mathbb{P}$ be $\mathcal{MN}_{\mathbb{P}}^{\mathcal{I}} := \{(\mathcal{M}_P^{\mathcal{I}}, \mathcal{N}_P^{\mathcal{I}}) : P \in \mathbb{P}\}$. Based on the definitions in [1] and [2], I make the following definitions in order to establish an approximation approach for intuitionistic fuzzy target sets. Let U be an arbitrary finite universe. For all $\alpha, \beta \in [0,1]^U$, we say $\alpha \geq \beta$ iff for all $u \in U(\alpha(u) \geq \beta(u))$ and $\alpha \leq \beta$ iff for all $u \in U(\alpha(u) \leq \beta(u))$. We use $\vec{0}$ and $\vec{1}$ to denote the zero and one constant function, respectively.

Definition 2. For all $\alpha, \beta \in [0,1]^U$, define $\alpha \cap^* \beta : U \to [0,1]$ by $(\alpha \cap^* \beta)(u) := min\{\alpha(u), \beta(u)\}$ and $\alpha \cup^* \beta : U \to [0,1]$ by $(\alpha \cup^* \beta)(u) := max\{\alpha(u), \beta(u)\}$.

Definition 3. For all $(\alpha, \beta), (\gamma, \delta) \in [0,1]^U \times [0,1]^U$, define $(\alpha, \beta)(\leq, \geq)(\gamma, \delta)$ iff $\alpha \leq \gamma$ and $\beta \geq \delta$ and we say (γ, δ) is an upper bound of (α, β) and (α, β) is a lower bound of (γ, δ). Moreover, define $(\alpha, \beta)(\geq, \leq)(\gamma, \delta)$ iff $\alpha \geq \gamma$ and $\beta \leq \delta$.

Definition 4. For all $(\alpha, \beta), (\gamma, \delta) \in [0,1]^U \times [0,1]^U$, define $(\alpha, \beta) \cap^{**} (\gamma, \delta) := (\alpha \cap^* \gamma, \beta \cup^* \delta)$. If $S \subseteq [0,1]^U \times [0,1]^U$, then let $\cap^{**} S$ denote the $\cap^{**}$-operation over every element in S.

Definition 5. For all $(\alpha, \beta), (\gamma, \delta) \in [0,1]^U \times [0,1]^U$, define $(\alpha, \beta) \cup^{**} (\gamma, \delta) := (\alpha \cup^* \gamma, \beta \cap^* \delta)$. If $S \subseteq [0,1]^U \times [0,1]^U$, then let $\cup^{**} S$ denote the $\cup^{**}$-operation over every element in S.

Any intuitionistic fuzzy target set $\{(u, \in(u), \notin(u)) : u \in U\}$ will be identified with a pair of functions: membership $\in$ and non-membership $\notin$ in the form of an ordered set

$$((\in(u_1), \in(u_2), ..., \in(u_n)), (\notin(u_1), \notin(u_2), ..., \notin(u_n))).$$

Define the set of all the lower and upper bounds of τ in $\mathcal{MN}_{\mathbb{P}}^{\mathcal{I}}$ by

$$lb(\tau) := \{\delta \in \mathcal{MN}_{\mathbb{P}}^{\mathcal{I}} : \delta(\leq, \geq)\tau\}$$

and

$$ub(\tau) := \{\delta \in \mathcal{MN}_{\mathbb{P}}^{\mathcal{I}} : \tau(\leq, \geq)\delta\},$$

respectively.

With all these operations on intuitionistic fuzzy sets, we can approximate an intuitionistic fuzzy target set as follows.

Definition 6 (Characterization). $Ch_{\mathbb{P}}^{\mathcal{I}} : [0,1]^U \times [0,1]^U \to [0,1]^U \times [0,1]^U$ by $Ch_{\mathbb{P}}^{\mathcal{I}}(\tau) := (\overline{lb}(\tau), \underline{ub}(\tau))$, where $\overline{lb} : [0,1]^U \to [0,1]^U$ is defined by supremum:

$$\overline{lb}(\tau) = \begin{cases} \cup^{**} lb(\tau) & \text{, if } lb(\tau) \neq \emptyset; \\ (\vec{0}, \vec{1}) & \text{, otherwise.} \end{cases}$$

and where $\underline{ub} : [0,1]^U \to [0,1]^U$ is defined by infimum:

$$\underline{ub}(\tau) = \begin{cases} \cap^{**}\{\delta \in \mathcal{MN}_{\mathbb{P}}^{\mathcal{I}} : \delta \in ub(\tau)\} & \text{, if } ub(\tau) \neq \emptyset; \\ (\vec{1}, \vec{0}) & \text{, otherwise.} \end{cases}$$

Example 4. Table 3 shows the user's knowledge specifications. Based on this, he converts the data from Table 2 into intuitionistic fuzzy granule knowledge in Table 4. The left-hand side of the table represents the membership function and the right-hand, the non-membership function. Both are plotted in Figure 1.

$\mathbb{P}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
P_1	low	short	0	green	hard	fly	cold	old	yes	2
P_2	high	short	0	red	hard	fly	warm	old	yes	3
P_3	low	long	0	black	soft	fly	hot	new	no	4
P_4	high	long	1	red	hard	swim	cold	new	no	0
P_5	medium	short	1	black	hard	walk	warm	old	no	2
P_6	high	short	0	green	hard	walk	cold	old	yes	3
P_7	low	short	0	black	soft	swim	cold	new	undecided	0
P_8	high	long	1	red	soft	swim	cold	new	yes	4
P_9	medium	long	0	black	hard	walk	warm	new	undecided	2
P_{10}	high	short	1	red	soft	swim	cold	old	undecided	0
P_{11}	high	short	0	green	hard	walk	cold	new	yes	4

Table 3: Classifier: Knowledge Specifications

$\mathcal{MN}^{\mathcal{I}}_{\mathbb{P}}$	u_1	u_2	u_3	u_4	u_1	u_2	u_3	u_4
$\mathcal{MN}^{\mathcal{I}}_{P_1}$	0.50	0.20	0.20	0.40	0.40	0.80	0.80	0.60
$\mathcal{MN}^{\mathcal{I}}_{P_2}$	0.40	0.60	0.30	0.10	0.50	0.40	0.70	0.90
$\mathcal{MN}^{\mathcal{I}}_{P_3}$	0.40	0.30	0.60	0.30	0.50	0.70	0.40	0.70
$\mathcal{MN}^{\mathcal{I}}_{P_4}$	0.30	0.30	0.30	0.70	0.60	0.70	0.70	0.30
$\mathcal{MN}^{\mathcal{I}}_{P_5}$	0.20	0.30	0.40	0.50	0.70	0.70	0.60	0.50
$\mathcal{MN}^{\mathcal{I}}_{P_6}$	0.30	0.50	0.10	0.30	0.60	0.50	0.90	0.70
$\mathcal{MN}^{\mathcal{I}}_{P_7}$	0.40	0.30	0.30	0.30	0.50	0.70	0.70	0.70
$\mathcal{MN}^{\mathcal{I}}_{P_8}$	0.20	0.40	0.60	0.50	0.70	0.60	0.40	0.50
$\mathcal{MN}^{\mathcal{I}}_{P_9}$	0.20	0.50	0.50	0.50	0.70	0.50	0.50	0.50
$\mathcal{MN}^{\mathcal{I}}_{P_{10}}$	0.20	0.50	0.20	0.30	0.70	0.50	0.80	0.70
$\mathcal{MN}^{\mathcal{I}}_{P_{11}}$	0.40	0.30	0.30	0.40	0.50	0.70	0.70	0.60

Table 4: Intuitionistic Fuzzy Classifier

Let the intuitionistic fuzzy target set $\tau = ((0.2, 0.32, 0.43, 0.5), (0.7, 0.61, 0.52, 0.5))$. From Figure 1, one observes that $lb(\tau) = \{\mathcal{MN}^{\mathcal{I}}_{P_5}\}$ and $ub(\tau) = \{\mathcal{MN}^{\mathcal{I}}_{P_8}, \mathcal{MN}^{\mathcal{I}}_{P_9}\}$. Thus

$$\overline{lb}(\tau) = \mathcal{MN}^{\mathcal{I}}_{P_5} = ((0.2, 0.3, 0.4, 0.5), (0.7, 0.7, 0.6, 0.5))$$

and

$$\underline{ub}(\tau) = \mathcal{MN}^{\mathcal{I}}_{P_8} \cap^{**} \mathcal{MN}^{\mathcal{I}}_{P_9} = ((0.2, 0.4, 0.5, 0.5), (0.7, 0.6, 0.5, 0.5))$$

and

$$Ch^{\mathcal{I}}_{\mathbb{P}}(\tau) := (\overline{lb}(\tau), \underline{ub}(\tau)).$$

Observe that $\overline{lb}(\tau)(\leq, \geq)\tau(\leq, \geq)\underline{ub}(\tau)$. Now I want to show how to measure the precision of an approximation. Let $\alpha, \beta, \gamma, \delta \in [0, 1]^U$ be arbitrary.

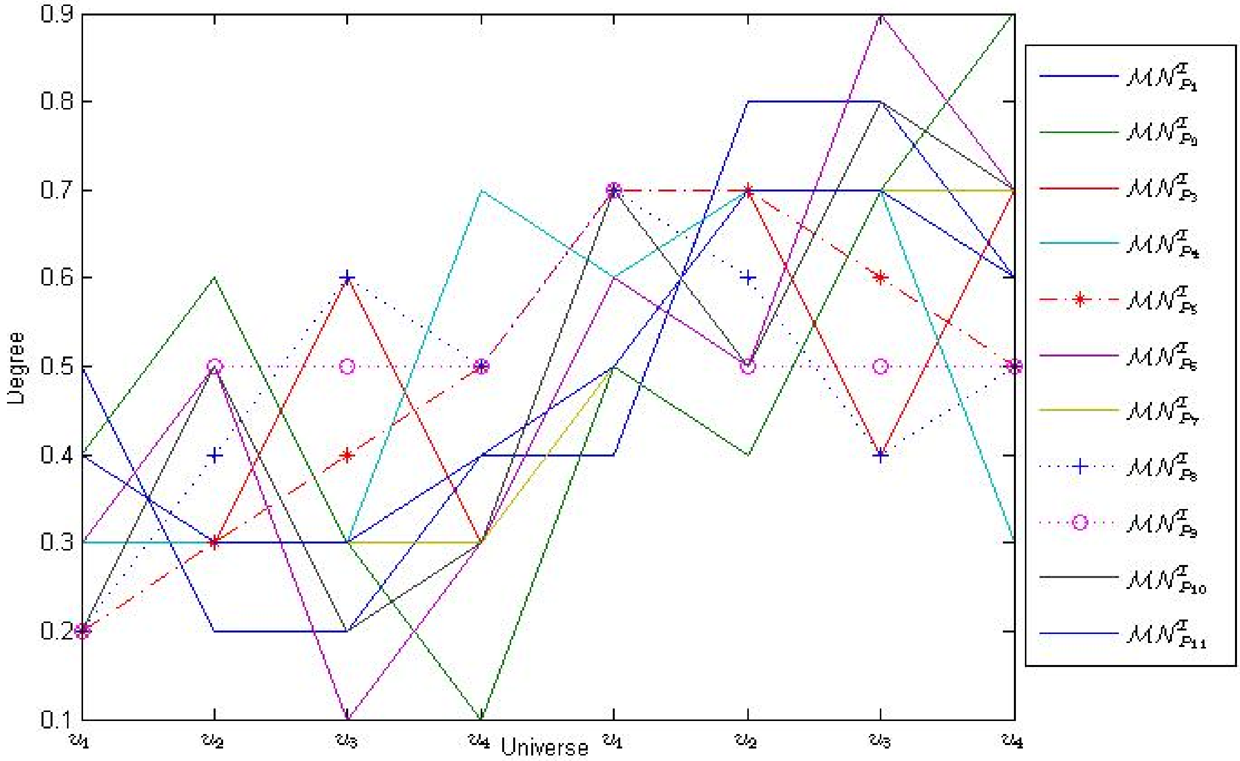

Figure 1: Membership and Non-membership Functions

Definition 7 (Precision)**.** Let $Ch^{\mathcal{I}}_{\mathbb{P}}(\tau) := (mn_2, mn_1)$, where $mn_2 = (\gamma, \delta)$ and $mn_1 = (\alpha, \beta)$. Define

$$PCS^{\mathcal{I}}_{\mathbb{P}}(mn_2, mn_1) := \begin{cases} 1 - \frac{\sum\limits^{u \in U} (\alpha(u) - \gamma(u) + \delta(u) - \beta(u))}{\sum\limits^{u \in U} (\alpha(u) + 1 - \beta(u))} & \text{, if it is defined;} \\ 1 & \text{, otherwise.} \end{cases}$$

Example 5. Let us continue the results in Example 4. Then

$$PCS^{\mathcal{I}}_{\mathbb{P}}(Ch^{\mathcal{I}}_{\mathbb{P}}(\tau)) = 1 - \frac{0.1 + 0.1 + 0.1 + 0.1}{(0.2 + 0.3) + (0.4 + 0.4) + (0.5 + 0.5) + (0.5 + 0.5)} = \frac{29}{33}.$$

Under the definition of $(\leq, \geq)$, the smallest element in $[0,1]^U \times [0,1]^U$ is $(\vec{0}, \vec{1})$ and the largest one is $(\vec{1}, \vec{0})$. If $mn_2 = mn_1 \neq (\vec{0}, \vec{1})$, then $PCS^{\mathcal{I}}_{\mathbb{P}}(mn_1, mn_2) = 1 - \frac{0}{|U|} = 1$. If $mn_2 = mn_1 = (\vec{0}, \vec{1})$, the only case when

$$\frac{\sum\limits^{u \in U} (\alpha(u) - \gamma(u) + \delta(u) - \beta(u))}{\sum\limits^{u \in U} (\alpha(u) + 1 - \beta(u))}$$

is undefined, then under our definition

$$PCS^{\mathcal{I}}_{\mathbb{P}}(mn_1, mn_2) = 1.$$

If $mn_2 = (\vec{0}, \vec{1})$ and $mn_1 = (\vec{1}, \vec{0})$, then

$$PCS^{\mathcal{I}}_{\mathbb{P}}(mn_1, mn_2) = 1 - \frac{2 \times |U|}{2 \times |U|} = 0.$$

For any $\alpha \in [0,1]^U$, define $||\alpha|| := \sum^{u \in U} \alpha(u)$. Then the above definition can be redefined by

$$PCS^{\mathcal{I}}_{\mathbb{P}}(mn_1, mn_2) := \frac{|U| + ||\gamma|| - ||\delta||}{|U| + ||\alpha|| - ||\beta||},$$

if it is defined, and 1, otherwise.

4 Data restoration

Data missing or corrupted occurs very often in data transformation or storage. How to recover or repair it becomes an important issue. For example, the messy information system shown in Table 1 could be restored as follows (Example 6):

Universe	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
u_1	low	short	1	red	hard	fly	hot	new	yes	0
u_2	high	long	0	red	soft	walk	warm	old	undecided	3
u_3	medium	long	1	black	soft	fly	warm	old	yes	4
u_4	medium	short	1	green	hard	swim	cold	new	no	0

Table 5: Restored Information System

The concept of an intuitionistic fuzzy set provides a natural approach to restore the data. Suppose the attribute domains to be restored are known to the user. Then he could restore the blank cells via the following approach. Let $BK^{\mathcal{J}} = \{\rho^{(m)}(\mathcal{J}(u_n))\}$ be the set of all the blank cells in a filtered information system $\mathcal{J}$. Let the user's restored domain be RD, where RD is a product of sets in which each set corresponds to the user's estimation of the blank cell. Let the intuitionistic fuzzy target set be $t \in [0,1]^U \times [0,1]^U$. Let $\mathcal{H}'$ be the information system in which all the blank cells are filled by $\mathcal{H} \in RD$. Then the optimal restoration for the user is

$$\underset{\mathcal{H} \in RD}{min} \{|PSC^{\mathcal{H}'}_{\mathbb{P}}(t) - PSC^{\mathcal{J}}_{\mathbb{P}}(t)|\}.$$

However, if the attribute domains are unknown to the user, then he could restore some parts of the blank cells based on Boolean logic. In this paper, I will solely study the case that the restoring attribute domains are known to the user and show how he can restore the blank cells.

Example 6. To recover the data of Table 2, suppose $\rho^{(3)}(\mathcal{I}(u_1)) \in$ {red, black}, $\rho^{(1)}(\mathcal{I}(u_2)) \in$ {low, medium, high}, $\rho^{(8)}(\mathcal{I}(u_3)) \in$ {old, new} and $\rho^{(2)}(\mathcal{I}(u_4)) \in$ {short, long}, intuitionistic fuzzy target $t = ((0.31, 0.4, 0.2, 0.4), (0.52, 0.6, 0.8, 0.6))$. Let the restoration domain $RD =$ {red, black} $\times$ {low, medium, high} $\times$ {old, new} $\times$ {short, long}.

Based on Table 4, one computes $PCS^{\mathcal{I}}_{\mathbb{P}}(t) = 0$ and thus restores the information system to be one of $\{\mathcal{I}_9, \mathcal{I}_{10}, \mathcal{I}_{11}, \mathcal{I}_{12}, \mathcal{I}_{21}, \mathcal{I}_{22}, \mathcal{I}_{23}, \mathcal{I}_{24}\}$ – e.g., Table 5 (i.e., $\mathcal{I}_9$) is one of the optimal restorations.

Information System	RD	Precision
$\mathcal{I}_1$	(red, low, old, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_1}(t) = \frac{4+1.3-2.7}{4+4-0} = \frac{13}{40}$
$\mathcal{I}_2$	(red, low, old, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_2}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_3$	(red, low, new, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_3}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_4$	(red, low, new, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_4}(t) = \frac{4+1.1-2.9}{4+4-0} = \frac{11}{40}$
$\mathcal{I}_5$	(red, medium, old, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_5}(t) = \frac{4+1.3-2.7}{4+4-0} = \frac{13}{40}$
$\mathcal{I}_6$	(red, medium, old, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_6}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_7$	(red, medium, new, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_7}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_8$	(red, medium, new, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_8}(t) = \frac{4+1.1-2.9}{4+4-0} = \frac{11}{40}$
$\mathcal{I}_9$	(red, high, old, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_9}(t) = 0$
$\mathcal{I}_{10}$	(red, high, old, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{10}}(t) = 0$
$\mathcal{I}_{11}$	(red, high, new, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{11}}(t) = 0$
$\mathcal{I}_{12}$	(red, high, new, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{12}}(t) = 0$
$\mathcal{I}_{13}$	(black, low, old, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{13}}(t) = \frac{4+1.3-2.7}{4+1.5-2.5} = \frac{13}{15}$
$\mathcal{I}_{14}$	(black, low, old, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{14}}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_{15}$	(black, low, new, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{15}}(t) = \frac{4+1.3-2.7}{4+1.6-2.4} = \frac{13}{16}$
$\mathcal{I}_{16}$	(black, low, new, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{16}}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_{17}$	(black, medium, old, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{17}}(t) = \frac{4+1.3-2.7}{4+4-0} = \frac{13}{40}$
$\mathcal{I}_{18}$	(black, medium, old, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{18}}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_{19}$	(black, medium, new, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{19}}(t) = \frac{4+1.3-2.7}{4+4-0} = \frac{13}{40}$
$\mathcal{I}_{20}$	(black, medium, new, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{20}}(t) = \frac{4+1.2-2.8}{4+4-0} = \frac{3}{10}$
$\mathcal{I}_{21}$	(black, high, old, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{21}}(t) = 0$
$\mathcal{I}_{22}$	(black, high, old, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{22}}(t) = 0$
$\mathcal{I}_{23}$	(black, high, new, short)	$PCS_{\mathbb{P}}^{\mathcal{I}_{23}}(t) = 0$
$\mathcal{I}_{24}$	(black, high, new, long)	$PCS_{\mathbb{P}}^{\mathcal{I}_{24}}(t) = 0$

Table 6: Recovering messy information system

References

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