Notes on Intuitionistic Fuzzy Sets
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Transversals of intuitionistic fuzzy directed hypergraphs

K. K. Myithili[1] and R. Parvathi[2]

[1] Department of Mathematics, Vellalar College for Women
Erode - 638 012, Tamilnadu, India
e-mail: myth_maths@rediffmail.com

[2] Department of Mathematics, Vellalar College for Women
Erode - 638 012, Tamilnadu, India
e-mail: paarvathis@rediffmail.com

Abstract: Hypergraph is a graph in which an edge can connect more than two vertices. Directed hypergraphs are much like standard directed graphs. In usual directed graph, standard arcs connect a single tail node to a single head node whereas in the intuitionistic fuzzy directed hypergraph, hyperarcs connect a set of tail nodes to a set of head nodes. A transversal is a line that intersects two lines whereas in intuitionistic fuzzy directed hypergraph the transversals, is a hyperarc that intersects two or more hyperedges. In this paper, operations on intuitionistic fuzzy transversals of intuitionistic fuzzy directed hypergraphs are introduced and some of their properties are discussed. Further, operations like union, join, intersection, structural subtraction, composition and cartesian product on intuitionistic fuzzy directed hypergraphs are defined and studied with minimal intuitionistic fuzzy transversals as the edge set.
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1 Introduction

The notion of graph theory was introduced by Euler in 1736. The theory of graphs is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. In order to expand the application base, the notion of graph was generalized to that of a hypergraph, that is, a set V of vertices together with a collection of subsets of V. In 1976, Berge [5] introduced the concepts of graph and hypergraph. In [6], the concepts of fuzzy graph and fuzzy hypergraph were introduced. Fuzzy graph theory is now finding numerous applications in modern science and technology. In 1986, Atanassov[1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Intuitionistic fuzzy graph and intuitionistic fuzzy hypergraph (IFHG) were introduced in [7, 9]. In [2, 8], index matrix representation and operations on intuitionistic fuzzy graphs have been discussed. Hypergraphs have vast applications in system analysis, circuit clustering and pattern recognition. In mathematical and computer science problems, hypergraphs also arise naturally in important practical problems, including circuit layout, boolean satisfiability, numerical linear algebra. Directed hypergraphs are a generalization of directed graphs (digraphs) and they can model binary relations among subsets of a given set. In [16] transversals of intuitionistic fuzzy directed hypergraphs (IFDHGs) and minimal transversals of IFDHG were initiated. In this way, the authors got motivated to extend their work on operations in transversals of intuitionistic fuzzy directed hypergraph. Hence in this paper, operations such as union, join, intersection, structural subtraction, composition and cartesian product of transversals of intuitionistic fuzzy directed hypergraphs (TIFDHGs) have been introduced and studied.

2 Notations and Preliminaries

$H = (V, E)$	- Hypergraph with vertex set V and edge set E
$h(H)$	- Height of a hypergraph H
$F(H)$	- Fundamental sequence of H
$C(H)$	- Core set of H
$I(H)$	- Induced fundamental sequence of H
$H^{(r_i,s_i)}$	- (r_i, s_i) - level intuitionistic fuzzy hypergraph
$Tr(H)$	- Intuitionistic fuzzy transversals (IFT) of H
T	- Minimal IFT of H
μ_{t_i}, ν_{t_i}	- Degrees of membership and non-membership of the vertex v_i of $Tr(H)$
$\mu_{t_{ij}}, \nu_{t_{ij}}$	- Degrees of membership and non-membership of the edge e_{ij} of $Tr(H)$

In this section, basic definitions relating to intuitionistic fuzzy sets, intuitionistic fuzzy graphs, IFDHGs are dealt with.

Definition 2.1. [1] Let a set E be fixed. An *intuitionistic fuzzy set (IFS) V* in E is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle / v_i \in E\}$, where the function $\mu_i : E \to [0, 1]$ and $\nu_i : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E$, $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$.

Definition 2.2. [4] The six cartesian products of the two IFSs V_1, V_2 of V over E is defined as

$$
\begin{aligned}
&V_1 \times_1 V_2 = \{\langle (v_1, v_2)\,, \mu_1.\mu_2, \nu_1.\nu_2 \rangle | v_1 \in V_1, v_2 \in V_2\},\\
&V_1 \times_2 V_2 = \{\langle (v_1, v_2)\,, \mu_1 + \mu_2 - \mu_1\mu_2, \nu_1.\nu_2 \rangle | v_1 \in V_1, v_2 \in V_2\},\\
&V_1 \times_3 V_2 = \{\langle (v_1, v_2)\,, \mu_1.\mu_2, \nu_1 + \nu_2 - \nu_1.\nu_2 \rangle | v_1 \in V_1, v_2 \in V_2\},\\
&V_1 \times_4 V_2 = \{\langle (v_1, v_2)\,, \min(\mu_1, \mu_2), \max(\nu_1, \nu_2) \rangle | v_1 \in V_1, v_2 \in V_2\},\\
&V_1 \times_5 V_2 = \{\langle (v_1, v_2)\,, \max(\mu_1, \mu_2), \min(\nu_1, \nu_2) \rangle | v_1 \in V_1, v_2 \in V_2\}.\\
&V_1 \times_6 V_2 = \{\langle (v_1, v_2)\,, \tfrac{\mu_1+\mu_2}{2}, \tfrac{\nu_1+\nu_2}{2} \rangle) | v_1 \in V_1, v_2 \in V_2\}.
\end{aligned}
$$

It must be noted that $V_1 \times_s V_2$ is an IFS, where $s = 1, 2, 3, 4, 5, 6$.

Definition 2.3. [17] An *intuitionistic fuzzy graph (IFG)* is of the form $G = \langle V, E \rangle$ where
(i) $V = \{v_1, v_2, ...v_n\}$ such that $\mu_i : V \to [0, 1]$ and $\nu_i : V \to [0, 1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and

$$0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$$

for every $v_i \in V, i = 1, 2, ..., n$
(ii) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0, 1]$ and $\nu_{ij} : V \times V \to [0, 1]$ are such that

$$
\begin{aligned}
\mu_{ij} &\leq \mu_i \oslash \mu_j\\
\nu_{ij} &\leq \nu_i \oslash \nu_j
\end{aligned}
$$

and

$$0 \leq \mu_{ij} + \nu_{ij} \leq 1$$

where μ_{ij} and ν_{ij} are the degrees of membership and non-membership of the edge (v_i, v_j); the values of $\mu_i \oslash \mu_j$ and $\nu_i \oslash \nu_j$ can be determined by one of the cartesian products $\times_s$, $s = 1, 2, ..., 6$ for all i and j given in Definition 2.2.

Note: Throughout this paper, it is assumed that the fifth cartesian product

$$
\begin{aligned}
V_1 \times_5 V_2 \times_5 V_3 ... \times_5 V_n = \{\langle (v_1, v_2, \cdots, v_n)\,, &\max(\mu_1, \mu_2, \cdots, \mu_n),\\
&\min(\nu_1, \nu_2, \cdots, \nu_n) \rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}.
\end{aligned}
$$

is used to determine the degrees of membership (μ_{ij}) and non-membership (ν_{ij}) of the edge e_{ij}.

Definition 2.4. [9] An *intuitionistic fuzzy hypergraph* (IFHG) is an ordered pair $H = (V, E)$ where
(i) $V = \{v_1, v_2, ..., v_n\}$, is a finite set of intuitionistic fuzzy vertices,
(ii) $E = \{E_1, E_2, ..., E_m\}$ is a family of crisp subsets of V,
(iii) $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_j)) : \mu_j(v_i), \nu_j(v_i) \geq 0$ and $\mu_j(x_i) + \nu_j(x_i) \leq 1\}, j = 1, 2, ..., m$,
(iv) $E_j \neq \phi, j = 1, 2, ..., m$,
(v) $\bigcup_j supp(E_j) = V, j = 1, 2, ..., m$.
Here, the hyperedges E_j are crisp sets of intuitionistic fuzzy vertices, $\mu_j(v_i)$ and $\nu_j(v_i)$ denote the degrees of membership and non-membership of vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(v_{ij}, \mu_j(v_i), \nu_j(v_j))$. The sets V, E are crisp sets.

Notations:
1. Hereafter,$\langle \mu(v_i), \nu(v_i)\rangle$ or simply $\langle \mu_i, \nu_i\rangle$ denote the degrees of membership and non-membership of the vertex $v_i \in V$, such that $0 \leq \mu_i + \nu_i \leq 1$.
2. $\langle \mu(e_{ij}), \nu(e_{ij})\rangle$ or simply $\langle \mu_{ij}, \nu_{ij}\rangle$ denote the degrees of membership and non-membership of the edge $(v_i, v_j) \in V \times V$, such that $0 \leq \mu_{ij} + \nu_{ij} \leq 1$. That is, μ_{ij} and ν_{ij} are the degrees of membership and non-membership of i^{th} vertex in j^{th} edge.
Note: The support of an IFS V in E is denoted by $supp(E_j) = \{v_i/\mu_{ij} > 0 \text{ and } \nu_{ij} > 0\}$.

Definition 2.5. [11] An *intuitionistic fuzzy directed hypergraph* (IFDHG) H is a pair (V, E), where V is a non empty set of vertices and E is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $E_i \in E$ is defined as a pair $(t(E_i), h(E_i))$, where $t(E_i) \subset V$, with $t(E_i) \neq \emptyset$, is its tail, and $h(E_i) \in V - t(E_i)$ is its head. A vertex s is said to be a *source vertex* in H if $h(E_i) \neq s$, for every $E_i \in E$. A vertex d is said to be a *destination vertex* in H if $d \neq t(E_i)$, for every $E_i \in E$.

Definition 2.6. [16] Let $H = (V, E)$ be an intuitionistic fuzzy directed hypergraph. Suppose $E_j, E_k \in E$ and $0 < \alpha \leq 1, 0 < \beta \leq 1$. The (α, β)-*level* is defined by

$$(E_j, E_k)^{(\alpha,\beta)} = \Big\{v_i \in V/ \max(\mu^{\alpha}_{ij}(v_i) \geq \alpha, \min(\nu^{\beta}_{ij}(v_i) \leq \beta\Big\} \tag{1}$$

Definition 2.7. [16] Let $H = (V, E)$ be an intuitionistic fuzzy directed hypergraph, for $0 < (r_i, s_i) \leq h(H)$, let $H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})$ be the (r_i, s_i) - level intuitionistic fuzzy hypergraph of H. The sequence of real numbers $\{r_1, r_2, ..., r_n; s_1, s_2, ..., s_n\}$, such that $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$, satisfying the properties:
(i) If $r_1 < \alpha \leq 1$ and $0 \leq \beta < s_1$ then $E^{\alpha,\beta} = \emptyset$,
(ii) If $r_{i+1} \leq \alpha \leq r_i$; $s_i \leq \beta \leq s_{i+1}$ then $E^{\alpha,\beta} = E^{r_i,s_i}$,
(iii) $E^{r_i,s_i} \sqsubset E^{r_{i+1},s_{i+1}}$
is called the *fundamental sequence of* H, and is denoted by $F(H)$.
The core set of H is denoted by $C(H)$ and is defined by $C(H) = \{H^{r_1,s_1}, H^{r_2,s_2}, ..., H^{r_n,s_n}\}$. The corresponding set of (r_i, s_i) - level hypergraphs $H^{r_1,s_1} \subset H^{r_2,s_2} \subset .... \subset H^{r_n,s_n}$ is called the H *induced fundamental sequence* and is denoted by ***I(H)***. The (r_n, s_n) level is called the *support level* of H and the H^{r_n,s_n} is called the *support* of H.

Definition 2.8. [16] Let $H = (V, E)$ be an intuitionistic fuzzy directed hypergraph. An *intuitionistic fuzzy transversal* T of H is an intuitionistic fuzzy subset of V with the property that $T^{(E_j,E_k)} \bigcap A^{(E_j,E_k)} \neq \emptyset$ for each $A \in E$, where $E_j = \max(\mu_{ij})$ and $E_k = \min(\nu_{ij})$, for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$. Also μ_{ij} is the membership value of i^{th} vertex in j^{th} edge and ν_{ij} is the non-membership value of i^{th} vertex in j^{th} edge.

Definition 2.9. [16] A *minimal intuitionistic fuzzy transversal* T for H is a transversal of H with the property that if $T_1 \subset T$, then T_1 is not an intuitionistic fuzzy transversal of H.

3 Operations on transversals of IFDHG

Proposition 3.1. Let E be the fixed set and $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle | v_i \in V\}$ be an IFS. Let $V_1, V_2, \cdots, V_n$ be n subsets of V over E. Then the following six cartesian products of intuitionistic fuzzy sets are:

$$(i) V_1 \times_1 V_2 \times_1 V_3 ... \times_1 V_n = \{\langle (v_1, v_2, \cdots, v_n), \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i\rangle \\ | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\},$$

$$(ii) V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} ... \times_2 V_{i_n} = \{\langle (v_1, v_2, \cdots, v_n), \sum_{i=1}^{n} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \\ \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \cdots \neq n} \mu_i \mu_j \mu_k \cdots \mu_n + \\ (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$$

$$(iii) V_{i_1} \times_3 V_{i_2} \times_3 V_{i_3} ... \times_3 V_{i_n} = \{\langle (v_1, v_2, \cdots, v_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \nu_i - \sum_{i \neq j} \nu_i \nu_j + \\ \sum_{i \neq j \neq k} \nu_i \nu_j \nu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \cdots \neq n} \nu_i \nu_j \nu_k \cdots \nu_n + \\ (-1)^{n-1} \prod_{i=1}^{n} \nu_i\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$$

$$(iv) V_1 \times_4 V_2 \times_4 V_3 ... \times_4 V_n = \{\langle (v_1, v_2, \cdots, v_n), \min(\mu_1, \mu_2, \cdots, \mu_n), \\ \max(\nu_1, \nu_2, \cdots, \nu_n)\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$$

$$(v) V_1 \times_5 V_2 \times_5 V_3 ... \times_5 V_n = \{\langle (v_1, v_2, \cdots, v_n), \max(\mu_1, \mu_2, \cdots, \mu_n), \\ \min(\nu_1, \nu_2, \cdots, \nu_n)\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}.$$

$$(vi) V_1 \times_6 V_2 \times_6 V_3 ... \times_6 V_n = \{\langle (v_1, v_2, \cdots, v_n), \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \nu_i}{n}\rangle \\ | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}.$$

Proof:
(i) Claim:$V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} ... \times_2 V_{i_n} = \{\langle (v_1, v_2, \cdots, v_n), \sum_{i=1}^{n} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \cdots \neq n} \mu_i \mu_j \mu_k \cdots \mu_n + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$ is an intuitionistic fuzzy set.
When $n = 2$, the proof is obvious.
When $n = 3, V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} = \{\langle (v_1, v_2, v_3), \sum_{i=1}^{3} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k, \prod_{i=1}^{3} \nu_i\rangle | v_1 \in V_1, v_2 \in V_2, v_3 \in V_3$. The proposition is true for $n = 3$. Assume that the proposition is true for $n = m - 1$. Therefore, for $n = m$, $(V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} \cdots \times_2 V_{i_{m-1}}) \times_2 V_{i_m} = \{\langle (v_1, v_2, \cdots, v_{m-1}, v_m), \sum_{i=1}^{m} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{m-2} \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k + (-1)^{m-1} \prod_{i=1}^{m} \mu_i, \prod_{i=1}^{m} \nu_i\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_m \in V_m\}$.
Obviously $\sum_{i=1}^{n} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-1} \prod_{i=1}^{n} \mu_i + \prod_{i=1}^{n} \nu_i \geq 0$.
Now to prove $\sum_{i=1}^{n} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-1} \prod_{i=1}^{n} \mu_i + \prod_{i=1}^{n} \nu_i \leq 1$.

For $n = 2$,

$$\begin{aligned} 0 &\leq \nu_1.\nu_2 \\ &\leq \mu_1 + \mu_2 - \mu_1.\mu_2 + \nu_1.\nu_2, \text{ since } \mu_1, \mu_2 \in [0,1] \\ &\leq \mu_1 + \mu_2 - \mu_1.\mu_2 + (1-\mu_1)(1-\mu_2) \\ &\leq \mu_1 + \mu_2 - \mu_1.\mu_2 + 1 - \mu_1 - \mu_2 + \mu_1.\mu_2 \\ &= 1. \end{aligned}$$

For $n = 3$,

$$\begin{aligned} 0 &\leq \nu_1.\nu_2\nu_3 \\ &\leq \mu_1 + \mu_2 + \mu_3 - \mu_1.\mu_2 - \mu_2.\mu_3 - \mu_3.\mu_1 + \mu_1.\mu_2.\mu_3 - \mu_1\mu_2\mu_3 + \nu_1.\nu_2.\nu_3, \\ &\quad \text{since } \mu_i \in [0,1],\ i = 1,2,3. \\ &\leq \mu_1 + \mu_2 + \mu_3 - \mu_1.\mu_2 - \mu_2.\mu_3 - \mu_3.\mu_1 + \mu_1.\mu_2.\mu_3 - \mu_1\mu_2\mu_3 + (1-\mu_1)(1-\mu_2)(1-\mu_3) \\ &\leq \mu_1 + \mu_2 + \mu_3 - \mu_1.\mu_2 - \mu_2.\mu_3 - \mu_3.\mu_1 + \mu_1.\mu_2.\mu_3 - \mu_1\mu_2\mu_3 + 1 - \mu_2 - \mu_1 + \mu_1\mu_2 - \mu_3 \\ &+ \mu_2\mu_3 + \mu_1\mu_3 - \mu_1.\mu_2.\mu_3 \\ &= 1. \end{aligned}$$

Assume that the result holdsgood for $n = m - 1$.

$$\begin{aligned} 0 &\leq \nu_1.\nu_2 \cdots \nu_{m-1} \\ &\leq \sum_{i=1}^{m-1} \mu_i - \sum_{i \neq j} \mu_i.\mu_j + \sum_{i \neq j \neq k} \mu_i\mu_j\mu_k - \cdots + (-1)^{(m-2)} \sum_{i \neq j \neq \cdots \neq m-1} \mu_{i_1}\mu_{i_2} \cdots \mu_{i_{m-1}} + \prod_{i=1}^{m-1} \nu_i \\ &= 1. \end{aligned}$$

Therefore for $n = m$,

$$\begin{aligned} 0 &\leq \prod_{i=1}^{m} \nu_i \\ &\leq \nu_1.\nu_2 \cdots \nu_m \\ &\leq \sum_{i=1}^{m} \mu_i - \sum_{i \neq j} \mu_i.\mu_j + \sum_{i \neq j \neq k} \mu_i\mu_j\mu_k - \cdots + (-1)^{(m-1)} \sum_{i \neq j \neq \cdots \neq m} \mu_{i_1}\mu_{i_2} \cdots \mu_{i_m} + \prod_{i=1}^{m} \nu_i \\ &= 1. \end{aligned}$$

Hence the result is true for all n.

The other results can be proved in a similar way. □

Throughout this chapter the following notations were considered.
Let $H_1 = (V_1, E_1, \langle \mu_{t_i}, \nu_{t_i} \rangle, \langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle)$ and $H_2 = (V_2, E_2, \langle \mu_{t'_i}, \nu_{t'_i} \rangle, \langle \mu_{t'_{ij}}, \nu_{t'_{ij}} \rangle)$ be two IFDHGs.
Then T_1 and T_2 be the transversals of H_1 and H_2 respectively.

Definition 3.1. The *union* of T_1 and T_2, denoted by $T_1 \cup T_2$, is defined as

$$T = T_1 \cup T_2 = \{V_1 \cup V_2, E_1 \cup E_2, \langle \mu_{t_r} = \mu_{t_i \cup t'_i}, \nu_{t_r} = \nu_{t_i \cup t'_i} \rangle, \langle \mu_{t_{rs}} = \mu_{t_{ij} \cup t'_{ij}}, \nu_{t_{rs}} = \nu_{t_{ij} \cup t'_{ij}} \rangle\}$$

$$\langle \mu_{t_r}, \nu_{t_r} \rangle = \begin{cases} \langle \mu_{t_i}, \nu_{t_i} \rangle & \text{if} \quad v \in V_1 - V_2 \\ \langle \mu_{t'_i}, \nu_{t'_i} \rangle & \text{if} \quad v \in V_2 - V_1 \\ \langle \max(\mu_{t_i}, \mu_{t'_i}), \min(\nu_{t_i}, \nu_{t'_i}) \rangle & \text{if} \quad v \in V_1 \cap V_2 \end{cases}$$

$$\langle \mu_{t_{rs}}, \nu_{t_{rs}} \rangle = \begin{cases} \langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle & \text{if} \quad e_{ij} \in E_1 - E_2 \\ \langle \mu_{t'_{ij}}, \nu_{t'_{ij}} \rangle & \text{if} \quad e_{ij} \in E_2 - E_1 \\ \langle \max(\mu_{t_{ij}}, \mu_{t'_{ij}}), \min(\nu_{t_{ij}}, \nu_{t'_{ij}}) \rangle & \text{if} \quad e_{ij} \in E_1 \cap E_2 \\ \langle 0, 1 \rangle & \text{otherwise} \end{cases}$$

Example 1. Consider an IFDHGs, $H_1 = (V_1, E_1)$ and $H_2 = (V_2, E_2)$. Its adjacency matrix is given by

$$H_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cc} E_1 & E_2 \end{array} \\ \begin{pmatrix} \langle 0.4, 0.6 \rangle & \langle 0, 1 \rangle \\ \langle 0.6, 0.2 \rangle & \langle 0.6, 0.2 \rangle \\ \langle 0.3, 0.3 \rangle & \langle 0.3, 0.3 \rangle \\ \langle 0, 1 \rangle & \langle 0.5, 0.4 \rangle \end{pmatrix} \end{array}$$

and

$$H_2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{pmatrix} \langle 0.5, 0.4 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0.3, 0.5 \rangle & \langle 0.3, 0.5 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.4, 0.3 \rangle & \langle 0.4, 0.3 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.6, 0.1 \rangle \end{pmatrix} \end{array}$$

The corresponding graph is shown in Figure 1.

$$Tr(H_1) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \\ \begin{pmatrix} \langle 0.5, 0.4 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.6, 0.2 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.4, 0.3 \rangle \\ \langle 0.6, 0.1 \rangle & \langle 0.6, 0.1 \rangle & \langle 0.6, 0.1 \rangle \end{pmatrix} \end{array}$$

and

$$Tr(H_2) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cc} T_1 & T_2 \end{array} \\ \begin{pmatrix} \langle 0, 1 \rangle & \langle 0.5, 0.4 \rangle \\ \langle 0.6, 0.2 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.4, 0.3 \rangle \\ \langle 0.6, 0.1 \rangle & \langle 0.6, 0.1 \rangle \end{pmatrix} \end{array}$$

The corresponding graph is shown in Figure 2.

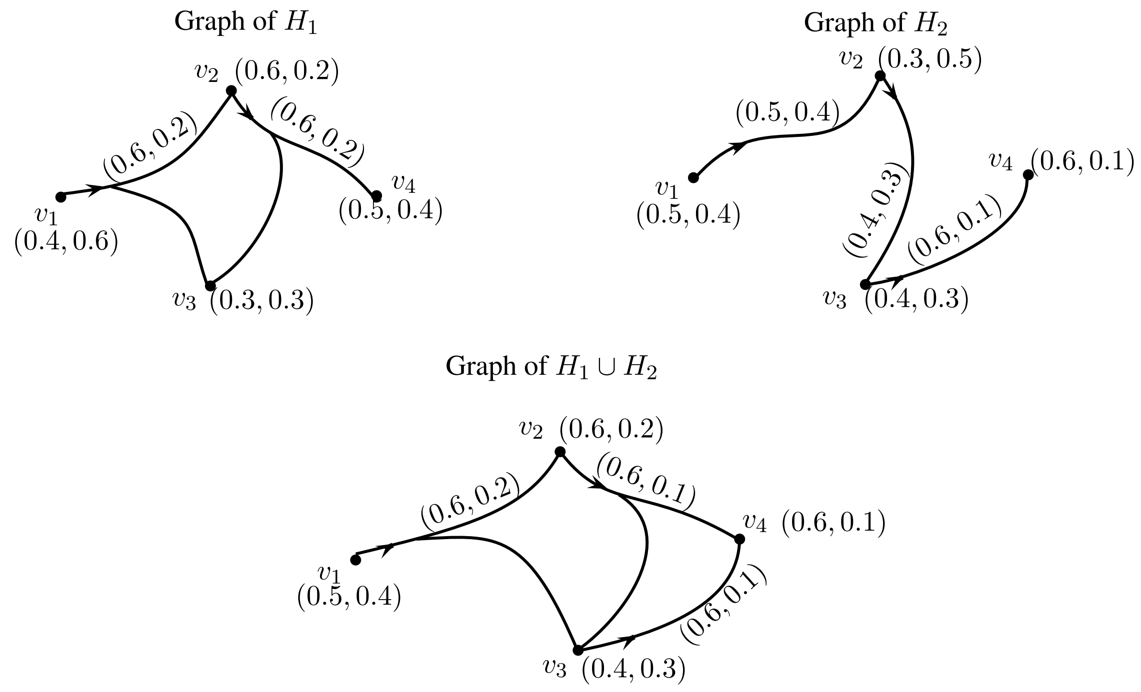

Figure 1: Union of IFDHG.

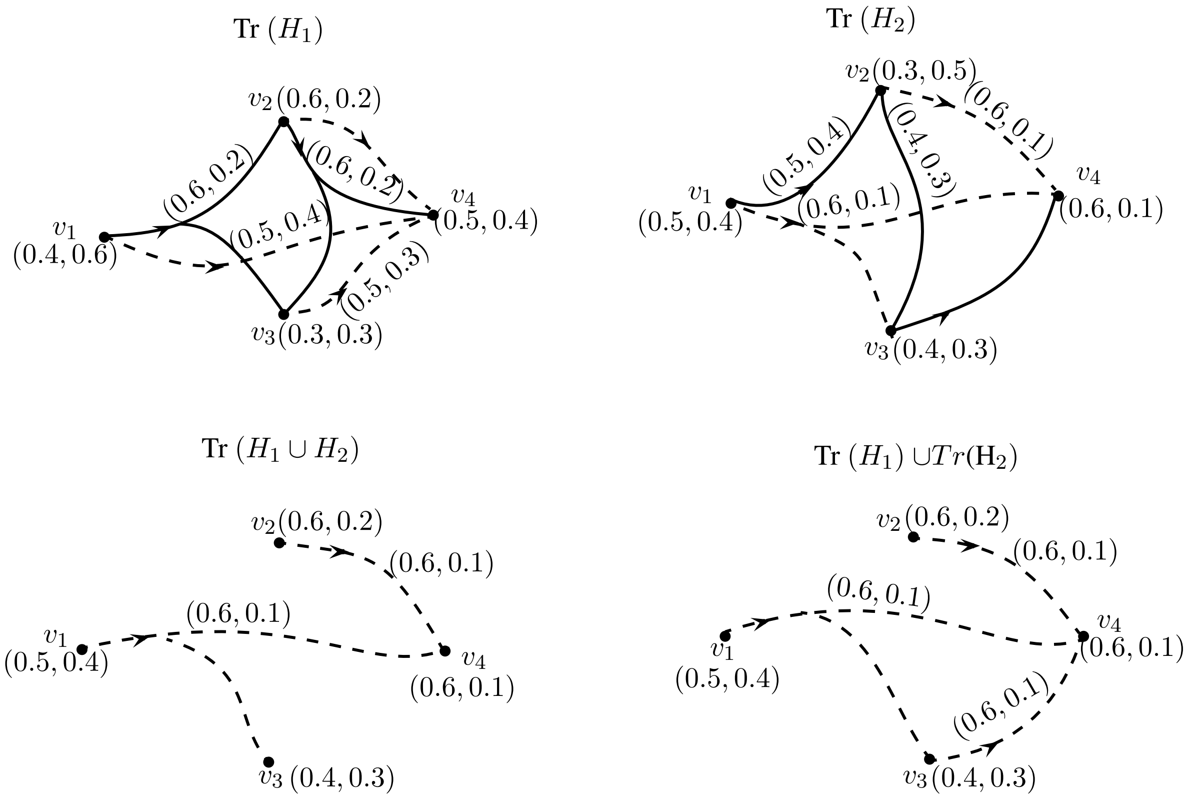

Figure 2: Transversals of Union of IFDHG.

Definition 3.2. The *intersection* of T_1 and T_2, denoted by $T_1 \cap T_2$, is defined as

$$T = T_1 \cap T_2 = \{V_1 \cap V_2, E_1 \cap E_2, \langle \mu_{t_r} = \mu_{t_i \cap t'_i}, \nu_{t_r} = \nu_{t_i \cap t'_i} \rangle, \langle \mu_{t_{rs}} = \mu_{t_{ij} \cap t'_{ij}}, \nu_{t_{rs}} = \nu_{t_{ij} \cap t'_{ij}} \rangle\}$$

and defined by

$$\langle \mu_{t_r}, \nu_{t_r} \rangle = \begin{cases} \langle \mu_{t_i}, \nu_{t_i} \rangle & \text{if} \quad v \in V_1 - V_2 \\ \langle \mu_{t'_i}, \nu_{t'_i} \rangle & \text{if} \quad v \in V_2 - V_1 \\ \langle \min(\mu_{t_i}, \mu_{t'_i}), \max(\nu_{t_i}, \nu_{t'_i}) \rangle & \text{if} \quad v \in V_1 \cap V_2 \end{cases}$$

$$\langle \mu_{t_{rs}}, \nu_{t_{rs}} \rangle = \begin{cases} \langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle & \text{if} \quad e_{ij} \in E_1 - E_2 \\ \langle \mu_{t'_{ij}}, \nu_{t'_{ij}} \rangle & \text{if} \quad e_{ij} \in E_2 - E_1 \\ \langle \min(\mu_{t_{ij}}, \mu_{t'_{ij}}), \max(\nu_{t_{ij}}, \nu_{t'_{ij}}) \rangle & \text{if} \quad e_{ij} \in E_1 \cap E_2 \\ \langle 0, 1 \rangle & \text{otherwise} \end{cases}$$

Definition 3.3. The *join* of T_1 and T_2, denoted by $T_1 + T_2$, is defined as $T = T_1 + T_2 = \{V_1 \cup V_2, E_1 \cup E_2 \cup E', \langle \mu_{t_i + t'_i}, \nu_{t_i + t'_i} \rangle, \langle \mu_{t_{ij} + t'_{ij}}, \nu_{t_{ij} + t'_{ij}} \rangle\}$ and defined by

$$(\mu_{t_i + t'_i})(v) = (\mu_{t_i} \vee \mu_{t'_i})(v) \; if \; v \in V_1 \cup V_2$$

$$(\nu_{t_i + t'_i})(v) = (\nu_{t_i} \wedge \nu_{t'_i})(v) \; if \; v \in V_1 \cup V_2$$

$$(\mu_{t_{ij} + t'_{ij}})(v_i v_j) = (\mu_{t_{ij}} \vee \mu_{t'_{ij}})(v_i v_j) \; if \; v_i v_j \in E_1 \cup E_2$$

$$= (\mu_{t_{ij}}(v_i) \cdot \mu_{t'_{ij}}(v_j)) \; if \; v_i v_j \in E'.$$

$$(\nu_{t_{ij} + t'_{ij}})(v_i v_j) = (\nu_{t_{ij}} \wedge \nu_{t'_{ij}})(v_i v_j) \; if \; v_i v_j \in E_1 \cup E_2$$

$$= (\nu_{t_{ij}}(v_i) \cdot \nu_{t'_{ij}}(v_j)) \; if \; v_i v_j \in E'.$$

Definition 3.4. The *structural subtraction* of T_1 and T_2, denoted by $T_1 \ominus T_2$, is defined as $T = T_1 \ominus T_2 = \{V_1 - V_2, \langle \mu_{t_r}, \nu_{t_r} \rangle, \langle \mu_{t_{rs}}, \nu_{t_{rs}} \rangle\}$ where ‘ $-$ ‘ is the set theoretical difference operation and

$$\langle \mu_{t_r}, \nu_{t_r} \rangle = \begin{cases} \langle \mu_{t_i}, \nu_{t_i} \rangle & \text{if} \quad v_i \in V_1 \\ \langle \mu_{t_j}, \nu_{t_j} \rangle & \text{if} \quad v_j \in V_2 \\ \langle 0, 1 \rangle & \text{otherwise.} \end{cases}$$

$$\langle \mu_{t_{rs}}, \nu_{t_{rs}} \rangle = \begin{cases} \langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle & \text{for} \quad v_r = v_i \in V_1 - V_2 \\ & \qquad v_s = v_j \in V_1 - V_2 \end{cases}$$

where $V_1 - V_2 = \emptyset$.

Definition 3.5. The *cartesian product* of T_1 and T_2, denoted by $T_1 \times T_2$, is defined as $T = T_1 \times T_2 = (V, E')$ where $V = V_1 \times V_2$ and $E' = \{(u, u_j), (u, v_j) : u \in V_1, u_j v_j \in E_2\} \cup \{(u_i, w)(v_i, w) : w \in v_2, u_i v_j \in E_1\}$. Then,

$(\mu_{t_i \times t'_i})(u_i, u_j) = (\mu_{t_i} . \mu_{t'_i})(u_j)$ for every (u_i, u_j)in V and

$(\nu_{t_i \times t'_i})(u_i, u_j) = (\nu_{t_i}.\nu_{t'_i})(u_j)$ for every (u_i, u_j)in V.

$(\mu_{t_{ij} \times t'_{ij}})(u, u_j), (u, v_j) = (\mu_{t_i}(u).\mu_{t_{ij}})(v_i v_j)$ for every $u \in V_1$ and $u_j v_j \in E_2$

$(\nu_{t_{ij} \times t'_{ij}})(u, u_j), (u, v_j) = (\nu_{t_i}(u).\nu_{t_{ij}})(v_i v_j)$ for every $u \in V_1$ and $u_j v_j \in E_2$

$(\mu_{t_{ij} \times t'_{ij}})(u_i, w), (v_i, w)) = (\mu_{t_i}(w).\mu_{t_{ij}})(u_i v_i)$ for every $w \in V_2$ and $u_i v_i \in E_1$

$(\nu_{t_{ij} \times t'_{ij}})(u_i, w), (v_i, w)) = (\nu_{t_i}(w).\nu_{t_{ij}})(u_i v_i)$ for every $w \in V_2$ and $u_i v_i \in E_1$

Definition 3.6. The *composition* of T_1 and T_2, denoted by $T_1 \circ T_2$, is defined as $T = T_1 \circ T_2 = (V_1 \times V_2, E)$ where $V = V_1 \times V_2$ and $E = \{(u, u_j), (u, v_j) : u \in V_1, u_j v_j \in E_2\} \cup \{(u_i, w)(v_i, w) : w \in v_2, u_i v_i \in E_1\} \cup \{(u_i, u_j)(v_i, v_j) : u_i v_i \in E_1, u_j \neq v_j\}$. Then,

$(\mu_{t_i \circ t'_i})(u_i, u_j) = (\mu_{t_i}.\mu_{t'_i})(u_j)$ for every (u_i, u_j)in $V_1 \times V_2$ and

$(\nu_{t_i \circ t'_i})(u_i, u_j) = (\nu_{t_i}.\nu_{t'_i})(u_j)$ for every (u_i, u_j)in $V_1 \times V_2$.

$(\mu_{t_{ij} \circ t'_{ij}})(u, u_j), (u, v_j) = (\mu_{t_i}(u).\mu_{t_{ij}})(u_j v_j)$ for every $u \in V_1$ and $u_j v_j \in E_2$

$(\nu_{t_{ij} \circ t'_{ij}})(u, u_j), (u, v_j) = (\nu_{t_i}(u).\nu_{t_{ij}})(u_j v_j)$ for every $u \in V_1$ and $u_j v_j \in E_2$

$(\mu_{t_{ij} \circ t'_{ij}})(u_i, w), (v_i, w)) = (\mu_{t_i}(w).\mu_{t_{ij}})(u_i v_i)$ for every $w \in V_2$ and $u_i v_i \in E_1$

$(\nu_{t_{ij} \circ t'_{ij}})(u_i, w), (v_i, w)) = (\nu_{t_i}(w).\nu_{t_{ij}})(u_i v_i)$ for every $w \in V_2$ and $u_i v_i \in E_1$

$(\mu_{t_{ij} \circ t'_{ij}})(u_i, u_j), (v_i, v_j)) = (\mu_{t'_i}(u_j).\mu_{t'_i}(v_j).\mu_{t_{ij}})(u_i v_i)$ for every $(u_i, u_j), (v_i, v_j) \in E - E'$

$(\nu_{t_{ij} \circ t'_{ij}})(u_i, u_j), (v_i, v_j)) = (\nu_{t'_i}(u_j).\nu_{t'_i}(v_j).\nu_{t_{ij}})(u_i v_i)$ for every $(u_i, u_j), (v_i, v_j) \in E - E'$

where $E' = \{(u, u_j), (u, v_j) : u \in V_1, u_j v_j \in E_2\} \cup \{(u_i, w)(v_i, w) : w \in v_2, u_i v_j \in E_1\}$.

4 Properties of TIFDHGs

Theorem 4.1. $Tr(H_1 \cup H_2) \subseteq Tr(H_1) \cup Tr(H_2)$. That is, union of IFTs of H_1 and H_2 contains the IFT of union of H_1 and H_2.
Proof: Let $H_1 = (V_1, E_1, \langle \mu_i, \nu_i \rangle, \langle \mu_{ij}, \nu_{ij} \rangle)$ and $H_2 = (V_2, E_2, \langle \mu_p, \nu_p \rangle, \langle \mu_{pq}, \nu_{pq} \rangle)$ be two IFDHGs with $i, j = 1, 2, ..., m$ and $p, q = 1, 2, ..., n$ vertices respectively.

Then, $H_1 \cup H_2 = (V_1 \cup V_2, E_1 \cup E_2, \langle \mu_{i \cup p}, \nu_{i \cup p} \rangle, \langle \mu_{(ij) \cup (pq)}, \nu_{(ij) \cup (pq)} \rangle)$.

Hence, $Tr(H_1 \cup H_2) = (V_T, E_T, \langle \mu_t, \nu_t \rangle, \langle \mu_{t'}, \nu_{t'} \rangle)$ where

$$\left.\begin{aligned} V_T &= V_1 \cup V_2 - \{v_k\}_k, k < m \\ E_T &= E_1 \cup E_2 - \{e_{pq}\}_{p,q}, p, q < n \end{aligned}\right\}, \tag{2}$$

$$\langle \mu_t, \nu_t \rangle = \langle max(\mu_i, \mu_p), min(\nu_i, \nu_p) \rangle \text{ and}$$
$$\langle \mu_{t'}, \nu_{t'} \rangle = \langle max(\mu_{ij}, \mu_{pq}), min(\nu_{ij}, \nu_{pq}) \rangle$$

Also, $Tr(H_1) = (V_{T_1}, E_{T_1}, \langle \mu_{t_i}, \nu_{t_i} \rangle, \langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle)$ and $Tr(H_2) = (V_{T_2}, E_{T_2}, \langle \mu_{t_p}, \nu_{t_p} \rangle, \langle \mu_{t_{pq}}, \nu_{t_{pq}} \rangle)$ where

$$V_{T_1} = V_1 - \{v_a\}_a, a < m,$$
$$E_{T_1} = E_1 - \{e_{ls}\}_{l,s}, l, s < n,$$
$$\langle \mu_{t_i}, \nu_{t_i} \rangle = \langle max(\mu_i), min(\nu_i) \rangle \text{ and}$$
$$\langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle = \langle max(\mu_{ij}), min(\nu_{ij}) \rangle$$

and

$$V_{T_2} = V_2 - \{v_b\}_b, b < m,$$
$$E_{T_2} = E_2 - \{e_{uv}\}_{u,v}, u, v < n,$$
$$\langle \mu_{t_p}, \nu_{t_p} \rangle = \langle max(\mu_p), min(\nu_p) \rangle \text{ and}$$
$$\langle \mu_{t_{pq}}, \nu_{t_{pq}} \rangle = \langle max(\mu_{pq}), min(\nu_{pq}) \rangle$$

Therefore, $Tr(H_1) \cup Tr(H_2) = (V_{T_1} \cup V_{T_2}, E_{T_1} \cup E_{T_2}, \langle \mu_x, \nu_x \rangle, \langle \mu_y, \nu_y \rangle$ where

$$\left.\begin{aligned} V_{T_1} \cup V_{T_2} &= V_1 \cup V_2 - \{v_a\}_a - \{v_b\}_b \\ E_{T_1} \cup E_{T_2} &= E_1 \cup E_2 - \{e_{ls}\}_{l,s} - \{e_{uv}\}_{u,v} \end{aligned}\right\}, \tag{3}$$

$$\langle \mu_x, \nu_x \rangle = \langle max(\mu_i, \mu_p), min(\nu_i, \nu_p) \rangle \text{ and}$$
$$\langle \mu_y, \nu_y \rangle = \langle max(\mu_{ij}, \mu_{pq}), min(\nu_{ij}, \nu_{pq}) \rangle$$

such that

a and b take at least one value of k
l and s take at least one value of p and
u and v take at least one value of q.

From (2) and (3), it is clear that

$$V_T \subseteq V_{T_1} \cup V_{T_2}$$
$$E_T \subseteq E_{T_1} \cup E_{T_2}$$

Hence $Tr(H_1 \cup H_2) \subseteq Tr(H_1) \cup Tr(H_2)$. □

Note: Similarly, the following properties can also be verified.
(i) $Tr(H_1 \cap H_2) \supseteq Tr(H_1) \cap Tr(H_2)$
Every IFT of intersection of H_1 and H_2 contains intersection of IFTs of H_1 and H_2.
(ii) $Tr(H_1 \ominus H_2) \supseteq Tr(H_1) \ominus Tr(H_2)$

Theorem 4.2. Transversals of addition of two IFDHGs is always a null IFHG. That is $Tr(H_1 + H_2) = \emptyset$.

Corollary: $Tr(H_1 + H_2) \neq Tr(H_1) + Tr(H_2)$.

Proof: Let $H_1 = (V_1, E_1, \langle \mu_i, \nu_i \rangle, \langle \mu_{ij}, \nu_{ij} \rangle)$ and $H_2 = (V_2, E_2, \langle \mu_p, \nu_p \rangle, \langle \mu_{pq}, \nu_{pq} \rangle)$ be two IFDHGs with $i, j = 1, 2, ..., m$ and $p, q = 1, 2, ..., n$ vertices respectively.
Then, $H_1 + H_2 = (V_1 \cup V_2, E_1 \cup E_2 \cup E', \langle \mu_{i+p}, \nu_{i+p} \rangle, \langle \mu_{ij+pq}, \nu_{ij+pq} \rangle)$.
Hence, by Defnition 3.3 and Theorem 4.2, $Tr(H_1 + H_2) = (V_T, E_T, \langle \mu_t, \nu_t \rangle, \langle \mu_{t'}, \nu_{t'} \rangle)$ where

$$\left.\begin{aligned} V_T = V_1 \cup V_2 - \{v_k\}_k, k < n \\ E_T = \emptyset \end{aligned}\right\}, \tag{4}$$

$$\langle \mu_t, \nu_t \rangle = \langle max(\mu_i, \mu_p), min(\nu_i, \nu_p) \rangle, \text{ and}$$
$$\langle \mu_{t'}, \nu_{t'} \rangle = \langle 0, 0 \rangle$$

Also, $Tr(H_1) = (V_{T_1}, E_{T_1}, \langle \mu_{t_i}, \nu_{t_i} \rangle, \langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle)$ and $Tr(H_2) = (V_{T_2}, E_{T_2}, \langle \mu_{t_p}, \nu_{t_p} \rangle, \langle \mu_{t_{pq}}, \nu_{t_{pq}} \rangle)$ where

$$V_{T_1} = V_1 - \{v_a\}_a, a < m,$$
$$E_{T_1} = E_1 - \{e_{ls}\}_{l,s}, l, s < n,$$
$$\langle \mu_{t_i}, \nu_{t_i} \rangle = \langle max(\mu_i), min(\nu_i) \rangle, \text{ and}$$
$$\langle \mu_{t_{ij}}, \nu_{t_{ij}} \rangle = \langle max(\mu_{ij}), min(\nu_{ij}) \rangle$$

and

$$V_{T_2} = V_2 - \{v_b\}_b, b < m,$$
$$E_{T_2} = E_2 - \{e_{uv}\}_{u,v}, u, v < n,$$
$$\langle \mu_{t_p}, \nu_{t_p} \rangle = \langle max(\mu_p), min(\nu_p) \rangle, \text{ and}$$
$$\langle \mu_{t_{pq}}, \nu_{t_{pq}} \rangle = \langle max(\mu_{pq}), min(\nu_{pq}) \rangle$$

Therefore, $Tr(H_1) + Tr(H_2) = (V_{T_1} \cup V_{T_2}, E_{T_1} \cup E_{T_2} \cup E'_T, \langle \mu_x, \nu_x \rangle, \langle \mu_y, \nu_y \rangle$ where

$$\left.\begin{aligned} V_{T_1} \cup V_{T_2} = V_1 \cup V_2 - \{v_a\}_a - \{v_b\}_b \\ E_{T_1} \cup E_{T_2} \cup E'_T \neq \emptyset \end{aligned}\right\}, \tag{5}$$

$$\langle \mu_x, \nu_x \rangle = \langle max(\mu_i, \mu_p), min(\nu_i, \nu_p) \rangle, \text{ and}$$
$$\langle \mu_y, \nu_y \rangle = \langle max(\mu_{ij}, \mu_{pq}), min(\nu_{ij}, \nu_{pq}) \rangle$$

such that a and b assume at least one value of k .

From (4) and (5),

$$V_T \subseteq V_{T_1} \cup V_{T_2}$$
$$E_T \neq E_{T_1} \cup E_{T_2} \cup E'_T$$

Hence $Tr(H_1 + H_2) \neq Tr(H_1) + Tr(H_2)$. This completes the proof. □

Note: Similarly, the following properties can also be verified.
(i) $Tr(H_1 \otimes H_2) \neq Tr(H_1) \otimes Tr(H_2)$.
(ii) $Tr(H_1 \circ H_2) \neq Tr(H_1) \circ Tr(H_2)$.
(iii) $Tr(H^c) = (Tr(H))^c$.

5 Conclusion

In this paper, the operations on TIFDHG are defined and discussed. Also, some interesting properties like union, intersection, addition, structural subtraction, multiplication and complement are dealt with. There is abundant scope for future research on this topic. Further, the authors proposed to work on truncation of an IFDHG and its applications in coloring of intuitionistic fuzzy hypergraphs.

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