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Short remark on interval type-2 fuzzy sets and intuitionistic fuzzy sets

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Abstract: Short comparison between concepts of interval type-2 fuzzy sets and intuitionistic fuzzy sets is given.
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1 Introduction

In this short remark a comparison between two of the extensions of the Zadeh's fuzzy sets [1, 2] is given. The first extension of the concept of a fuzzy set is the *L*-fuzzy set [3] of J. Goguen. Some years later Z. Pawlak introduced the concept of rough set [4]. The next two extensions are interval type-2 fuzzy sets [5, 6] and intuitionistic fuzzy sets [7,8]. The first two authors work intensively on the first of the later concepts (see, e.g. [9,10]), while the second two – on the second one of these fuzzy set extensions (see [11,12,13]). The present remark is the first attempt to discuss both concepts in parallel.

2 Main results

First, following [B4], we mention that if for a type-1 membership function, as in Fig. 1, we blur it to the left and to the right, as illustrated in Fig. 2, then a type-2 membership function is obtained. In this case, for a specific value x', the membership function (u'), takes on different values, which are not all weighted the same, so we can assign an amplitude distribution to all of those points.

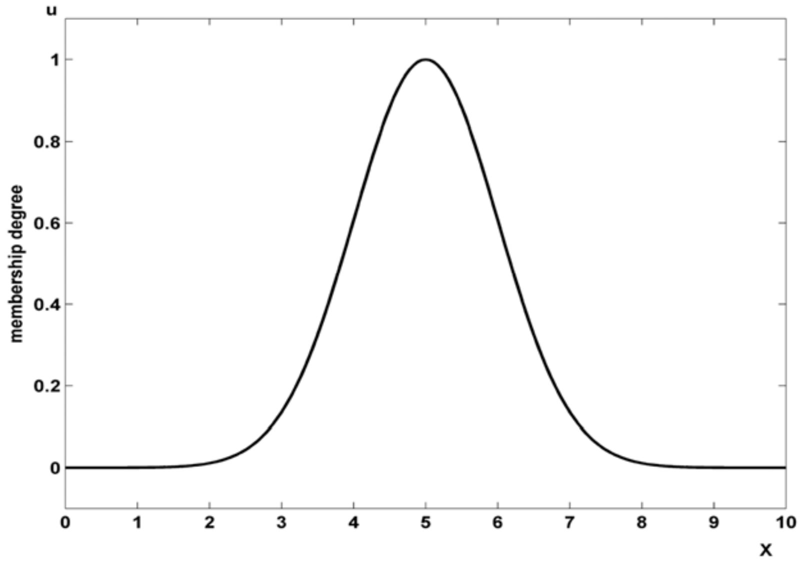

Figure 1. Type-1 membership function

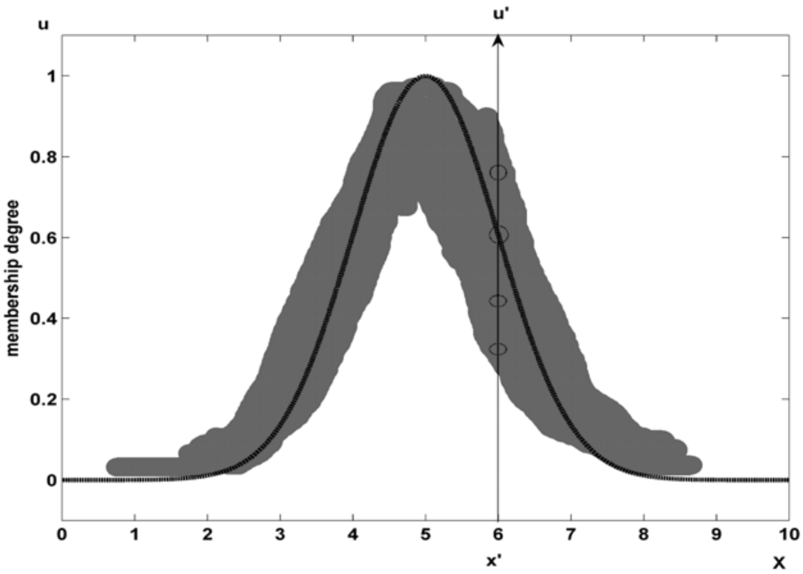

Figure 2. Blurred type-1 membership function

A type-2 fuzzy set $\tilde{A}$, is characterized by the membership function [15, 16]:

$$\tilde{A} = \left\{ \left((x,u), \mu_{\tilde{A}}(x,u) \right) \mid x \in X, u \in J_x \subseteq [0,1] \right\} \tag{1}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. Another expression for $\tilde{A}$ is

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \tag{2}$$

for $J_x \subseteq [0,\ 1]$, where $\int\int$ denotes the union over all admissible input variables *x* and *u*. For discrete universes of discourse $\int$ is replaced by $\sum$, [17].

In the same case, the intuitionistic fuzzy set $\tilde{A}$, is characterized by the membership and non-membership function and has the form:

$$\tilde{A} = \left\{ \left((x,u), \mu_{\tilde{A}}(x,u), \nu_{\tilde{A}}(x,u) \right) \mid x \in X, u \in J_x \subseteq [0,1] \right\} \tag{3}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$, $0 \le \nu_{\tilde{A}}(x,u) \le 1$, $0 \le \mu_{\tilde{A}}(x,u) + \nu_{\tilde{A}}(x,u) \le 1$. The analogous of the second expression (2), now is:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u) \times \nu_{\tilde{A}}(x,u) / (x,u) \tag{4}$$

for $J_x \subseteq [0,\ 1]$, where, as above, $\int\int$ denotes the union over all admissible input variables *x* and *u*. For discrete universes of discourse $\int$ is replaced by $\sum$.

Let *X* be a universe, $g : \mathrm{X} \to [0,\ 1]$ be an interval function and $X_g = \bigcup_{x \in X} x \times g(x)$.

Let us consider the IFS

$$A^*_g = \{ \langle \mathrm{z}, \mu_{\tilde{A}}(\mathrm{z}), \nu_{\tilde{A}}(\mathrm{z}) \rangle \mid \mathrm{z} \in X_g \}.$$

Then any type-2 fuzzy set $\tilde{A}$ can be represented by IFS A^*_g with a suitable choice of *g*.

In fact $J_x \subseteq [0,\ 1]$ represents the primary membership of *x*, and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in [0, 1], i.e. the primary membership. Corresponding to each primary membership, there is a secondary membership (which can also be in [0, 1]) that defines the possibilities for the primary membership. Uncertainty is represented by a region, which is called the footprint of uncertainty (FOU).

When $\mu_{\tilde{A}}(x,u) = 1$, for every $u \in J_x \subseteq [0,\ 1]$ we have an interval type-2 membership function, as shown in Figure 3. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\bar{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

Let us consider IFS

$$A^* = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \},$$

where $\mu_A(x) = \underline{\mu}_{\tilde{A}}(x)$ and $\nu_A(x) = 1 - \bar{\mu}_{\tilde{A}}(x)$.

Then the thickness of interval type-2 membership function the second axis in each point *x* will be described by the function

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

If μ_A and ν_A are Riemann integrable functions on [*a*, *b*], where $\text{supp}(\overline{\mu}_{\tilde{A}}) = [a,b]$, then the area of the thickness of interval type-2 membership function is equal to $\int_a^b \pi_A(x)dx$.

In the general case when we have a monotonously measurable space (X, F, m) and $\pi_A(x)$ is measurable, then the nearest picture of the area of the thickness of interval type-2 membership function is given by Choquet integral

$$(C)\int_X \pi_A dm .$$

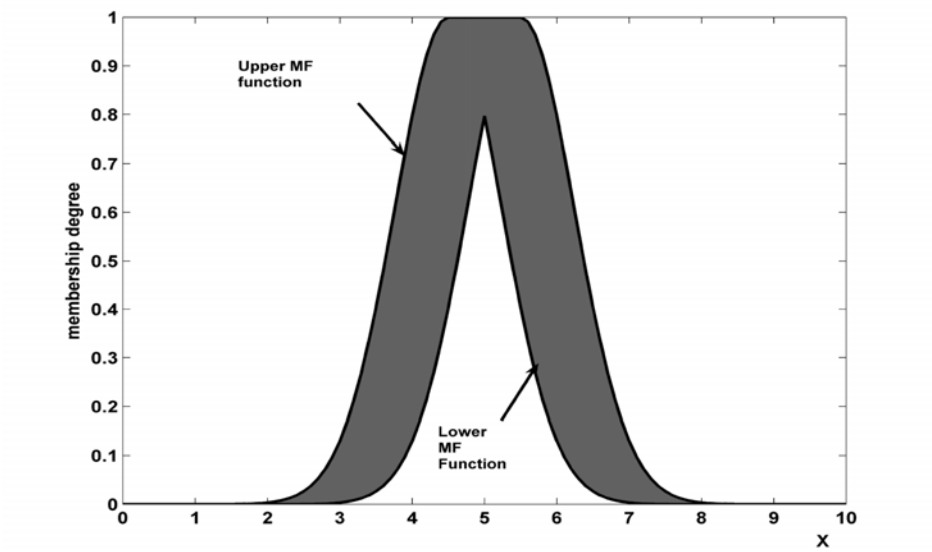

Figure 3. Interval type-2 membership function

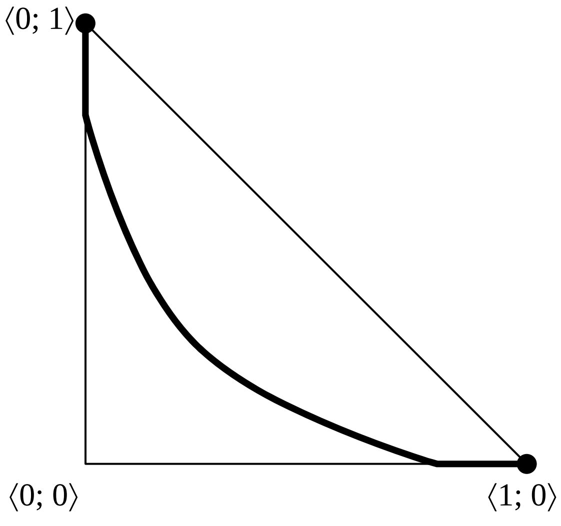

Figure 4. Intuitionistic fuzzy interpretation

In IFS theory there are different geometrical interpretations (see, e.g. [11, 12]). Another one is shown on Fig. 4, where the interval type-2 membership function from Fig. 3 now has an essentially different form. If we like to observe the development of the process of constructing the values of interval type-2 membership function in time, we can use a temporal IFS (see, e.g., [12]).

In conclusion, we mention that in future a detail comparison between both concepts will be prepared and relationships between the operations, relations and especially the operators will be compared.

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