Modifications of the weight-center operator, defined over intuitionistic fuzzy sets. Part 3

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Introduction

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

$$\begin{aligned}
A \subseteq B \quad & iff \quad (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)),\\
A \subset_{\Box} B \quad & iff \quad (\forall x \in E)(\mu_A(x) \leq \mu_B(x)),\\
A \subset_{\Diamond} B \quad & iff \quad (\forall x \in E)(\nu_A(x) \geq \nu_B(x)),\\
A = B \quad & iff \quad (\forall x \in E)(\mu_A(x) = \mu_B(x) \ \& \ \nu_A(x) = \nu_B(x)),\\
\overline{A} \quad & = \quad \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\},
\end{aligned}$$

$$
\begin{aligned}
C(A) &= \{\langle x, K, L\rangle | x \in E\}, \\
I(A) &= \{\langle x, k, l\rangle | x \in E\}, \\
C_\nu(A) &= \{\langle x, \mu_A(x), L\rangle | x \in E\}, \\
I_\mu(A) &= \{\langle x, k, \nu_A(x)\rangle | x \in E\},
\end{aligned}
$$

where

$$
K = \sup_{y \in E} \mu_A(y),
$$

$$
L = \inf_{y \in E} \nu_A(y),
$$

$$
k = \inf_{y \in E} \mu_A(y),
$$

$$
l = \sup_{y \in E} \nu_A(y).
$$

$$W(A) = \{\langle x, \frac{\sum\limits_{y\in E} \mu_A(y)}{card(E)}, \frac{\sum\limits_{y\in E} \nu_A(y)}{card(E)}\rangle | x \in E\}, \qquad (1)$$

where $card(E)$ is the number of the elements of a finite universe E.

The operators $H_{\alpha,\beta}$ and $J_{\alpha,\beta}$ are defined over IFS A by

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\}.$$

Obviously, for every IFS X over universe E,

$$H_{0,0}(X) = \{\langle x, 0, \nu_X(x)\rangle | x \in E\},$$

$$J_{0,0}(X) = \{\langle x, \mu_X(x), 0\rangle | x \in E\}.$$

Let the IFS $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$. Therefore,

$$\sum_{y \in E} \mu_B(y) > 0,$$

$$\sum_{y \in E} \nu_B(y) > 0,$$

$$\sum_{y \in E} (\mu_B(y) + \nu_B(y)) > 0.$$

$$W_B^1(A) = \{\langle x, \frac{(\sum_{y\in E} \mu_A(y)).\mu_B(x)}{card(E)\sum_{y\in E} \mu_B(y)}, \frac{(\sum_{y\in E} \nu_A(y)).\nu_B(x)}{card(E)\sum_{y\in E} \nu_B(y)}\rangle | x \in E\}$$

and

$$W_B^2(A) = \{\langle x, \frac{(\sum_{y\in E} \mu_A(y)).\mu_B(x)}{2\max(\sum_{y\in E} \mu_B(y), \sum_{y\in E} \nu_B(y))}, \frac{(\sum_{y\in E} \nu_A(y)).\nu_B(x)}{2\max(\sum_{y\in E} \mu_B(y), \sum_{y\in E} \nu_B(y))}\rangle | x \in E\}$$

respectively, where $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$.

Third modification of the weight-center operator

Let

$$||X|| = \frac{\sum\limits_{y \in E} (\mu_X(y) + \nu_X(y))}{card(E)}$$

be a norm of X.

Let A and B be two IFSs over the finite universe E, so that $||A|| \leq ||B||$.

Now, we introduce *"the third modified weight-center operator"* over IFSs A and B over the finite universe E.

Let everywhere below, $B \neq H_{0,0}(B)$, $B \neq J_{0,0}(B)$ and $||A|| \leq ||B||$.

The new operator has the form

$$W_B^3(A) = \{\langle x, \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{\sum\limits_{y \in E} (\mu_B(y) + \nu_B(y))}, \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y \in E} (\mu_B(y) + \nu_B(y))} \rangle | x \in E\}$$

First, we see that the definition is correct, i.e., $W_B^3(A)$ is an IFS. Really, for every $x \in E$,

$$0 \leq \frac{(\sum\limits_{y\in E} \mu_A(y)).\mu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \leq \frac{\sum\limits_{y\in E} \mu_A(y)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \leq 1,$$

$$0 \leq \frac{(\sum\limits_{y\in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \leq \frac{\sum\limits_{y\in E} \nu_A(y)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \leq 1$$

and

$$\frac{(\sum\limits_{y\in E} \mu_A(y)).\mu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} + \frac{(\sum\limits_{y\in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \leq 1.$$

Theorem 1. For every two IFSs A and B over the finite universe E, so that $B \neq H_{0,0}(B)$, $B \neq J_{0,0}(B)$ and $||A|| \leq ||B||$:

(a) $\overline{W_B^3(\overline{A})} = W_B^3(A)$,

(b) $I(W_B^3(A)) = W_B^3(I(A))$,

(c) $C(W_B^3(A)) = W_B^3(C(A))$,

(d) $I_\mu(W_B^3(A)) = W_B^3(I_\mu(A))$,

(c) $C_\nu(W_B^3(A)) = W_B^3(C_\nu(A))$.

Theorem 2. For every two IFSs A and $B \neq U^*$ over the finite universe E:

$$W_B^1(A)A \subset_\Box W_B^2(A) \subset_\Box W_B^3(A),$$
$$W_B^1(A)A \subset_\Diamond W_B^2(A) \subset_\Diamond W_B^3(A).$$

Thank You For Your Attention!