CANTOR'S NORMS FOR INTUITIONISTIC FUZZY SETS

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Abstract
New norms, called "Cantor's norms", are introduced and some of their properties are discussed.

The Intuitionistic Fuzzy Set (IFS, see [2])

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

is defined over a fixed universe E, where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

IFSs have different (already, more than 10) geometrical interpretations (see [2]). Two of them are shown on Fig. 1 and 2. The first interpretation can be called "standard". If was the first really useful IFS-interpretation, because after its construction a lot of properties of the IFSs became visible and it generated a lot of new ideas in IFS theory. About ten years later, Eulalia Szmidt and Janusz Kacprzyk introduced a new interpretation that proved its usefullness in the years [3, 4, 5]. Let us call it "Szmidt-Kacprzyk interpretation".

Here, we shall introduce new norms. They will be related to the two mentioned IFS-interpretations. These two norms will be defined on the basis of one of the very important Georg Cantor's ideas in set theory and by this reason below we shall call them "Cantor norms" for IFSs. They are essentially different than the Euclidean and Hamming norms existing in fuzzy set theory.

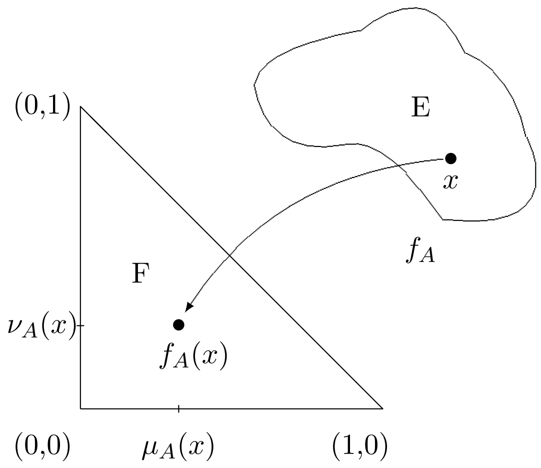

Figure 1.

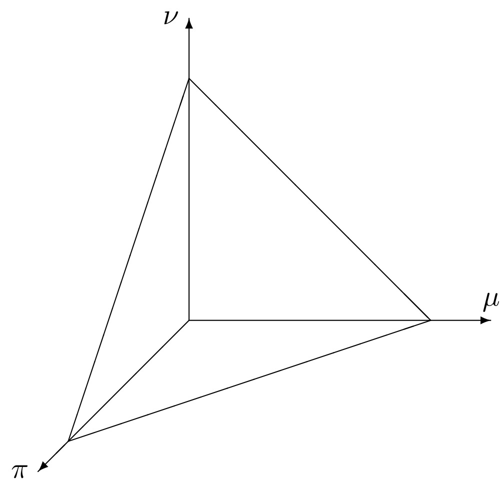

Figure 2.

Let $x \in E$ be fixed and let

$$\mu_A(x) = 0, a_1 a_2 ...$$

$$\nu_A(x) = 0, b_1 b_2 ...$$

Then, we bijectively construct the numbers

$$||x||_{\mu,\nu} = 0, a_1 b_1 a_2 b_2 ...$$

and

$$||x||_{\nu,\mu} = 0, b_1 a_1 b_2 a_2 ...$$

for which we see that
1. $||x||_{\mu,\nu}, ||x||_{\nu,\mu} \in [0, 1]$
2. having each one of them, we can directly reconstruct numbers $\mu_A(x)$ and $\nu_A(x)$.

Let us call numbers $||x||_{\mu,\nu}$ and $||x||_{\nu,\mu}$ Cantor norm of element $x \in E$. In some cases these norms will be denoted by $||x||_{2,\mu,\nu}$ and $||x||_{2,\nu,\mu}$ with aim to draw that they are related to the two-dimensional IFS-interpretation.

In the case of Szmidt-Kacprzyk interpretation, i.e., when we like to use a three-dimensional IFS-interpretation, for point x we have

$$\mu_A(x) = 0, a_1 a_2 ...$$

$$\nu_A(x) = 0, b_1 b_2 ...$$

$$\pi_A(x) = 0, c_1 c_2 ...$$

where, of course, $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$. Now, in principle, we can introduce six different Cantor norms:

$$||x||_{3,\mu,\nu,\pi} = 0, a_1 b_1 c_1 a_2 b_2 c_2 ...,$$

$$||x||_{3,\mu,\pi,\nu} = 0, a_1 c_1 b_1 a_2 c_2 b_2 ...,$$

$$||x||_{3,\nu,\mu,\pi} = 0, b_1 a_1 c_1 b_2 a_2 c_2 ...,$$

$$||x||_{3,\nu,\pi,\mu} = 0, b_1 c_1 a_1 b_2 c_2 a_2 ...,$$

$$||x||_{3,\pi,\mu,\nu} = 0, c_1 a_1 b_1 c_2 a_2 b_2 ...,$$

$$||x||_{3,\pi,\nu,\mu} = 0, c_1 b_1 a_1 c_2 b_2 a_2 ....$$

Therefore, for a given three-dimensional Cantor norm we can again reconstruct bijectively the three degrees of element $x \in E$.

In each of these cases the norm has a standard (from mathematical point of view) form.

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