Notes on Intuitionistic Fuzzy Sets
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Interval-valued intuitionistic fuzzy sets over universes with special forms

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Abstract: An interval-valued intuitionistic fuzzy set (IVIFS) whose universe is an intuitionistic fuzzy set (IFS) is introduced here and its basic properties are discussed. One step further, it is discussed the case when the universe represents an IVIFS itself. Again, some basic properties of the so constructed object are formulated and proved. Ideas for further research are presented.
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1 Introduction

In [5], the idea for an Intuitionistic Fuzzy Set (IFS) whose universe is an IFS with respect to another universe, is discussed. Here we will develop this idea for the case of an Interval-Valued Intuitionistic Fuzzy Set (IVIFS).

Following [5, 6], we mention that for a fixed universe E and a subset A of E, the set

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\},$$

where

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

is called IFS and the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ represent the *degree of membership (validity, etc.)* and *non-membership (non-validity, etc.) of element* $x \in E$ *to a fixed set* $A \subseteq E$. Thus, we can also define function $\pi_A : E \to [0, 1]$ by means of

$$\pi(x) = 1 - \mu(x) - \nu(x)$$

and it corresponds to *the degree of indeterminacy (uncertainty, etc.).*

An IVIFS A over E is an object of the form:

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in E\},$$

where

$$M_A(x) \subseteq [0, 1] \text{ and } N_A(x) \subseteq [0, 1]$$

are intervals and for all $x \in E$:

$$\sup M_A(x) + \sup N_A(x) \leq 1$$

(see [3, 5, 7].

This definition is analogous to the definition of an IFS. It can be however rewritten to become an analogue of the definition of IFS, namely, if M_A and N_A are interpreted as functions. Then, an IVIFS A (over a universe set E) is given by the above inequality and the functions

$$M_A : E \to INT([0, 1]) \text{ and } N_A : E \to INT([0, 1]).$$

For every two IVIFSs A and B, the following relation holds:

$$\begin{aligned}
A \subset B \quad \text{iff} \quad & (\forall x \in E)(\inf M_A(x) \leq \inf M_B(x) \;\&\; \sup M_A(x) \leq \sup M_B(x)),\\
& \;\&\inf N_A(x) \geq \inf N_B(x) \;\&\; \sup N_A(x) \geq \sup N_B(x)),\\
A = B \quad \text{iff} \quad & (\forall x \in E)(\inf M_A(x) = \inf M_B(x) \;\&\; \sup M_A(x) = \sup M_B(x),\\
& \;\&\inf N_A(x) = \inf N_B(x) \;\&\; \sup N_A(x) = \sup N_B(x)).
\end{aligned}$$

2 Main results

In [4] the idea for a new IFS-construction was proposed. It was extended in [5] but only for the IFS-case. Here, we will do this for the IVIFS case. Partially, the idea was generated with regards to the definition of a soft set and some of its modifications [1,2,8–14]. But in the present research, we will keep only the IFS- and IVIFS-forms of the sets without adding any soft-set component.

Let E be a fixed universe and let A be an IVIFS over E.

Let F be another universe and let the set E be an IFS over F having the form:

$$E = \{\langle y, \mu_E(y), \nu_E(y)\rangle | y \in F\}.$$

Therefore, the element $x \in E$ has the form:

$$x = \langle y, \mu_E(y), \nu_E(y)\rangle,$$

i.e., $x \in F \times [0,1] \times [0,1]$,

$$\begin{aligned} A =& \{\langle\langle y, \mu_E(y), \nu_E(y)\rangle, M_A(\langle y, \mu_E(y), \nu_E(y)\rangle), N_A(\langle y, \mu_E(y), \nu_E(y)\rangle)\rangle \\ & |\langle y, \mu_E(y), \nu_E(y)\rangle \in E\} \end{aligned}$$

and there exists a bijection between the E- and F- elements of x- and y-types, respectively.

Thus we can use the symbol "y" for both y- and x-elements.

Let A // E stands for *"A is an IVIFS over E"* and A // (E/F) stands for *"A is an IVIFS over E, which is an IFS over F"*.

2.1 IVIFS whose universe is an IFS

If the degrees of membership and non-membership of an element y to a set A in the frames of a universe E are $M_A(y)$ and $N_A(y)$, and the element $\langle y, M_A(y), N_A(y)\rangle$ has degrees of membership and non-membership to the set E within the universe F that are $\mu_E(y)$ and $\nu_E(y)$, then we can define the first batch of six forms of the transformation of the universe of the IVIFS A:

$$\begin{aligned}
A //_1^i (E/F) = \{\langle y, [&\mu_E(y) + \inf M_A(y) - \mu_E(y). \inf M_A(y), \mu_E(y) \\
&+ \sup M_A(y) - \mu_E(y). \sup M_A(y)], \\
&[\nu_E(y). \inf N_A(y), \nu_E(y). \sup N_A(y)]\rangle | y \in F\}, \\
A //_2^i (E/F) = \{\langle y, [&\max(\mu_E(y), \inf M_A(y)), \max(\mu_E(y), \sup M_A(y))], \\
&[\min(\nu_E(y), \inf N_A(y)), \min(\nu_E(y), \sup M_A(y))]\rangle | y \in F\}, \\
A //_3^i (E/F) = \Bigg\{\Bigg\langle y, &\left[\frac{\mu_E(y) + \inf M_A(y)}{2}, \frac{\mu_E(y) + \sup M_A(y)}{2}\right], \\
&\left[\frac{\nu_E(y) + \inf N_A(y)}{2}, \frac{\nu_E(y) + \sup N_A(y)}{2}\right]\Bigg\rangle | y \in F\Bigg\}, \\
A //_4^i (E/F) = \{\langle y, [&\min(\mu_E(y), \inf M_A(y)), \min(\mu_E(y), \sup M_A(y))], \\
&[\max(\nu_E(y), \inf N_A(y)), \max(\nu_E(y), \sup N_A(y))]\rangle | y \in F\}, \\
A //_5^i (E/F) = \{\langle y, [&\mu_E(y). \inf M_A(y), \mu_E(y). \sup M_A(y)], \\
&[\nu_E(y) + \inf N_A(y) - \nu_E(y). \inf N_A(y), \nu_E(y) \\
&+ \sup N_A(y) - \nu_E(y). \sup N_A(y)]\rangle | y \in F\}, \\
A //_6^i (E/F) = \{\langle y, [&\mu_E(y). \inf M_A(y), \mu_E(y). \sup M_A(y)], \\
&[\nu_E(y). \inf N_A(y), \nu_E(y). \sup N_A(y)]\rangle | y \in F\}.
\end{aligned}$$

We can call all these transformations, respectively, *very optimistic, optimistic, average, pessimistic, very pessimistic, standard.* Last name is given for the older transformation described for the IFS-case in [5].

Here, for the first time we formulate and prove the following

Theorem 1. *For each set A in the universe E, that is an IFS over the universe F,*

$$A \mathbin{//}_5^i (E/F) \subseteq A \mathbin{//}_4^i (E/F) \subseteq A \mathbin{//}_3^i (E/F) \subseteq A \mathbin{//}_2^i (E/F) \subseteq A \mathbin{//}_1^i (E/F),$$

$$A \mathbin{//}_5^i (E/F) \subseteq A \mathbin{//}_6^i (E/F) \subseteq A \mathbin{//}_1^i (E/F).$$

Proof. Let A be a set in the universe E, that is an IFS over the universe. For it, the following inequalities are valid for each $y \in F$:

$$\mu_E(y).\inf M_A(y) \leq \min(\mu_E(y), \inf M_A(y)) \leq \frac{\mu_E(y) + \inf M_A(y)}{2}$$
$$\leq \max(\mu_E(y), \inf M_A(y)) \leq \mu_E(y) + \inf M_A(y) - \mu_E(y).\inf M_A(y),$$

$$\mu_E(y).\sup M_A(y) \leq \min(\mu_E(y), \sup M_A(y)) \leq \frac{\mu_E(y) + \sup M_A(y)}{2}$$
$$\leq \max(\mu_E(y), \sup M_A(y)) \leq \mu_E(y) + \sup M_A(y) - \mu_E(y).\sup M_A(y),$$

and

$$\nu_E(y) + \inf N_A(y) - \nu_E(y).\inf N_A(y) \geq \max(\nu_E(y), \inf N_A(y))$$
$$\geq \frac{\nu_E(y) + \inf N_A(y)}{2} \geq \min(\nu_E(y), \inf N_A(y)) \geq \nu_E(y).\inf N_A(y),$$

$$\nu_E(y) + \sup N_A(y) - \nu_E(y).\sup N_A(y) \geq \max(\nu_E(y), \sup N_A(y))$$
$$\geq \frac{\nu_E(y) + \sup N_A(y)}{2} \geq \min(\nu_E(y), \sup M_A(y)) \geq \nu_E(y).\sup N_A(y)$$

that proves both assertions in the Theorem. □

Let F be a universe and let the set E be an IVIFS over F having the form:

$$E = \{\langle y, M_E(y), N_E(y)\rangle | y \in F\}.$$

Therefore the element $x \in E$ has the form:

$$x = \langle y, M_E(y), N_E(y)\rangle.$$

2.2 IVIFS whose universe is an IVIFS

Now, we can define the second batch of six new forms of the transformation of the universe of the IVIFS A:

$$\begin{aligned}
A //_1^{iv} (E/F) = \{\langle y, [&\inf M_E(y) + \inf M_A(y) - \inf M_E(y).\inf M_A(y),\\
&\sup M_E(y) + \sup M_A(y) - \sup M_E(y).\sup M_A(y)],\\
&[\inf N_E(y).\inf N_A(y), \sup N_E(y).\sup N_A(y)]\rangle | y \in F\},\\
A //_2^{iv} (E/F) = \{\langle y, [&\max(\inf M_E(y), \inf M_A(y)), \max(\sup M_E(y), \sup M_A(y))],\\
&[\min(\inf N_E(y), \inf N_A(y)), \min(\sup N_E(y), \sup N_A(y))]\rangle | y \in F\},\\
A //_3^{iv} (E/F) = \Bigg\{\Bigg\langle y, &\left[\frac{\inf M_E(y) + \inf M_A(y)}{2}, \frac{\sup M_E(y) + \sup M_A(y)}{2}\right],\\
&\left[\frac{\inf N_E(y) + \inf N_A(y)}{2}, \frac{\sup N_E(y) + \sup N_A(y)}{2}\right]\Bigg\rangle | y \in F\Bigg\},\\
A //_4^{iv} (E/F) = \{\langle y, [&\min(\inf M_E(y), \inf M_A(y)), \min(\sup M_E(y), \sup M_A(y))],\\
&[\max(\inf N_E(y), \inf N_A(y)), \max(\sup N_E(y), \sup N_A(y))]\rangle | y \in F\},\\
A //_5^{iv} (E/F) = \{\langle y, [&\inf M_E(y).\inf M_A(y), \sup M_E(y).\sup M_A(y)],\\
&[\inf N_E(y) + \inf N_A(y) - \inf N_E(y).\inf N_A(y),\\
&\sup N_E(y) + \sup N_A(y) - \sup N_E(y).\sup N_A(y)]\rangle | y \in F\},\\
A //_6^{iv} (E/F) = \{\langle y, [&\inf M_E(y).\inf M_A(y), \sup M_E(y).\sup M_A(y)],\\
&[\inf N_E(y).\inf N_A(y), \sup N_E(y).\sup N_A(y)]\rangle | y \in F\}.
\end{aligned}$$

We can formulate and prove a theorem similar to the above one:

Theorem 2. *For each set A in the universe E, that is an IFS over the universe F,*

$$A //_5^{iv} (E/F) \subseteq A //_4^{iv} (E/F) \subseteq A //_3^{iv} (E/F) \subseteq A //_2^{iv} (E/F) \subseteq A //_1^{iv} (E/F),$$

$$A //_5^{iv} (E/F) \subseteq A //_6^{iv} (E/F) \subseteq A //_1^{iv} (E/F).$$

3 Conclusion

Out next research will fork in two directions. On the one hand, we will study the possibility to represent a soft set by some of the above defined new types of sets. On the other hand, we will search for new ways of constructing of IVIFSs over universes with with IFS- or IVIFS-forms. For this aim, we plan to modify the extended interval-valued intuitionistic fuzzy modal operators, similarly to the modification, discussed in [6], in which the parameters α and β of the operators were changes with whole IFSs.

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References

[1] Agarwal, M., Biswas, K. K., & Hanmandlu, M. (2011). Relations in generalized intuitionistic fuzzy soft sets. I*EEE International Conference on Computational Intelligence for Measurement Systems and Applications Proceedings*, art. no. 6059919, 2011, 140–145.

[2] Agarwal, M., Hanmandlu, M., & Biswas, K. (2011). Generalized Intuitionistic Fuzzy Soft Set and its Application in Practical Medical Diagnosis Problem, *Conference: IEEE International Conference on Fuzzy Systems*, Taipei, June 27–30, 2972–2978.

[3] Atanassov, K. (1988). Review and new results on intuitionistic fuzzy sets. *Preprint IM-MFAIS-1-88*, Sofia, 1988. Reprinted: *Int. J. Bioautomation*, 2016, 20 (S1), S7–S16.

[4] Atanassov, K. (1992). Remarks on the intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 51 (1), 117–118.

[5] Atanassov, K. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer, Heldelberg.

[6] Atanassov, K. (2012). *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin.

[7] Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 31 (3), 343–349.

[8] Bora, M., Neog, T. J., & Sut, D. K. (2012). Some Results on Intuitionistic Fuzzy Soft Sets, *International Journal of Mathematics Trends and Technology*, 3 (2), 63–69.

[9] Gong, Z.-T., Xie, T., Shi, Z.-H., & Pan, W.-Q. (2011). A multiparameter group decision making method based on the interval-valued intuitionistic fuzzy soft set. *Proceedings of International Conference on Machine Learning and Cybernetics*, 1, art. no. 6016727, 125–130.

[10] Jiang, Y., Tang, Y. & Chen, Q. An adjustable approach to intuitionistic fuzzy soft sets based decision making. *Applied Mathematical Modelling*, 35 (2), 824–836

[11] Jiang, Y., Y. Tang, Chen, Q. Liu, H. & Yang, J. (2010). Interval-valued intuitionistic fuzzy soft sets and their properties, Computers & Mathematics with Applications, 60 (3), 906–918.

[12] Kalayathankal, S. J., Singh, G. S., Vinodkumar, P. B., Joseph, S., & Thomas, J. (2011). Ordered intuitionistic fuzzy soft model of flood alarm. *Iranian Journal of Fuzzy Systems*, 8 (1), 29–39

[13] Maji, P. K. (2009). More on Intuitionistic Fuzzy Soft Sets, *Lecture Notes in Artificial Intelligence*, Vol. 5908, 231–240.

[14] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Intuitionistic fuzzy soft sets. *The Journal of Fuzzy Mathematics*, 9 (3), 677–692.