Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy random variable

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Abstract: Information available in real life is, sometimes not precise, affected by various source of uncertainty, imprecision and vagueness. In a statistical survey, uncertainty contained in the data is itself an obstacle. Fuzzy set is the most effective tool to describe imprecise data through linguistic variables. Prof.Atanassov further generalized fuzzy sets into intuitionistic fuzzy sets to model vagueness present in the natural language. In this paper, intuitionistic fuzzy number is defined as a generalization of Wu's fuzzy number. Further, intuitionistic fuzzy random variable is defined and some of its properties are discussed. In addition, intuitionistic fuzzy statistical tools are described with suitable illustrations.
Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy number, Intuitionistic fuzzy random variable, Intuitionistic fuzzy statistical tools, Accuracy.
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1 Introduction

In real life, due to the inevitable measurement and inaccuracy, the exact values of the measured quantities are not known and so the problem involved in that situation are usually defined in an uncertain way. Therefore, it is desirable to consider the knowledge of experts about the vague data as fuzzy data. Lotfi Zadeh (1965), introduced the concept of fuzzy sets to model imprecision, ambiguity and uncertainty [9]. Out of several higher order fuzzy sets, Krassimir Atanassov (1985) gave a description of this situation through intuitionistic fuzzy sets (IFSs) using not only degrees of membership but the degrees of non-membership and hesitancy too [1]. Hence, IFS theory is still more suitable to deal with such problems. While collecting data, experts may have opinion with some degree of hesistancy. For example, a student says that a particular tutor can be given 0.6 degree of membership for punctuality but if we ask him whether he can be given 0.3 non-membership grade for punctuality, he will surely show some hesitation. To deal with these types of situations in real life, it is necessary to develop the theory of intuitionistic fuzzy statistics.

Now, there are more researches focusing on the fuzzy statistical analysis and applications [3, 6]. New statistical approach for fuzzy data was given by Ching-min Sun and Berlin Wu [4]. Wu and Sun illustrated about interval-valued statistics, fuzzy logic and its applications. Liu (2004) gave an illustration on uncertainty theory based on fuzzy set. Fuzzy mean, fuzzy median, fuzzy mode were defined by Hung T. Nguen and Berlin Wu [7]. As interval data is a generalized form of single valued data it is important in decision making situations. Further, interval valued intuitionistic fuzzy number (IVIFN) has been extended from IFN to handle vague information which is expressed as range of values than exact values [2, 5].

Recently, fuzzy statistical tools capture the attention of researchers more effectively [12]. Computation of information based on IF statistics is more futuristic and are widely used in decision making. Hence, the authors are motivated to design IF statistical tools. This paper is an extension of fuzzy statistical tools to IF environment and these new techniques can extract people's thought in a more precise way.

IFN introduced in this paper, can be viewed as an alternative approach to model reality with uncertainty for solving problems in IF system and it can be applied to many practical problems arising on economical, social survey, pollution control etc.,

In this paper, an attempt has been made to define intuitionistic fuzzy random variable (IFRV) with some properties.

The remaining part of the paper is organized as follows: Section 2, describes the basic definitions and notations of IFSs and IFNs. In Section 3, IFRV is defined and properties of IFRV are analysed. Section 4 deals with IF statistical tools for IF data and for IVIFN with suitable illustrations. Section 5 concludes the paper.

2 Definitions

In this section, some basic definitions that are necessary in the preparation of this paper are recollected and reviewed.

Definition 2.1. [1] Let the universal set U be fixed. An *intuitionistic fuzzy set* A in U is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\}$ where the functions $\mu_A : U \to [0, 1]$ and $\nu_A : U \to [0, 1]$ define the degrees of membership and non-membership of the element $x \in U$ respectively, and for every $x \in U$ in A, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds. Let for every $x \in U$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ denotes the degree of uncertainty.

Definition 2.2. Let U be the universal set and let $A = \{A_1, A_2, ..., A_n\}$ be the subset of discussion factors in U, then for any $x \in U$, its degrees of membership and non-membership corresponding to $\{A_1, A_2, ..., A_n\}$ are respectively $\{\mu_1(x), \mu_2(x), ..., \mu_n(x)\}$ and $\{\nu_1(x), \nu_2(x), ..., \nu_n(x)\}$ where $\mu : U \to [0, 1]$ and $\nu : U \to [0, 1]$ are real, then the *intuitionistic fuzzy number* is an object of the form $\{\langle x, \mu_U(x), \nu_U(x)\rangle : x \in U\}$ and can be written as

$$\mu_U(x) = \frac{\mu_1(x)}{A_1} + \frac{\mu_2(x)}{A_2} + ..., \frac{\mu_n(x)}{A_n},$$

$$\nu_U(x) = \frac{\nu_1(x)}{A_1} + \frac{\nu_2(x)}{A_2} +, ..., \frac{\nu_n(x)}{A_n}$$

where "+" stands for "or" and "÷" stands for the membership $\mu_i(x)$ on A_i. It can also be written as

$$A_i(\mu, \nu) = \frac{\langle \mu_1(x), \nu_1(x) \rangle}{A_1} + \frac{\langle \mu_2(x), \nu_2(x) \rangle}{A_2} + ... + \frac{\langle \mu_n(x), \nu_n(x) \rangle}{A_n}.$$

Example 2.3. A survey about favourite subjects using IFN:
Consider an intuitionistic fuzzy set of like subjects S_1, S_2, S_3, S_4, S_5 of a person as shown in Table 1 below. When the degree is given as 1 or 0 , that is *like* or *dislike*, a standard *Yes* or *No* are in complementary relationship as in binary logic.

Like Subjects	**IF Perception**		**Binary Perception**	
	$A_1(\mu, \nu)$	$A_2(\mu, \nu)$	$A_1 = like$	$A_2 = dislike$
Mathematical Science (S_1)	$(0.1, 0.8)$	$(0.8, 0.1)$		Yes
Physical Science (S_2)	$(0.5, 0.2)$	$(0.1, 0.4)$		No
Social Science (S_3)	$(0.6, 0.3)$	$(0.2, 0.6)$	Yes	
Biological Science (S_4)	$(0.8, 0.1)$	$(0.2, 0.4)$	Yes	
Life Science (S_5)	$(0.2, 0.6)$	$(0.6, 0.3)$	No	

Table 1: Comparison of IF perception and binary perception on favourite subjects

Let A_1 represents *like subjects*, A_2 represents *dislike* subjects. $A_1(\mu, \nu)$ denotes IF response of *like subjects*. $A_2(\mu, \nu)$ denotes IF response of *dislike subjects*. The intuitionistic fuzzy number of these two statements can be represented as

$$A_1(\mu, \nu) = \langle \frac{(0.1, 0.8)}{S_1} + \frac{(0.4, 0.5)}{S_2} + \frac{(0.6, 0.3)}{S_3} + \frac{(0.8, 0.1)}{S_4} + \frac{(0.2, 0.6)}{S_5} \rangle$$

$$A_2(\mu, \nu) = \langle \frac{(0.8, 0.1)}{S_1} + \frac{(0.3, 0.4)}{S_2} + \frac{(0.2, 0.6)}{S_3} + \frac{(0.2, 0.4)}{S_4} + \frac{(0.6, 0.3)}{S_5} \rangle$$

If someone uses membership and non-membership functions to express their degree of feeling based on human perception, the result will be very closer to the human thought. IFN gives a better representation of favoritism of subjects of a person by giving the degree of belongingness or non-belongingness instead of Yes or No answer.

Definition 2.4. [5] Let $A(\mu, \nu)$ be an IFN. Then $S(A) = \mu - \nu$ is called as the *score* of A where $S(A) \in [-1, 1]$.

Definition 2.5. [5] Let $A(\mu, \nu)$ be an IFN. Then $L(A) = \mu + \nu$ be the *accuracy* of A where $L(A) \in [0, 1]$.

Note. Let A and B be any two IFNs. According to their scores and accuracies, the ranking order of A and B is stipulated as follows:

(i) If $S(A) > S(B)$, then A is greater than B, denoted by $A \succ B$.

(ii) If $S(A) < S(B)$, then A is smaller than B, denoted by $A \prec B$.

(iii) If $S(A) = S(B)$, then there arises three situations:

(a) If $L(A) = L(B)$, then A is equal to B, denoted by $A = B$.

(b) If $L(A) > L(B)$, then A is greater than B, denoted by $A \succ B$.

(c) If $L(A) < L(B)$, then A is smaller than B, denoted by $A \prec B$.

Note. Let $A = ([a, b], [c, d])$, denote an IVIFN.

Definition 2.6. [5] Let $A_i = ([a_i, b_i], [c_i, d_i])$ be an interval valued intuitionistic fuzzy number, accuracy function L of A_i is defined as

$$L(A_i) = \frac{a_i + b_i - d_i(1 - b_i) - c_i(1 - a_i)}{2}.$$

Definition 2.7. [5] Let $A_i = ([a_i, b_i], [c_i, d_i])$ be an IVIFN. The following are the steps involved in ranking interval-valued intuitionistic fuzzy numbers:

Step 1: Find the score function of A_i using

$$L(A_i) = \frac{a_i + b_i - d_i(1 - b_i) - c_i(1 - a_i)}{2}.$$

Step 2: Calculate $B_{A_i(x_i,x_j)} = \{a_i \in A_i/x_i > x_j\}$, $C_{A_i(x_i,x_j)} = \{a_i \in A_i/x_i = x_j\}$ and fuzzy dominance relation using the formula

$$R_{A_i(x_i,x_j)} = \frac{|B_{A_i(x_i,x_j)}| + |C_{A_i(x_i,x_j)}|}{2|A_i|}.$$

Step 3: Calculate the entire dominance degree using

$$R_{A_i(x_i)} = \frac{1}{|E|} \sum_{i=1}^{|E|} R_{A_i(x_i,x_j)}.$$

Step 4: Now, the IVIFNs are ranked based on the value of entire dominance degree.

3 Intuitionistic fuzzy random variable

Definition 3.1. Let U be the universal set and let $L = \{L_1, L_2, ..., L_k\}$ be the set of k linguistic variables. Then an *intuitionistic fuzzy random variable* X is characterized by the map $X : U \to F$ such that $x_i \in U$ is associated with an ordered pair in $[0, 1] \times [0, 1]$ of the form $F = \langle \mu_{ij}, \nu_{ij} \rangle$ for every $x_i \in U$ where the function $\mu_{ij} : U \to [0, 1]$ and $\nu_{ij} : U \to [0, 1]$ define the degrees of membership and non-membership of x_i with respect to L_j, such that $0 \leq \mu_{ij} + \nu_{ij} \leq 1$.

Example 3.2. A farmer wants to adapt a new farming style for cultivating a crop from traditional techniques. He invites 5 experts for evaluation. After they tested the crop, they are asked to give a IF grading as Very Unsatisfactory $= L_1$, Unsatisfactory $= L_2$, No difference $= L_3$, Satisfactory $= L_4$, Very Satisfactory $= L_5$. Table 2 shows the evaluation of the five experts.

$Expert$	L_1	L_2	L_3	L_4	L_5
A	$(0.6, 0.2)$	$(0.5, 0.1)$	$-$	$-$	$(0.2, 0.6)$
B	$(0.1, 0.6)$	$-$	$(0.4, 0.1)$	$(0.7, 0.2)$	$-$
C	$-$	$(0.2, 0.6)$	$-$	$(0.4, 0.3)$	$(0.7, 0.2)$
D	$(0.8, 0.1)$	$(0.5, 0.2)$	$-$	$(0.2, 0.6)$	$-$
E	$(0.1, 0.6)$	$-$	$-$	$(0.7, 0.1)$	$(0.8, 0.2)$

Table 2: An example of IFRV

Though, the responses are available, still there remains an uncertainty about precise meaning of the response. IFRV extends the case to model not only uncertainty but also to model the hesitation which is naturally present in the uncertainty.

Properties of IFRV

1. Let U be the universal set. Let X_1 and X_2 be the set of two intuitionistic fuzzy random variables over X, where

$$X_1 = \{\langle x_i, L_k, \mu_{ik}, \nu_{ik}\rangle : x_i \in X\}$$

and

$$X_2 = \{\langle x_j, L_k, \mu_{jk}, \nu_{jk}\rangle : x_j \in X\},$$

$i = 1, 2, ..., m$ and $k = 1, 2, ..., n$ then

   (a) $X_1 + X_2$ is an object of the form $X_1 + X_2 = \{\langle x_i + x_j, L_k, \mu_{ij}, \nu_{ij}\rangle : x_i, x_j \in X\}$, where $\mu_{ij} = \mu_{ik}(x_i) + \mu_{jk}(x_j) - \mu_{ik}(x_i).\mu_{jk}(x_j), \nu_{ij} = \nu_{ik}(x_i).\nu_{jk}(x_j)$ and $0 \le \mu_{ij} + \nu_{ij} \le 1$ is also an IFRV.

   (b) $X_1.X_2 = \{\langle x_i.x_j, L_k, \mu_{ij}, \nu_{ij}\rangle : x_i, x_j \in X\}$, where $\mu_{ij} = \mu_{ik}(x_i) + \mu_{jk}(x_j)$ and $\nu_{ij} = \nu_{ik}(x_i) + \nu_{jk}(x_j) - \nu_{ik}(x_i).\nu_{jk}(x_j)$ is also an IFRV.

2. If X is an IFRV and n is a scalar, then $nX = \{\langle x, 1 - (1 - \mu_{ik}(x))^n, \nu_{ik}(x)^n\rangle\}$ is also an IFRV.

3. If X_1 and X_2 are IFRVs, then $max(X_1, X_2)$ and $min(X_1, X_2)$ are also IFRVs, where

$$\max(X_1, X_2) = \{\langle x, \max(\mu_{ik}(x_i), \mu_{jk}(x_j)), \min(\nu_{ik}(x_i), \nu_{jk}(x_j))\rangle\}$$

and

$$\min(X_1, X_2) = \{\langle x, \min(\mu_{ik}(x_i), \mu_{jk}(x_j)), \max(\nu_{ik}(x_i), \nu_{jk}(x_j))\rangle\}.$$

4 Intuitionistic fuzzy statistical tools

The following notations are used throughout this section. Let U be the universal set, $L = \{L_1, L_2, ..., L_k\}$ be a set of k linguistic variables on U and let X be an *intuitionistic fuzzy random variable*.

Throughout this paper, whenever the notation $\frac{\langle .,. \rangle}{L_j}$ is used, it denotes the degrees of membership and nonmembership of the linguistic variable L_j.

Definition 4.1. The *intuitionistic fuzzy mean* of X, denoted by IF_M, is defined as

$$IF_M = \frac{\left\langle \frac{\sum_{i=1}^{n} \mu_{i1}}{n}, \frac{\sum_{i=1}^{n} \nu_{i1}}{n} \right\rangle}{L_1} + \frac{\left\langle \frac{\sum_{i=1}^{n} \mu_{i2}}{n}, \frac{\sum_{i=1}^{n} \nu_{i2}}{n} \right\rangle}{L_2} + ... + \frac{\left\langle \frac{\sum_{i=1}^{n} \mu_{ik}}{n}, \frac{\sum_{i=1}^{n} \nu_{ik}}{n} \right\rangle}{L_k}.$$

Note. Since $0 \leq \mu_{ij} \leq 1$ and $0 \leq \nu_{ij} \leq 1$ for all i and j,

$$0 \leq \frac{\sum_{i=1}^{n} \mu_{ij}}{n} \leq 1 \text{ and } 0 \leq \frac{\sum_{i=1}^{n} \nu_{ij}}{n} \leq 1,$$

and hence

$$\frac{\sum_{i=1}^{n} \mu_{ij}}{n} + \frac{\sum_{i=1}^{n} \nu_{ij}}{n} \in [0, 1].$$

Hence, IF_M is also an IFS.

Example 4.2. A manufacturing company is launching new product for the forthcoming year. The company asked seven experts to give their grading based on the quality of the product. The quality of the product are classified as E = Excellent, G = Good, F = Fair, B = Bad, W = Worst. Find the expected value of the product, based on quality criteria.

Expert analysis	E	G	F	B	W
E_1	$(0.6, 0.2)$	$(0.4, 0.1)$	$(0.3, 0.3)$	$-$	$-$
E_2	$-$	$-$	$(0.4, 0.1)$	$(0.6, 0.3)$	$(0.8, 0.1)$
E_3	$-$	$(0, 0.8)$	$(0.1, 0.2)$	$(0.3, 0.2)$	$(0.7, 0.2)$
E_4	$(0.1, 0.6)$	$-$	$-$	$(0.5, 0.2)$	$(0.3, 0.1)$
E_5	$(0.8, 0.1)$	$(0.6, 0.3)$	$(0.4, 0.1)$	$(0.1, 0.7)$	$-$
E_6	$-$	$-$	$(0.3, 0.6)$	$(0.4, 0.3)$	$(0.6, 0.1)$
E_7	$(0.7, 0.1)$	$(0.5, 0.2)$	$-$	$(0.2, 0.7)$	$(0.1, 0.8)$
$(\sum_{i=1}^{5} \mu_{i1}, \sum_{i=1}^{5} \nu_{i1})$	$(2.2, 1.0)$	$(1.5, 1.4)$	$(1.5, 1.3)$	$(2.1, 2.4)$	$(2.5, 1.3)$
IF_M	$(0.44, 0.14)$	$(0.21, 0.2)$	$(0.21, 0.18)$	$(0.3, 0.34)$	$(0.35, 0.18)$
Score function	0.3	0.01	0.03	-0.04	0.17

Table 3: Expert analysis on quality

Let $L = \{E, G, F, B, W\}$ be the set of the linguistic variables. Then IF expected value is given by

$$IF_M = \frac{\left\langle \frac{1.5}{5}, \frac{0.9}{5} \right\rangle}{E} + \frac{\left\langle \frac{1}{5}, \frac{1.2}{5} \right\rangle}{G} + \frac{\left\langle \frac{1.2}{5}, \frac{0.7}{5} \right\rangle}{F} + \frac{\left\langle \frac{1.5}{5}, \frac{1.4}{5} \right\rangle}{B} + \frac{\left\langle \frac{1.8}{5}, \frac{0.4}{5} \right\rangle}{W}$$
$$= \frac{\langle 0.3, 0.2 \rangle}{E} + \frac{\langle 0.2, 0.24 \rangle}{G} + \frac{\langle 0.2, 0.14 \rangle}{F} + \frac{\langle 0.3, 0.28 \rangle}{B} + \frac{\langle 0.3, 0.08 \rangle}{W}$$

Based on the values of the score function of IF_M, it is expected that quality of the product is *Excellent* and the company is supposed to continue the production retaining the same quality.

Definition 4.3. Let X be an intuitionistic fuzzy random variable. Then:

(i) *intuitionistic fuzzy geometric mean* of X, denoted by $IF_{G.M}$, is defined as

$$IF_{G.M} = \frac{\left\langle (\prod_{i=1}^{n} \mu_{i1})^{\frac{1}{n}}, 1 - (\prod_{i=1}^{n} (1 - \nu_{i1}))^{\frac{1}{n}} \right\rangle}{L_1} + \frac{\left\langle (\prod_{i=1}^{n} \mu_{i2})^{\frac{1}{n}}, 1 - (\prod_{i=1}^{n} (1 - \nu_{i2}))^{\frac{1}{n}} \right\rangle}{L_2} + \ldots + \frac{\left\langle (\prod_{i=1}^{n} \mu_{ik})^{\frac{1}{n}}, 1 - (\prod_{i=1}^{n} (1 - \nu_{ik}))^{\frac{1}{n}} \right\rangle}{L_k}$$

(ii) *intuitionistic fuzzy harmonic mean* of X, denoted by $IF_{H.M}$, is defined as

$$IF_{H.M} = \frac{\left\langle \frac{n}{\sum_{i=1}^{n} \frac{1}{\mu_{i1}}}, 1 - \frac{n}{\sum_{i=1}^{n} \frac{1}{1-\nu_{i1}}} \right\rangle}{L_1} + \frac{\left\langle \frac{n}{\sum_{i=1}^{n} \frac{1}{\mu_{i2}}}, 1 - \frac{n}{\sum_{i=1}^{n} \frac{1}{1-\nu_{i2}}} \right\rangle}{L_2} + \ldots + \frac{\left\langle \frac{n}{\sum_{i=1}^{n} \frac{1}{\mu_{ik}}}, 1 - \frac{n}{\sum_{i=1}^{n} \frac{1}{1-\nu_{ik}}} \right\rangle}{L_k}$$

Example 4.4. A production firm wants to launch a new cosmetic item. The company director asks five experts to grade after introducing the product with the classification of the profit as HS = Highly Satisfied, S = Satisfied, M = Moderate, BM = Below Moderate, DS = Dissatisfied. The opinion of the experts is given as IF grading. Give your suggestions to the company to launch the new product based on IF geometric mean and IF harmonic mean.

Here $L = \{HS, S, M, BM, DS\}$ is the set of linguistic variables.

Expert analysis	HS	S	M	BM	DS
E_1	$(0.8, 0.1)$	$(0.6, 0.2)$	$(0.5, 0.3)$	$(0.2, 0.4)$	$(0.2, 0.7)$
E_2	$(0.6, 0.2)$	$(0.5, 0.2)$	$(0.4, 0.3)$	$(0.2, 0.7)$	$(0.1, 0.8)$
E_3	$(0.1, 0.8)$	$(0.2, 0.6)$	$(0.4, 0.2)$	$(0.7, 0.2)$	$(0.8, 0.1)$
E_4	$(0.2, 0.7)$	$(0.1, 0.6)$	$(0.3, 0.2)$	$(0.5, 0.2)$	$(0.6, 0.3)$
E_5	$(0.8, 0.1)$	$(0.5, 0.3)$	$(0.3, 0.3)$	$(0.2, 0.6)$	$(0.1, 0.7)$
$IF_{G.M}$	$(0.37, 0.5)$	$(0.3, 0.4)$	$(0.37, 0.26)$	$(0.28, 0.45)$	$(0.2, 0.6)$
Score function	-0.13	-0.1	0.11	-0.17	-0.4
$IF_{H.M}$	$(0.26, 0.5)$	$(0.24, 0.4)$	$(0.36, 0.26)$	$(0.27, 0.5)$	$(0.2, 0.6)$
Score function	-0.31	-0.2	0.1	-0.23	-0.4

Table 4: Opinion of experts on launching new products

Then IF geometric mean is

$$IF_{G.M} = \frac{\left\langle (0.007)^{\frac{1}{5}}, 1-(0.38)^{\frac{1}{5}} \right\rangle}{HS} + \frac{\left\langle (0.003)^{\frac{1}{5}}, 1-(0.07)^{\frac{1}{5}} \right\rangle}{S} + \frac{\left\langle (0.007)^{\frac{1}{5}}, 1-(0.021)^{\frac{1}{5}} \right\rangle}{M}$$

$$+ \frac{\left\langle (0.002)^{\frac{1}{5}}, 1-(0.04)^{\frac{1}{5}} \right\rangle}{BM} + \frac{\left\langle (0.0009)^{\frac{1}{5}}, 1-(0.011)^{\frac{1}{5}} \right\rangle}{DS}$$

$$= \frac{\langle 0.37, 0.5 \rangle}{HS} + \frac{\langle 0.3, 0.4 \rangle}{S} + \frac{\langle 0.37, 0.26 \rangle}{M} + \frac{\langle 0.28, 0.45 \rangle}{BM} + \frac{\langle 0.24, 0.45 \rangle}{DS}$$

and IF harmonic mean is

$$IF_{H.M} = \frac{\left\langle \frac{5}{19.1}, 1-\frac{5}{11.8} \right\rangle}{HS} + \frac{\left\langle \frac{5}{20.66}, 1-\frac{5}{8.92} \right\rangle}{S} + \frac{\left\langle \frac{5}{13.66}, 1-\frac{5}{6.78} \right\rangle}{M} + \frac{\left\langle \frac{5}{18.42}, 1-\frac{5}{10} \right\rangle}{BM} + \frac{\left\langle \frac{5}{27.9}, 1-\frac{5}{14.2} \right\rangle}{DS}$$

$$= \frac{\langle 0.26, 0.57 \rangle}{HS} + \frac{\langle 0.24, 0.44 \rangle}{S} + \frac{\langle 0.36, 0.26 \rangle}{M} + \frac{\langle 0.27, 0.5 \rangle}{BM} + \frac{\langle 0.17, 0.64 \rangle}{DS}$$

Based on the values of the score function S_j of $IF_{G.M}$ and $IF_{H.M}$, it is inferred that after introducing the new product, the company will get a *moderate* profit. Hence, it is suggested to launch the new product.

Definition 4.5. Let U be the universal set. Let X be an intuitionistic fuzzy random variable on U and let $L = \{L_1, L_2, ..., L_k\}$ be a set of k linguistic variables. Denote $I_j = \sum_{i=1}^{n} \mu_{ij}$ and $J_j = \sum_{i=1}^{n} \nu_{ij}$. Assume that $\langle L_j, NI_j, NJ_j \rangle$ denote the normalised sum of membership and non-membership of I_j and J_j with respect to L_j such that $NI_j = \frac{I_j}{supI_j}$ and $NJ_j = \frac{J_j}{supJ_j}$. Let S_j be the score function. Then:

(i) *Intuitionistic fuzzy median* of X is defined as the median of S_j. That is, $IF_{Med} = L_j$ corresponding to median of S_j.

(ii) *Intuitionistic fuzzy mode* of X is defined as the maximum of S_j. That is, $IF_{Mo} = L_j$ corresponding to maximum of S_j.

Example 4.6. In a newly started dietary centre, a study was made to analyse the diets. Five experts are asked to evaluate the nature of diet of a person. The table below shows the diet analysis by experts. L_1 = Very healthy, L_2 = Healthy, L_3 = Normal, L_4 = Weak, L_5 = Poor be the set of linguistic variables. Find the IF median and IF mode for the diet of a person.

Diet analysis	L_1	L_2	L_3	L_4	L_5
D_1	$(0.8, 0.1)$	$(0.7, 0.3)$	$-$	$(0.1, 0.7)$	$-$
D_2	$-$	$-$	$(0.4, 0.2)$	$(0.6, 0.1)$	$(0.8, 0.1)$
D_3	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.3, 0.1)$	$(0.2, 0.7)$	$-$
D_4	$(0.6, 0.2)$	$(0.5, 0.1)$	$(0.3, 0.2)$	$-$	$(0.2, 0.7)$
D_5	$-$	$(0.1, 0.6)$	$(0.5, 0.3)$	$(0.6, 0.1)$	$(0.8, 0.1)$
(I_j, J_j)	$(2.1, .5)$	$(1.9, 1.1)$	$(1.5, 0.8)$	$(1.5, 1.6)$	$(1.8, 0.9)$
Normalized Sum	$(1.0, 0.3)$	$(0.9, 0.6)$	$(0.7, 0.5)$	$(0.7, 1.0)$	$(0.9, 0.5)$
Score S_j	0.7	0.3	0.2	-0.3	0.4

Table 5: Opinion of five experts about the diet

In analysing the nature of diet of a person, *Very Healthy* and *Healthy* are the most analysed factor. *Healthy* is the IF Median and *Very Healthy* is the IF mode which suggest that the diet of a person is Healthy and he is advised to continue the same diet.

Definition 4.7. Let U be the universal set. Let X be an intuitionistic fuzzy random variable on U and let $L = \{L_1, L_2, ..., L_k\}$ be a set of k linguistic variables. Denote $I_j = \sum_{i=1}^{n} \mu_{ij}$ and $J_j = \sum_{i=1}^{n} \nu_{ij}$. Assume that $\langle L_j, NI_j, NJ_j \rangle$ denote the normalised sum of membership and non-membership of I_j and J_j with respect to L_j such that $NI_j = \frac{I_j}{supI_j}$ and $NJ_j = \frac{J_j}{supJ_j}$. Let S_j be the score function and let w_j be the weights. Then *intuitionistic fuzzy weighted mean* of X, denoted by *IF*$_W$ is defined as *IF*$_W = \langle \frac{\prod_{j=1}^{n} W_j S_j}{L_j} \rangle$.

Example 4.8. An organisation is analysing the economical condition of a country for the next period. Assume that the company operating in Europe and South Asia is analysing its general policy for next year, based on some strategy. The group of experts of the company considers the economical situation as the key factor. Depending on the situation, the expected benefits for the company will be different. The experts have considered five possible situations for the next year as $S_1 =$ Excellent, $S_2 =$ Good, S_3 = Fair, $S_4 =$ Bad, $S_5 =$ Worst. Find the weighted arithmetic mean for the analysing factor of the company.

Assume that the experts use the weight function for the calculation. Economical condition is the influencing factor for the progress of the company.

Expert analysis	S_1	S_2	S_3	S_4	S_5
D_1	$(0.6, 0.2)$	$(0.5, 0.1)$	$(0.3, 0.2)$	$-$	$(0.2, 0.7)$
D_2	$(0.8, 0.1)$	$(0.7, 0.3)$	$-$	$(0.1, 0.7)$	$-$
D_3	$-$	$(0.1, 0.6)$	$(0.5, 0.3)$	$(0.6, 0.1)$	$(0.8, 0.1)$
D_4	$-$	$-$	$(0.4, 0.2)$	$(0.6, 0.1)$	$(0.8, 0.1)$
D_5	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.3, 0.1)$	$(0.2, 0.7)$	$-$
(I_j, J_j)	$(2.1, 0.5)$	$(1.9, 1.1)$	$(1.5, 0.8)$	$(1.5, 1.6)$	$(1.8, 0.9)$
Normalized Sum	$(1.0, 0.3)$	$(0.9, 0.6)$	$(0.7, 0.5)$	$(0.7, 1.0)$	$(0.9, 0.5)$
Score S_j	0.7	0.3	0.2	-0.3	0.4
Weights	9	8	7	6	7
IFW_{mean}	1.2	0.48	.28	-0.36	0.64

Table 6: Expert analysis about economical condition of the company

From the table, it is infered that the economical condition of the company is *Excellent*. There will be a hike in the economical condition for the next year if it follows the existing strategies.

4.1 IF statistical tools for IVIFNs

Definition 4.9. Let U be the universal set and $X = \{\langle x_i, [a_i, b_i], [c_i, d_i] \rangle | x_i \in U\}$, $a_i, b_i, c_i, d_i \in \Re$ where $i = 1, 2, ..., n$ be the set of interval-valued intuitionistic fuzzy numbers, on U, then the *intuitionistic fuzzy mean of IVIFNs* denoted by $IVIF\bar{x}$ is defined as

$$\mathit{IVIF}\bar{x} = \left\{ \left\langle x_i, \left[\frac{\sum_{i=1}^{n} a_i}{n}, \frac{\sum_{i=1}^{n} b_i}{n} \right], \left[\frac{\sum_{i=1}^{n} c_i}{n}, \frac{\sum_{i=1}^{n} d_i}{n} \right] : x_i \in X \right\rangle \right\}$$

Notations: Let U be the universal set, $L = \{L_1, L_2, ..., L_k\}$ be a set of k linguistic variables on U and $X = \{\langle x_i, [a_i, b_i], [c_i, d_i]\rangle | x_i \in U, a_i, b_i, c_i, d_i \in \Re\}$ where $i = 1, 2, ..., n$ be the set of interval-valued intuitionistic fuzzy numbers on U. Let $\rho(x_j)$ be the ranks of interval-valued intuitionistic fuzzy numbers x_j.

Definition 4.10. The *intuitionistic fuzzy median* of X is defined as the median of $\rho(x_j)$. That is, $IVIF_{Med} = L_j$ corresponding to median of $\rho(x_j)$.

Definition 4.11. The *intuitionistic fuzzy mode* of X is defined as the maximum of $\rho(x_j)$. That is, $\mathit{IVIF}_{Mo} = L_j$ corresponding to maximum of $\rho(x_j)$.

Example 4.12. An enterprise plans to seek an adequate supplier for purchasing equipments needed for assembling the parts. Consider a problem of selection of the best supplier among five suppliers based on five attributes. Let $L_1 =$ Quality of the product, $L_2 =$ Social involvement, $L_3 =$ Performance of delivery, $L_4 =$ Legal issue, $L_5 =$ Customer relationship be the five linguistic variables.

Find also the IVIF_{Med} and IVIF_{Mode} for the best supplier.

Expert analysis	L_1	L_2	L_3	L_4	L_5
x_1	$([0, 0.2], [0.2, 0.4])$	$(0, [0.4, 0.6])$	$([0.2, 0.4], [0.4, 0.6])$	$(0.4, [0.2, 0.4])$	$(0.2, [0.4, 0.8])$
x_2	$(0, [0, 0.6])$	$(0, 1)$	$([0.2, 0.4], [0.4, 0.6])$	$(0.2, 0.6)$	$(0.4, 0.2)$
x_3	$(0.6, 0.4)$	$([0.4, 0.6], [0.2, 0.4])$	$(0.8, 0)$	$([0.2, 0.4], [0, 0.4])$	$([0.4, 0.6], [0, 0.2])$
x_4	$([0, 0.2], [0.2, 0.4])$	$([0.2, 0.4], [0, 0.4])$	$([0.2, 0.6], [0, 0.2])$	$([0.2, 0.4], [0.4, 0.6])$	$([0.4, 0.6], [0, 0.2])$
x_5	$(0.2, 0.8)$	$(0.2, 0.6)$	$(0.4, 0.4)$	$(0.2, 0.8)$	$(0.4, 0.2)$

Table 7: Evaluation of suppliers with respect to five attributes

Step 1: Find $R_A(x_i, x_j)$

$R_A(x_i, x_j)$	x_1	x_2	x_3	x_4	x_5
x_1	0.5	0.7	0.2	0.3	0.4
x_2	0.3	0.5	0	0	0.5
x_3	0.8	1	0.5	0.9	1
x_4	0.7	1	0.1	0.5	1
x_5	0.6	0.5	0	0	0.5

Step 2: Calculate rank

x_i	x_1	x_2	x_3	x_4	x_5
$R_A(x_i)$	0.42	0.28	0.84	0.66	0.32
$\rho(x_i)$	3	1	5	4	2

Step 3:
IF$_{Med} = L_1/\rho(x_j) =$ median of $\rho(x_i)$
IF$_{Med} = 0.42$ corresponding to quality of the product.
IF$_{Mo} = L_3/\rho(x_j) =$ maximum of $\rho(x_i)$
IF$_{Mo} = 0.84$ corresponding to performance of delivery.

Hence, it is inferred:

(i) From the value of *IF*$_{Median}$ that the best supplier will be selected based on the *Quality of the product*, they supply.

(ii) From *IF*$_{Mode}$ that the best supplier will be selected based on the *Performance of delivery*.

5 Conclusion

In this paper, intuitionistic fuzzy random variable is introduced and some of its properties are discussed. Also, an attempt has been made to define IF perception using IF numbers. These concepts can be used to reveal the complex human thoughts. IF grading will give a answer closer to human thought. Further, a few IF statistical tools are defined and illustrated. In addition, interval valued intuitionistic fuzzy mean and intuitionistic fuzzy mode of intuitionistic fuzzy numbers are proposed with suitable illustrations. The authors further propose to work on the applications of these statistical tools in designing filtering algorithms in image processing.

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