Ant Colony Algorithms for Combinatorial Optimization

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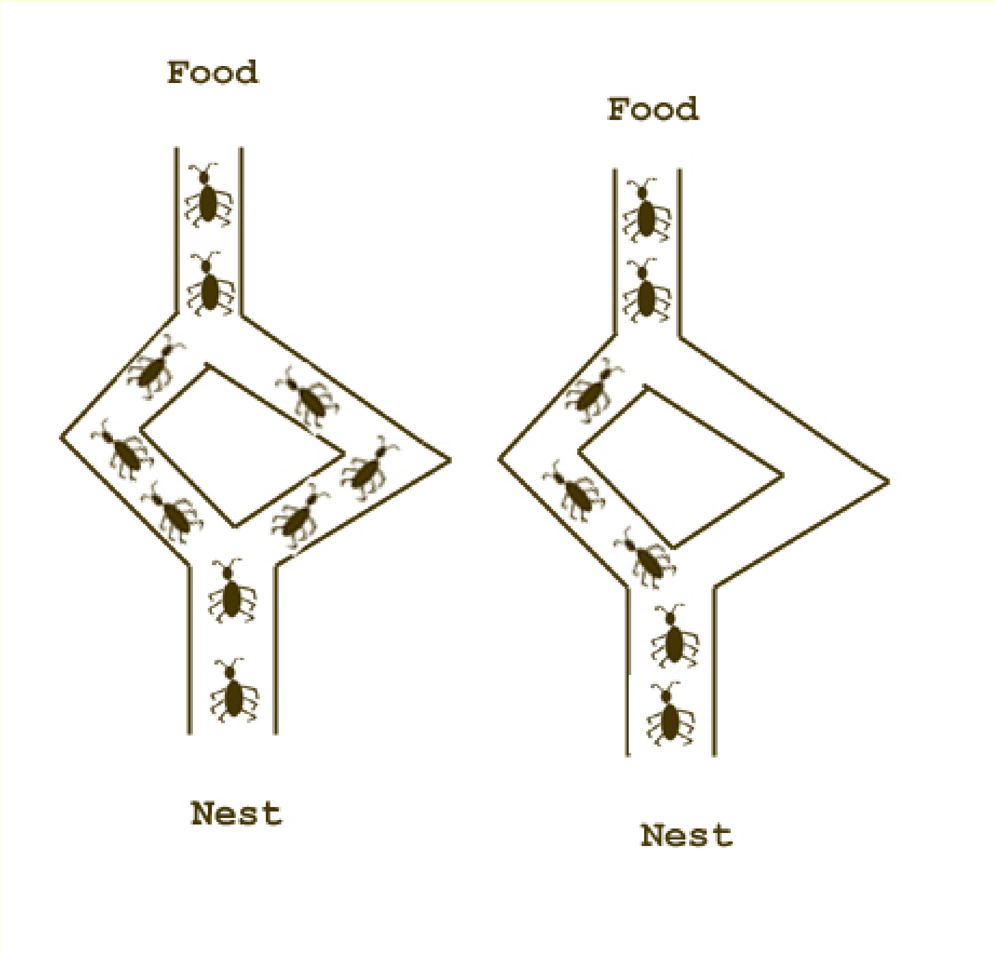
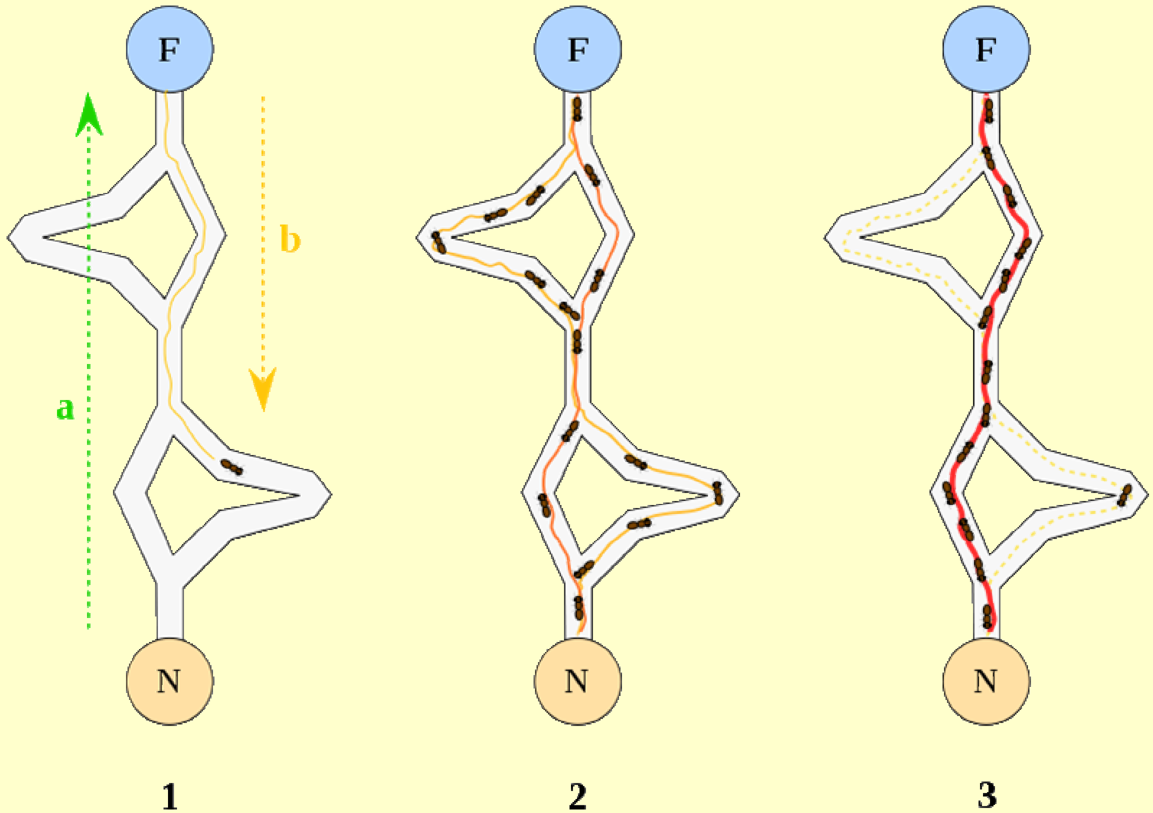
Metaheuristics

- A **metaheuristics** are methods for solving a very general class of computational problems by combining user-given black-box procedures in the hope of obtaining a more efficient or more robust procedure. The name combines the Greek prefix "meta" ("beyond", here in the sense of "higher level") and "heuristic" (from ευρισκειν, *heuriskein*, "to find").
- Metaheuristics are generally applied to problems for which there is no satisfactory problem-specific algorithm or heuristic; or when it is not practical to implement such a method. Most commonly used metaheuristics are targeted to combinatorial optimization problems, but of course can handle any problem that can be recast in that form, such as solving boolean equations

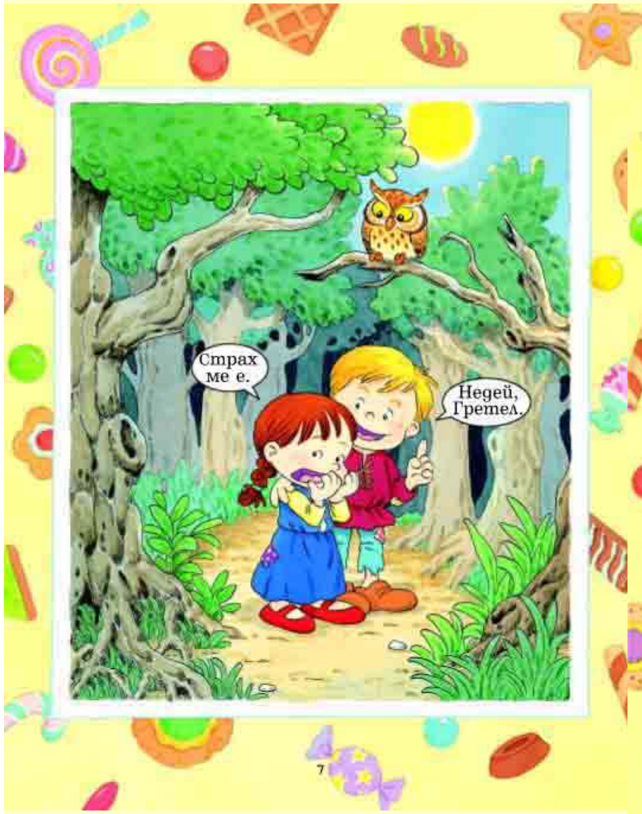
Ant Colony Optimization



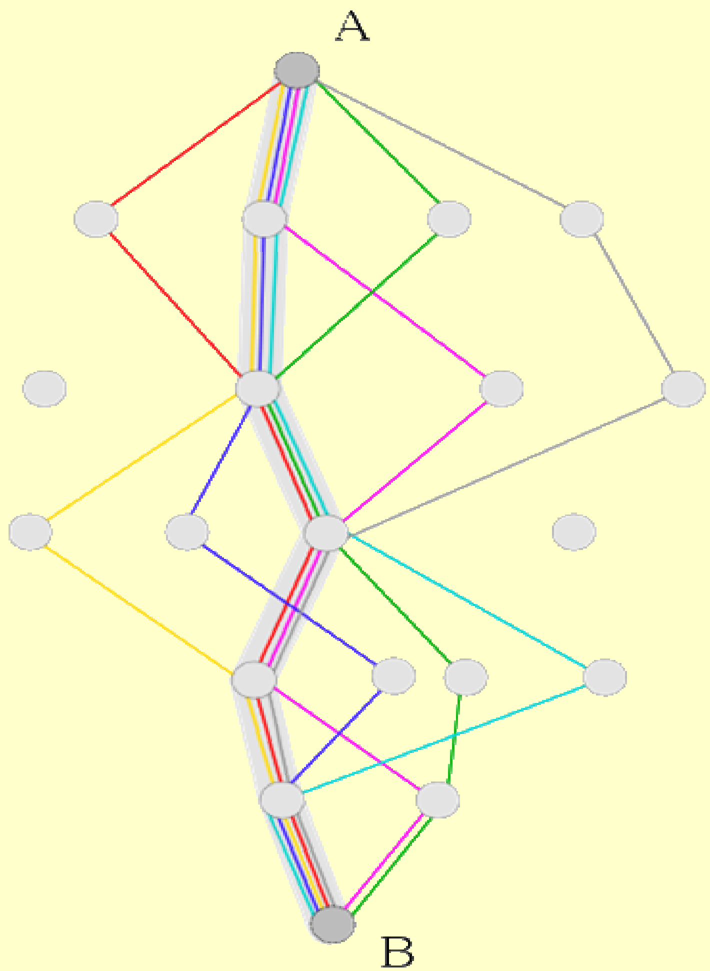
Ant Colony Optimization



Hensel and Grethen



Graph of the Problem



Ant Colony Optimization

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Procedure ACO
Begin
    initialize the pheromone
    while stopping criterion not satisfied do
        position each ant on a starting node
        repeat
            for each ant do
                chose next node
            end for
        until every ant has build a solution
        update the pheromone
    end while
end
```

Transition Probability

$$\mathrm{Prob}_{ij}^{k}(t) = \begin{cases} \dfrac{\tau_{ij}\eta_{ij}}{\sum_{b \in allowed_k(t)} \tau_{ib}\eta_{ib}} & if \ \ j \in allowed_k(t) \\ 0 & otherwise \end{cases}$$

Pheromone Updating

$$\tau_{ij} \leftarrow \rho\tau_{ij} + \Delta\tau_{ij}$$

$$0 < \rho < 1 \quad - \text{evaporation}$$

Ant Colony Algorithms

- Ant System
- Ant Colony System
- ACO with elitist ants
- Max-Min Ant System
- ACO with additional reinforcement

Ant System

$$\tau_{ij} \leftarrow \rho\tau_{ij} + (1-\rho)\Delta\tau_{ij}$$

$$\Delta\tau_{ij} = \begin{cases} 1/F & if \ \min imisation \ problem \\ F & if \ \max imization \ problem \end{cases}$$

$$0 < \rho < 1$$

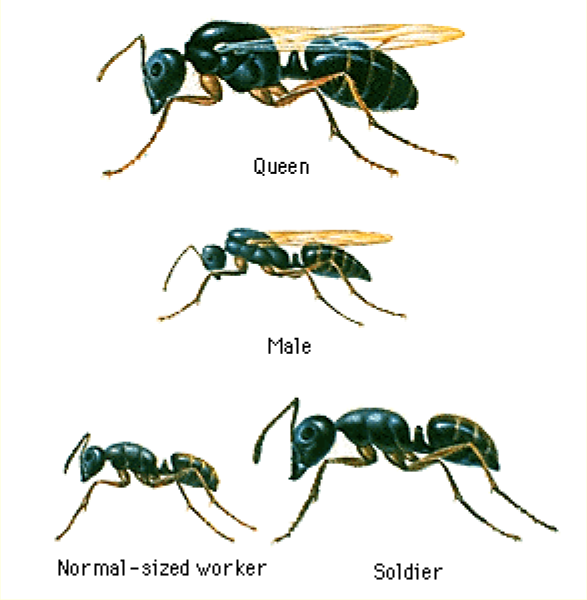
Ant Colony System

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\tau_0$$

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\Delta\tau_{ij}$$

$$\Delta\tau_{ij} = \begin{cases} L_{gb} & \text{if } (i,j) \in \text{global best} \\ 0 & \text{otherwise} \end{cases}$$

ACO with Elitist Ants



$$\tau_{ij} \leftarrow \rho\tau_{ij}$$

$$\tau_{ij} \leftarrow \tau_{ij} + (1-\rho)L \quad if\ the\ ant\ is\ elit$$

Max-Min Ant System

$\tau_{\max} \qquad \tau_{\min}$

$$If\ \tau_{ij} < \tau_{\min}\ then\ \tau_{ij} = \tau_{\min}$$

$$If\ \tau_{ij} > \tau_{\max}\ then\ \tau_{ij} = \tau_{\max}$$

ACO with Additional Reinforcement

$$\tau_{ij} \leftarrow \rho \tau_{ij} + \Delta \tau_{ij}$$

$$0 < \rho < 1 \quad - \text{evaporation}$$

$$\tau_{ij} \leftarrow \alpha \tau_{ij} + q \, \tau_{\max}$$

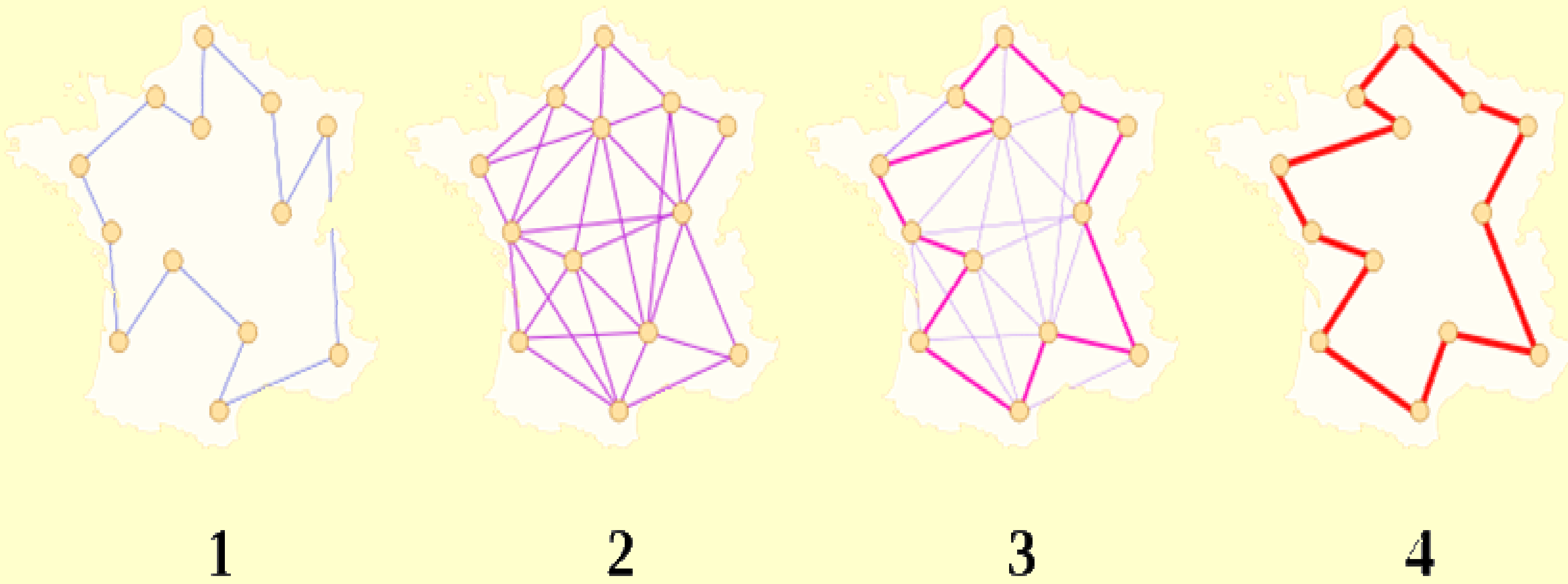
$$\alpha = \begin{cases} 1 - \text{if unused movements are evaporated} \\ \rho - \text{otherwise} \end{cases}$$

$$q \leq \rho$$

Traveling Salesman Problem

- It must visit each city exactly once;
- A distant city has less chance of being chosen (the visibility);
- The more intense the pheromone trail laid out on an edge between two cities, the greater the probability that that edge will be chosen;
- Having completed its journey, the ant deposits more pheromones on all edges it traversed, if the journey is short;
- After each iteration, trails of pheromones evaporate.

Traveling Salesman Problem



Subset of Nodes

- N1 -> S1
- N2 -> S2 …
- Nn -> Sn
- N(n+1) -> S1 …

Subset Estimation

$$D_j(i) = \varphi.D_j(i) + (1-\varphi).F_j(i) \qquad E_j(i) = \varphi.E_j(i) + (1-\varphi).G_j(i)$$

$$i \geq 1, \quad 1 \leq j \leq n,$$

$$F_j(i) = \begin{cases} \dfrac{f_{jA}}{k_j} & if \ k_j \neq 0 \\ F_j(i-1) & otherwise \end{cases} \qquad G_j(i) = \begin{cases} \dfrac{g_{jB}}{k_j} & if \ k_j \neq 0 \\ G_j(i-1) & otherwise \end{cases}$$

Start Strategies

1. If Ej(i)/Dj(i)>E then the subset j is forbidden for current iteration;
2. If Ej(i)/Dj(i)>E then the subset j is forbidden for current simulation
3. If Ej(i)/Dj(i)>E then the subset j is forbidden for K1 iterations
4. Let r1є(0.5,1) and r2є(0,1) are random numbers. If r2>r1 randomly choose a node from $\{j|D_j(i)>D\}$
5. The same as in case 4. but D increase with r3 every iteration

Thank for Your Attention