Notes on Intuitionistic Fuzzy Sets
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Aggregation operator, score function and accuracy function for multicriteria decision problems in intuitionistic fuzzy context

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Abstract: The notion of Intuitionistic Fuzzy Set (IFS for short) theory by Krassimir Atanassov strikes a paradigm shift in solving decision making problems, which is one of the crucial problems in our real life. Ranking of IFS and Interval Valued Intuitionistic Fuzzy Sets (IVIFS for short) is very often required in decision making. In this paper, we develop an aggregation operator for aggregating Intuitionistic fuzzy sets as well as interval valued intuitionistic fuzzy sets. It appears to be more elegant and simple than the existing aggregation operators. We also propose a score function and an accuracy function to rank the aggregated alternatives. It is illustrated with an example.

Keywords: Intuitionistic fuzzy sets, Interval-valued intuitionistic fuzzy sets, Aggregation operator.
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1 Introduction

Following the introduction of Fuzzy set by Zadeh in 1965, K. Atanassov introduced the notion of IFS (see [1, 2]) which has been found a better tool to model decision problems. Multicriteria decision making methods based on IFS theoretical tools were introduced in the decision theory in 2007 by Z. S. Xu, [5]. Xu introduced different types of aggregation operators. This was extended to IVIFS, [7].

In this paper, we propose an aggregation operator, score function and an accuracy function for an intuitionistic fuzzy set.

2 Preliminaries

Definition 2.1, [2]. Let X be a given set. An Intuitionistic fuzzy set A in X is given by $A = \{(x,\ \mu_A(x),\ \nu_A(x)) \mid x \in X\}$ where $\mu_A, \nu_A : X \to [0,1]$, $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of non-membership of x in A, and $0 \leq \mu_A(x)+\nu_A(x) \leq 1$. For each $x \in X$, $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$ is the degree of hesitation.

Definition 2.2, [2]. Let D [0, 1] be the set of all closed subintervals of the interval [0, 1]. Let $X \neq \phi$ be a given set. An interval valued intuitionistic fuzzy set A in X is given by $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where $\mu_A : X \to D[0,1]$, $\nu_A : X \to D[0,1]$ with the condition $0 \leq sup_x\ \mu_A(x) + sup_x\ \nu_A(x) \leq 1$. The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to the set A. Thus, for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals whose lower and upper end points are respectively, denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$ and $\nu_{AL}(x)$, $\nu_{AU}(x)$.

A can also be denoted by $A = \{(x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]) : x \in X\}$, where $0 \leq \mu_{AU}(x) + \nu_{AU}(x) \leq 1$, $\mu_{AL}(x) \geq 0$ and $\nu_{AL}(x) \geq 0$.

We will denote the set of all the IVIFS in X by IVIFS(X).

Now we define some aggregation operators which are already in literature.

Definition 2.3, [7]. The arithmetic average operator for alternatives A_j $(j = 1, 2, ..., n)$ is defined by $F(A_1, A_2, ..., A_n) = (1 - \prod(1 - \mu_{Aj}(x)), \prod(\nu_{Aj}(x)))$. And the geometric average operator is defined by $G(A_1, A_2, ..., A_n) = (\prod \mu_{Aj}(x), 1 - \prod(1 - \nu_{Aj}(x)))$.

Definition 2.4, [7]. The weighted arithmetic average operator for alternatives A_j $(j = 1, 2, ..., n)$ is defined by $F_w(A_1, A_2, ..., A_n) = (1 - \prod(1 - \mu_{Aj}(x))^{w_j}, \prod(\nu_{Aj}(x))^{w_j})$, where w_j is the weight of $A_j (j = 1, 2, ..., n)$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Also the weighted geometric average operator is defined by

$$G_w(A_1, A_2, ..., A_n) = (\prod(\mu_{Aj}(x))^{w_j}, 1 - \prod(1 - \nu_{Aj}(x))^{w_j}),$$

where w_j is the weight of $A_j (j = 1, 2, ..., n)$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 2.5, [7]. Let $A_j (j = 1, 2, ..., n) \in$ IVIFS(X). The weighted geometric average operator for IVIFSs is defined by

$$G_w(A_1, A_2, A_3, ..., A_n) = \Pi {A_j}^{w_j} =$$
$$= ([\Pi {\mu_{A_jL}}^{w_j}(x), \Pi {\mu_{A_jU}}^{w_j}(x)], [1 - \Pi(1 - \nu_{A_jL}(x))^{w_j}, 1 - \Pi(1 - \nu_{A_jU}(x))^{w_j}]),$$

where w_j is the weight of $A_j (j = 1, 2, ..., n)$, $w_j \in [0,1]$ and $\sum_{i=1}^{n} w_j = 1$.

By assuming $w_j = 1/n(j = 1, 2, ..., n)$ then G_w is called an geometric average operator for $A_1, A_2, ..., A_n$. Clearly G_w is an IVIFS.

Also, for $A_j(j = 1, 2, ..., n) \in$ IVIFS(X).The weighted arithmetic average operator is defined by

$$F_w(A_1, A_2, A_3, ..., A_n) = \sum_j w_j A_j$$

$$([1 - \Pi(1 - \mu_{A_jL}(x))^{w_j}, 1 - \Pi(1 - \mu_{A_jU}(x))^{w_j}], [\Pi\nu_{A_jL}{}^{w_j}(x), \Pi\nu_{A_jU}{}^{w_j}(x)]),$$

where w_j is the weight of A_j ($j = 1, 2, ..., n$), $w_j \in [0, 1]$ and $\sum_{i=1}^{n} w_j = 1$.

By assuming $w_j = 1/n(j = 1, 2, ..n)$ then F_w is called an arithmetic average operator for $A_1, A_2, ..A_n$.

3 New aggregation operators, score function and accuracy function

Definition 3.1. For IF alternatives A_i, $i = 1, 2, ..., n$ based on criteria C_j, $j = 1, 2, ..., m$ if a_{ij} indicates the degree that the alternative A_i satisfies the criterion C_j and b_{ij} indicates the degree that the alternative A_i does not satisfies the criterion C_j. Then the mean based aggregation operator denoted by $A_{im} = [A_{il}, A_{iu}]$, where, $A_{il} = \sum w_j a_{ij}$ and $A_{iu} = \sum w_j(1 - b_{ij})$. w_j are the weights for the criteria C_j, with $\sum_{j=1}^{m} w_j = 1$.

Definition 3.2. For IVIF alternatives A_i, $i = 1, 2, ..., n$ based on criteria C_j, $j = 1, 2, ..., m$, if $[a_{ij}, b_{ij}]$ indicates the degree that the alternative A_i satisfies the criterion C_j and $[c_{ij}, d_{ij}]$ indicates the degree that the alternative A_i does not satisfies the criterion C_j. Then, the mean based aggregation operator denoted by $A_{im} = [A_{il}, A_{iu}]$, where,

$$A_{il} = \frac{\sum w_j(a_{ij} + b_{ij})}{2}$$

and

$$A_{iu} = \sum w_j(1 - \frac{c_{ij} + d_{ij}}{2}),$$

where w_j are the weights for the criteria C_j, with $\sum w_j = 1$.

Note 3.3. $A_{im} = [A_{il}, A_{iu}] \subseteq [0, 1]$.

Definition 3.4. For an alternative A_i, whose mean based aggregated value given by $A_{im} = [A_{il}, A_{iu}]$, score for A_i is $S(A_i) = \frac{A_{il}+A_{iu}}{2}$.

Definition 3.5. For alternatives A_i, A_j, with score given by $S(A_i) = \frac{A_{il}+A_{iu}}{2}$, if score of A_i = Score of A_j, then we can compare them by comparing their A_{il}. In other words, if score function of two alternatives are equal, then their accuracy function can be given by $A_c(A_i) = A_{il}$ the lower limit of the aggregated value.

Theorem 3.6. For two comparable alternatives A_1 and A_2 based on criteria C_1 and C_2 such that $A_1 \supset A_2$, then their scores $S(A_1) > S(A_2)$.

Proof: Let the IVIF alternatives A_1 and A_2 be such that $a_{11} > a_{21},\ a_{12} > a_{22},\ b_{11} > b_{21}, b_{12} > b_{22},\ c_{11} < c_{21}, c_{12} < c_{22}$, and $d_{11} < d_{21},\ d_{12} < d_{22}$. Then the score of A_1 is

$$\begin{aligned} S(A_1) &= \frac{A_{1l} + A_{1u}}{2} = \frac{\sum w_j(a_{1j} + b_{1j})}{2} + \sum w_j(1 - (\frac{c_{1j} + d_{1j}}{2})) \\ &= \frac{w_1(a_{11} + b_{11})}{2} + \frac{w_2(a_{12} + b_{12})}{2} + w_1(1 - (\frac{c_{11} + d_{11}}{2})) + w_2(1 - (\frac{c_{12} + d_{12}}{2})) \end{aligned} \tag{1}$$

And the score of A_2 is

$$\begin{aligned} S(A_2) &= \frac{A_{2l} + A_{2u}}{2} = \frac{\sum w_j(a_{2j} + b_{2j})}{2} + \sum w_j(1 - (\frac{c_{2j} + d_{2j}}{2})) \\ &= \frac{w_1(a_{21} + b_{21})}{2} + \frac{w_2(a_{22} + b_{22})}{2} + w_1(1 - (\frac{c_{21} + d_{21}}{2})) + w_2(1 - (\frac{c_{22} + d_{22}}{2})) \end{aligned} \tag{2}$$

From (1) and (2), $S(A_1) - S(A_2)$ is positive. □

Theorem 3.7 For two comparable alternatives A_1 and A_2 based on criteria C_1 and C_2 such that $A_1 \supset A_2$, then accuracy value of A_1 is greater than that of A_2.

Proof: Let the alternatives A_1 and A_2 be such that $a_{11} > a_{21}, a_{12} > a_{22},\ b_{11} > b_{21}$, $b_{12} > b_{22}, c_{11} < c_{21}, c_{12} < c_{22}$, and $d_{11} < d_{21}, d_{12} < d_{22}$.

We denote the accuracy of A_1 by $A_c(A_1)$ and that of A_2 by $A_c(A_2)$.

Then

$$A_c(A_1) - A_c(A_2) =$$

$$= \left(\frac{w_1(a_{11} + b_{11})}{2} + \frac{w_2(a_{12} + b_{12})}{2}\right) - \left(\frac{w_1(a_{21} + b_{21})}{2} + \frac{w_2(a_{22} + b_{22})}{2}\right)$$

is a positive number. Which shows A_1 is better than A_2.

3.1 Illustration 1

For two IF alternatives A_1 and A_2 based on two weighted criteria C_1(weight $w_1 = 0.4$) and C_2 (weight $w_2 = 0.6$) as follows

	C_1	C_2
A_1	$(0.4, 0.5)$	$(0.55, 0.25)$
A_2	$(0.3, 0.6)$	$(0.5, 0.4)$

Based on our aggregation operator we can find the lower limit for the aggregated interval for A_1 as $A_{1l} = 0.4 \times 0.4 + 0.6 \times 0.55 = 0.49$. The upper limit for the aggregated interval for A_1, i.e., $A_{1u} = 0.4 \times 0.5 + 0.6 \times 0.75 = 0.65$. Therefore, aggregated interval corresponding to A_1 is $[0.49, 0.65]$. Similarly for A_2, $A_{2l} = 0.42$ and $A_{2u} = 0.52$. Therefore, aggregated interval corresponding to A_2 is $[0.42, 0.52]$. The score for A_1 is $\frac{0.49+0.65}{2} = 0.57$ and score for A_2 is 0.47. In this method, the alternative A_1 is better than A_2.

3.2 Illustration 2

For two IVIF alternatives A_1 and A_2 based on two weighted criteria C_1(weight $w_1 = 0.4$) and C_2 (weight $w_2 = 0.6$) as follows

	C_1	C_2
A_1	$[0.4, 0.5], [0.2, 0.3]$	$[0.6, 0.7], [0.1, 0.2]$
A_2	$[0.3, 0.4], [0.5, 0.55]$	$[0.4, 0.5], [0.3, 0.5]$

The lower limit for the aggregated interval for A_1 as $A_{1l} = 0.4 \times 0.45 + 0.6 \times 0.65 = 0.57$. The upper limit for the aggregated interval for A_1 as $A_{1u} = 0.4 \times 0.75 + 0.6 \times 0.85 = 0.81$. Therefore, aggregated interval corresponding to A_1 is $[0.57, 0.81]$. Similarly for A_2, the aggregated interval is $[0.41, 0.55]$. The score for A_1 is $\frac{0.57+0.81}{2} = 0.69$ and score for A_2 is 0.48. In this method, the alternative A_1 is better than A_2. In both the above cases, no need to find the accuracy values of the alternatives. If necessary, use Definition 3.5.

4 Conclusion

Decision making is one of the crucial problems in real life. Usually we have to deal with multicriteria decision making problems. In this paper, we propose an aggregation operator to aggregate the criteria for intuitionistic fuzzy alternatives as well as for interval valued intuitionistic fuzzy alternatives. We also propose score function and accuracy function to rank the alternatives.

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