NIFS 17 (2011), 3, 25–29

Information decomposition of intuitionistic fuzzy digital processes

Marin Marinov

College of Energetics and Electronics
Technical University of Sofia
Sofia, Bulgaria
e-mail: mmarinov@ecad.tu-sofia.bg

Abstract: A method of decomposition, based on intuitionistic fuzzy information is presented. Two ways of decomposition are applied, which lead to information processing organization with different effectiveness and reliability. Intuitionistic aspect of information decomposition is providing additional possibilities for improvement of the processing.
Keywords: Decomposition, Intuitionistic fuzzy information, Effectiveness.
AMS Classification: 03E72

1 Introduction

Decomposition is an element of information theory of systems, providing the alternatives for digital and in particular for computing processes organization.

So far it is widely used when processes are described by the help of probabilistic (Shannon) information [1, 3]. In the present article we will apply this approach to processes, represented in terms of intuitionistic fuzzy sets, as introduced in [2].

The process of information decomposition consists of entropy description of set partitions. For this purpose the notions of information – mutual and conditional have to be used.

2 Information decomposition of discrete random processes

Let in the universe of discourse R discrete random variables X_i (I = 1, …, s), $X_i \in X$ be defined. Also let s-partitioning be applied, i.e. $X_1 \cup X_2 \ldots\ldots \cup X_s$. The meaning of $\cup$ is supported by the definition of probability function $P(X)$ in R. Thus, the Shannon information, contained in X will be:

$$I(X) = \sum_{x \in X} p(x_i) \log p(x_i) \tag{1}$$

Let X, Y and Z be discrete random variables. Then the conditional information between X and Y under condition of Z will be:

$$I(X, Y/Z) = H(X/Z) - H(X/Y \cup Z) \tag{2}$$

where $I(X, Y/Z)$ is interpreted as an uncertainty decreasing of X thanks to the fact that Y is known – same as with the unconditional information $I(X, Y)$ when Z is known before and after Y becomes known. In other words, *a-posteriori* and *a-priori* uncertainty of X is taken under condition of always known Z.

Following [3], the mutual information of discrete random variables $X_1, ..., X_s$ can be determined recursively as:

$$I(X_1, \ldots, X_s) = I(X_1, \ldots, X_{s-1}) - I(X_1, \ldots, X_{s-1}/ X_s) \tag{3}$$

Respectively the conditional information is presented by the same recursive function:

$$I(X_1, \ldots, X_{s-1}/ X_s) = I(X_1, \ldots, X_{s-2}/ X_{s-1}) - I(X_1, \ldots, X_{s-2}/ X_{s-1} \cup X_s) \tag{4}$$

And finally the mutual information of set X (assuming the equivalence of $I(X) = H(X)$ – the entropy of X) is determined by the following expression:

$$\begin{aligned} I(X_1,...,X_s) = & \sum_i H(X_i) - \sum_{ij} H(X_i \cup X_j) + \sum_{ijk} H(X_i \cup X_j \cup X_k) - ... + \\ & + (-1)^{s+1} H(X_i \cup ... \cup X_s) \end{aligned} \tag{5}$$

The value of information $I(X_1, \ldots, X_s)$ is the reduction of uncertainty that each variable provides for the other.

Further, a graphical interpretation of the mutual information of four discrete random variables $X_1, \ldots, X_4$ is presented on Fig. 1.

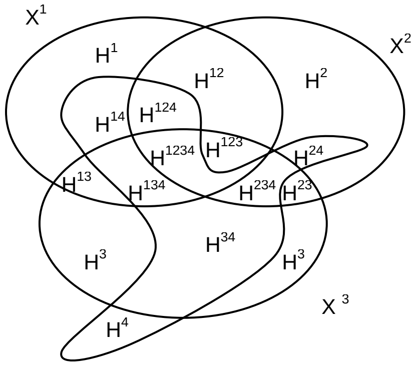

Figure 1. Presentation of four variables

The interpretation of the diagram is based on the following rules:

(a) Each area represents the entropy of corresponding variable;

(b) Intersection areas represent the mutual information of corresponding variables;

(c) Union of areas represent the entropy of the united variables;

(d) All areas are tagged with entropy denotation with superscripted indexes, corresponding to the indexes of the variables from the non-conditional part of every entropy term from expression (5), s being fixed.

All information and entropy areas can be defined in the way, shown below:

$$\begin{aligned} I(X_1, X_2) &= H^{12} + H^{123} + H^{124} + H^{1234} \\ I(X_1, X_2 / X_3 \cup X_4) &= H^{12} \\ I(X_1 / X_3 \cup X_4) &= H^1 + H^{12} \\ I(X_1 / X_2 \cup X_3 \cup X_4) &= H^1 \end{aligned} \tag{6}$$

3 Shannon information definition in intuitionistic fuzzy sets

In [4], we have shown an approach to Shannon information definition in intuitionistic fuzzy sets. For that purpose, we consider X as a set of probabilistic events x_i ($i = 1, \ldots, N$) with respective probabilities of occurrence p_i where

$$\sum_{i=1}^{N} p_i = 1 \tag{7}$$

Then, we decompose X into the following subsets:

$$\{x_1, \ldots, x_s\} \cup \{x_{s+1}, \ldots, x_q\} \cup \{x_{q+1}, \ldots, x_N\} \tag{8}$$

and the respective probabilities:

$$\{p_1, \ldots, p_s\} \cup \{p_{s+1}, \ldots, p_q\} \cup \{p_{q+1}, \ldots, p_N\}, \text{ where } 1 \le s \le q \le N \tag{9}$$

Next, we consider a subset $\{x_1, \ldots, x_s\}$ as one, which is probable to occur, while subset $\{x_{s+1}, \ldots, x_q\}$ is considered as one, which is not probable to occur.

Following Shannon approach, we determine normalized information on each subset in the following way:

$$\mu = \frac{\sum_{i=1}^{s} p_i \log p_i}{I(X)} \tag{10}$$

and

$$\nu = \frac{\sum_{i=s+1}^{q} p_i \log p_i}{I(X)} \tag{11}$$

Now we consider normalized information of both subsets as a degree of membership μ and a degree of non-membership ν, respectively, of a global set X, i.e. we can reconsider now this set as an intuitionistic one. It is clear that the condition $\mu + \nu \le 1$ is satisfied. And both indices are source of knowledge of the behaviour of X, i.e. could it occur or not. In that sense, we can apply again the Shannon information definition and calculate two values of information on the already intuitionistic set X:

(a) the minimum information, which can be obtained using the degree of membership μ :

$$I_{ifs} \min = \sum_{x} \mu \log \mu \tag{12}$$

(b) the maximum information obtained from joint degree of membership and intuitionism: $(1 - \nu)$

$$I_{ifs} \max = \sum_{x} (1-\nu) \log \nu \tag{13}$$

In [5], it has been proven that the notions of independence and conditional probability, as defined in classic fuzzy theory, are valid when applied to intuitionistic fuzzy set i.e.:

$$P(X_1X_2) = P(X_1) \text{ and } P(X_1X_2) = P(X_1 / X_2).P(X_2), \text{ respectively.} \tag{14}$$

Based on this result the conditional intuitionistic fuzzy information, as a ground notion in a decomposition process, can be formulated and in turn - the decomposition in the spirit of (2–5). What is specific in the case of intuitionistic fuzzy information is the opportunity to decompose it using either its minimum or maximum value, i.e. expression (5) can be described in terms of (12) or (13): $I_{\min}(X_1, \ldots, X_s)$ and $I_{\max}(X_1, \ldots, X_s)$.

4 Technical aspects of information decomposition applied to intuitionistic fuzzy processes

The value of the mutual IF information of discrete random processes $I(X_1, \ldots, X_s)$ can be obtained using two "border" approaches, independently of the used information definition (*min* or *max*), as follows:

(a) calculation of $I(X_1, \ldots, X_s)$ avoiding dependences between processes i.e.

$$I^{\max}(X_1, \ldots, X_s) = \sum H(X_i) \tag{15}$$

(the superscripted index *max* differs from subscripted one, dedicated to the applied information definition)

(b) calculation of $I(X_1, \ldots, X_s)$ taking into account all interprocess dependences i.e.

$$\begin{aligned} I^{\min}(X_1,\ldots,X_s) = {} & \sum_i H(X_i) - \sum_{ij} H(X_i \cup X_j) + \sum_{ijk} H(X_i \cup X_j \cup X_k) - \ldots + \\ & + (-1)^{s+1} H(X_i \cup \ldots \cup X_s) \end{aligned} \tag{16}$$

In the first case, the information value is a maximum as it comprises all repeated conditional information components (all doubled intersected parts of the set's areas on Fig. 1). In that sense, this approach of process decomposition is highly redundant. Technically, it could be a meaning of redundant resources for processing repeated conditional information components. On the other hand, this redundancy might be useful from the point of view of reliability reasons.

The other approach provides the minimal value of information, because it is constructed by not repeated conditional information components only (the intersected parts on Fig. 1 are not doubled). Technically, this decomposition ensures the maximum effectiveness of processing.

Thus, the information decomposition provides a space for various solutions between *max* redundancy and *max* effectiveness of the processing design.

Now, we will consider the influence of intuitionistic fuzzy interpretation of processes on its decomposition. For better understanding we present graphically the decomposition on two variables (processes), applying *min* (12) and *max* (13) definitions of the IF information (Fig. 2).

As it could be expected, the fuzziness and intuitionism provide more flexibility and freedom in achieving the trade-off between effectiveness and redundancy (reliability) of the decomposition of processes. In fact, they assure 'softer' grading of processing organization taking into account or not the interprocess dependences, described by the help of either *min* or *max* information definitions.

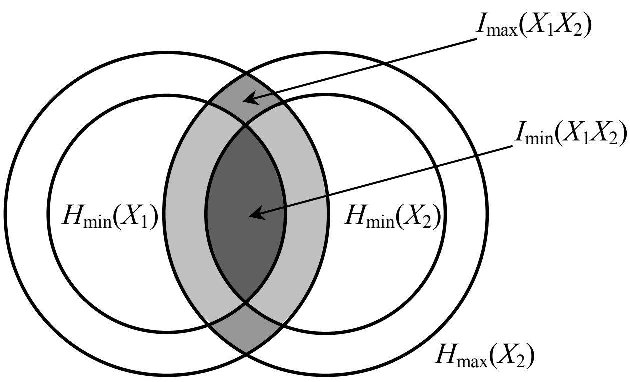

Figure 2. Presentation of two intuitionistic fuzzy processes

Graphically, it can be illustrated by the area of the mutual intuitionistic fuzzy information $I(X_1X_2)$, whose value (area) is ranging from *min* to *max*, depending on the used definitions (12), (13) for entropies $H(X_1)$ and $H(X_2)$. Nevertheless, it is not obligatory to use a unified definition, i.e. *min* and *max* definitions can be applied freely to all processes, participating in the decompositions. Thus, the designer gets more choices to balance processing. Graphically, this is represented by the four possible ways to "construct" $I(X_1X_2)$, compared to the single one, when the classical information approach is applied.

Conclusions

We propose a method of decomposition in the intuitionistic fuzzy space by the aid of minimum and maximum information definitions. We show that this decomposition is useful for the organization of effective and reliable information processing. The notion of intuitionism provides a third dimension of freedom when choosing the trade-off between effectiveness and reliability, which can improve the design and implementation of information processing systems.

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