Notes on Intuitionistic Fuzzy Sets
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Medical diagnosis in intuitionistic fuzzy context

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Abstract: In this paper, we introduce a distance formula for intuitionistic fuzzy sets called novel tangent inverse distance and found that it satisfies the properties of a distance measure. Application of tangent inverse distance in medical diagnosis is also discussed.
Keywords: Intuitionistic fuzzy sets, Tangent inverse distance, Medical diagnosis.
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1 Introduction

Intuitionistic fuzzy sets (IFSs) were proposed by K. T. Atanassov [1, 2] as an extension of Zadeh's fuzzy sets. They can express and process uncertainty in a better way, using hesitation degree. Most of the applications of IFS are implemented using some distance measures based approach. In literature many distance measures have been defined between IFSs like Hamming distance, normalized Hamming distance, Euclidean distance, normalized Euclidean distance etc. Among them tangent inverse distance has more accuracy and perfection [4].

In this paper, we propose the simplest form for tangent inverse distance given in [4] and discuss its properties and application in the field of medical diagnosis (see also [3, 7–10]).

2 Preliminaries

Definition 2.1. Intuitionistic fuzzy sets [1, 2]. Let X be a given set. An intuitionistic fuzzy set A in X is given by $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ where $\mu_A, \nu_A : X \to [0, 1]$, $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of non-membership of x in A, and $0 \le \mu_A(x) + \nu_A(x) \le 1$. For each $x \in X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the degree of hesitation.

Definition 2.2. Hamming distance [7, 8]. The Hamming distance between two intuitionistic fuzzy sets $A = (\mu_A(x_i), \nu_A(x_i), \pi_A(x_i))$ and $B = (\mu_B(x_i), \nu_B(x_i), \pi_B(x_i))$ is given by

$$d_{IFS}^1(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|).$$

Definition 2.3. Normalized Hamming distance [7, 8]. The normalized Hamming distance between two intuitionistic fuzzy sets $A = (\mu_A(x_i), \nu_A(x_i), \pi_A(x_i))$ and $B = (\mu_B(x_i), \nu_B(x_i), \pi_B(x_i))$ is given by

$$l_{IFS}^1(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|).$$

Definition 2.4. Tangent inverse distance [4]. Let $A = (\mu_A(x_i), \nu_A(x_i), \pi_A(x_i))$ and $B = (\mu_B(x_i), \nu_B(x_i), \pi_B(x_i))$ be two intuitionistic fuzzy sets. Then the tangent inverse distance is defined as

$$TID_{IFS}(A,B) = \frac{1}{2(n+1)}\sum_{i=1}^{n}\left[tan^{-1}\left[\frac{\pi}{4}[1 + |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|]\right]\right].$$

Propositions 2.5. [4].

(i) $TID_{IFS}(A,B) > 0$,

(ii) $TID_{IFS}(A,B) = TID_{IFS}(B,A)$,

(iii) If $A \subseteq B \subseteq C$ then $TID_{IFS}(A,C) \ge TID_{IFS}(A,B)$ & $TID_{IFS}(A,C) \ge TID_{IFS}(B,C)$.

3 A novel tangent inverse distance formula

Definition 3.1. Novel tangent inverse distance. Let $A = (\mu_A(x), \nu_A(x), \pi_A(x))$ and $B = (\mu_B(x), \nu_B(x), \pi_B(x))$ be two intuitionistic fuzzy sets. Then the tangent inverse distance is defined as

$$TID_{IFS}(A,B) = \frac{1}{2(n+1)}\sum_{i=1}^{n}\left[tan^{-1}[d_i]\right]$$

where

$$d_i = |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|$$

for $i = 1, 2, \ldots, n$.

Propositions 3.2.

(i) $TID_{IFS}(A,B) > 0$,

(ii) $TID_{IFS}(A,B) = TID_{IFS}(B,A)$,

(iii) If $A \subseteq B \subseteq C$ then $TID_{IFS}(A,C) \geq TID_{IFS}(A,B)$ & $TID_{IFS}(A,C) \geq TID_{IFS}(B,C)$.

Proof.

(i) Since $|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)| \geq 0$ we get $d_i \geq 0 \; \forall \; i = 1,2,\ldots,n$.

Therefore, $tan^{-1}[d_i] \geq 0 \qquad \forall \, i = 1,2,\ldots,n$.

Therefore, $\sum_{i=1}^{n}\left[tan^{-1}[d_i]\right] \geq 0$

Therefore, $\frac{1}{2(n+1)}\sum_{i=1}^{n}\left[tan^{-1}[d_i]\right] \geq 0$

i.e., $TID_{IFS}(A,B) > 0$

(ii) $TID_{IFS}(A,B) = \frac{1}{2(n+1)}\sum_{i=1}^{n}\left[tan^{-1}[d_i]\right]$

$$= \frac{1}{2(n+1)}\sum_{i=1}^{n}\left[tan^{-1}\left[|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right]\right]$$

$$= \frac{1}{2(n+1)}\sum_{i=1}^{n}\left[tan^{-1}\left[|\mu_B(x_i) - \mu_A(x_i)| + |\nu_B(x_i) - \nu_A(x_i)| + |\pi_B(x_i) - \pi_A(x_i)|\right]\right]$$

$$= TID_{IFS}(B,A)$$

(iii) We have if $A \subseteq B \subseteq C$ then $\begin{cases} \mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i) \\ \nu_A(x_i) \geq \nu_B(x_i) \geq \nu_C(x_i) \\ \pi_A(x_i) \geq \pi_B(x_i) \geq \pi_C(x_i) \end{cases}$

So,

$$|\mu_A(x_i) - \mu_B(x_i)| \leq |\mu_A(x_i) - \mu_C(x_i)|$$
$$|\mu_B(x_i) - \mu_C(x_i)| \leq |\mu_A(x_i) - \mu_C(x_i)|$$
$$|\nu_A(x_i) - \nu_B(x_i)| \leq |\nu_A(x_i) - \nu_C(x_i)|$$
$$|\nu_B(x_i) - \nu_C(x_i)| \leq |\nu_A(x_i) - \nu_C(x_i)|$$
$$|\pi_A(x_i) - \pi_B(x_i)| \leq |\pi_A(x_i) - \pi_C(x_i)|$$
$$|\pi_B(x_i) - \pi_C(x_i)| \leq |\pi_A(x_i) - \pi_C(x_i)|$$

From this we get $TID_{IFS}(A,C) \geq TID_{IFS}(A,B)$ & $TID_{IFS}(A,C) \geq TID_{IFS}(B,C)$.

4 Illustration

Following the numerical example in [3], let us consider four patients $\{P_1, P_2, P_3, P_4\}$ and the set of symptoms $S = \{S_1$ Temperature, S_2 Headache, S_3 Stomach pain, S_4 Cough, S_5 Chest pain$\}$. The intuitionistic fuzzy relation $Q(P \to S)$ is given as in Table 1. Let the set of diseases $D = \{D_1$ Viral fever, D_2 Malaria, D_3 Typhoid, D_4 Stomach problem, D_5 Chest problem$\}$. The intuitionistic fuzzy relation $R(S \to D)$ is given as in Table 2).

A	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.8,0.1,0.1)	(0.6,0.1,0.3)	(0.2,0.8,0.0)	(0.6,0.1,0.3)	(0.1,0.6,0.3)
P_2	(0.0,0.8,0.2)	(0.4,0.4,0.2)	(0.6,0.1,0.3)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
P_3	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.0,0.6,0.4)	(0.2,0.7,0.1)	(0.0,0.5,0.5)
P_4	(0.6,0.1,0.3)	(0.5,0.4,0.1)	(0.3,0.4,0.3)	(0.7,0.2,0.1)	(0.3,0.4,0.3)

Table 1. Patient–Symptom Relation

B	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.4,0.0,0.6)	(0.7,0.0,0.3)	(0.3,0.3,0.4)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Headache	(0.3,0.5,0.2)	(0.2,0.6,0.2)	(0.6,0.1,0.3)	(0.2,0.4,0.4)	(0.0,0.8,0.2)
Stomach pain	(0.1,0.7,0.2)	(0.0,0.9,0.1)	(0.2,0.7,0.1)	(0.8,0.0,0.2)	(0.2,0.8,0.0)
Cough	(0.4,0.3,0.3)	(0.7,0.0,0.3)	(0.2,0.6,0.2)	(0.2,0.7,0.1)	(0.2,0.8,0.0)
Chest Pain	(0.1,0.7,0.2)	(0.1,0.8,0.1)	(0.1,0.9,0.0)	(0.2,0.7,0.1)	(0.8,0.1,0.1)

Table 2. Symptom–Disease Relation

Using distance formula given in Definition 3.1 we get the following table.

T	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	0.2015	**0.1726**	0.1923	0.3244	0.3168
P_2	0.2464	0.2795	0.2164	**0.1127**	0.2627
P_3	0.2638	0.2851	**0.2176**	0.3019	0.3154
P_4	**0.2118**	0.2148	0.2659	0.2939	0.3393

Table 3. Novel tangent inverse distance

From the above table, P_1 is diagnosed with malaria, P_2 is diagnosed with stomach problem, P_3 is diagnosed with typhoid and P_4 is diagnosed with Viral fever.

Note: If the distance between a patient and a particular disease is the shortest, the patient is likely to have that disease.

5 Conclusions

We have introduced a novel distance formula for IFS called novel tangent inverse distance. It is elegant and simple than that given in [4]. It is useful to find the distance between IFS especially in medical diagnosis.

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