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Intuitionistic fuzzy optimization technique for the solution of an EOQ model

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Abstract: A purchasing inventory model with shortages where carrying cost, shortage cost, setup cost and demand quantity are considered as fuzzy numbers. The fuzzy parameters are transformed into corresponding interval numbers and then the interval objective function has been transformed into a classical multi-objective economic ordering quantity (EOQ) problem. To minimize the interval objective function, the order relation that represent the decision maker's preference between interval objective functions have been defined by the right limit, left limit, center and half width of an interval. Finally, the equivalent transformed problem has been solved by intuitionistic fuzzy programming technique. The proposed method is illustrated with a numerical example and sensitivity analysis has been done.

Keywords: Economic order quantity, Fuzzy demand, Fuzzy inventory cost parameters, Interval arithmetic, multi-objective programming, Intuitionistic fuzzy sets, Intuitionistic fuzzy optimization technique.
AMS Classification: 03E72

1 Introduction

Inventory problems are common in manufacturing, maintenance service and business operations in general. Often uncertainties may be associated with demand, various relevant costs, like, carrying cost, shortage cost and setup cost. In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory Zadeh [10], is applicable. Usually researchers considered different parameters of an inventory model either as constant or as dependent on time or probabilistic in nature for the development of the EOQ model. But, in real life situations, these parameters may have little deviations from the exact value, which may not follow any probability distribution. In these situations, if these parameters are treated as fuzzy parameters, then it will be more realistic.

Recently fuzzy concept is introduced in the inventory problems by several researchers. Park [5], Vujosevic [11] et al, Chang [4] et al, Lin [6] et al are proposed the EOQ model in the fuzzy sense where inventory parameters are triangular fuzzy number.

To deal with the ambiguous coefficients or parameters in an objective function, in mathematical programming inexact, fuzzy and interval programming techniques, Steuer [19], Tang [20], Ishibuchi and Tanaka [8] have been proposed. The programming technique is more flexible and allows to find the solutions which are more sufficient to the real problem. In fuzzy optimization the degree of acceptance of objectives and constraints are considered here. Now a days different modification and generalization form of fuzzy set theory have appeared. Intuitionistic fuzzy set is one of the generalization form of fuzzy set. The concept of an IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy sets

In this paper, we propose an inventory model with fuzzy inventory costs and fuzzy demand rate. The said fuzzy parameters are then converted into appropriate interval numbers following Grzegorzewski [13]. We propose a method to solve the EOQ inventory model using the concept of interval arithmetic. We have constructed an equivalent multi-objective deterministic model corresponding to the original problem with interval coefficients. To obtain the solution of this equivalent problem, we have used Intuitionistic fuzzy programming technique where the degree of acceptance and rejection of objectives are linear functions. Then this Intuitionistic fuzzy optimization is converted in to crisp one. It gives the $(\alpha - \beta)$ Pareto optimal solutions.

The advantage of the intuitionistic fuzzy optimization technique is twofold: they give the richest apparatus for formulation of optimization problems and, on the other hand, the solutions of intuionistic fuzzy optimization problems can satisfy the objective(s) with bigger degree than the analogous fuzzy optimization problem and the crisp one. In order to illustrate the solution method, numerical examples are provided. Sensitivity of the decision variables is examined to check the how far the output of the model is affected by changes or errors in its input parameters.

2 Intuitionistic fuzzy sets

Here we are to introduce first some relevant basic preliminaries, notations and definitions of IFS, in particular the works of Atanassov [2, 3].

Definition 1 Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An Atanasson's intuitionistic fuzzy set (IFS) in a given universal set X is an expression A given by

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\} \tag{1}$$

where the functions

$$\mu_A : X \to [0,1]$$
$$x_i \in X \to \mu_A(x_i) \in [0,1]$$

and

$$\nu_A : X \to [0,1]$$
$$x_i \in X \to \nu_A(x_i) \in [0,1]$$

define the degree of membership and the degree of non-membership of an element $x_i \in X$ to the set $A \subseteq X$, respectively, such that they satisfy the following condition: for every $x_i \in X$

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

Let

$$\pi_A(x_i) = 1 - \mu_A(x) - \nu_A(x)$$

which is called the Atanassov's [3] intuitionistic index of an element x_i in the set A. It is the degree of indeterminacy membership of the element x_i to the set A. Obviously,

$$0 \leq \pi_A(x_i) \leq 1$$

If an Atanassov's IFS C in X has only an element, then C is written as follows

$$C = \{\langle x_k, \mu_C(x_k), \nu_C(x_k)\rangle\}$$

which is usually denoted by $C = \{\langle \mu_C(x_k), \nu_C(x_k)\rangle\}$ for short.

Definition 2 Let A and B be two Atanassov's IFSs in the set X. $A \subset B$ iff $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$; for any $x_i \in X$.

Definition 3 Let A and B be two Atanassov's IFSs in the set X. $A = B$ iff $\mu_A(x_i) = \mu_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$; for any $x_i \in X$. Namely, $A = B$ iff $A \subset B$ and $B \subset A$.

Definition 4 Let A and B be two Atanassov's IFSs in the set X. The intersection of A and B is defined as follows:

$$A \cap B = \{\langle x_i, \min(\mu_A(x_i), \mu_B(x_i)), \max(\nu_A(x_i), \nu_B(x_i))\rangle | x_i \in X\}.$$

Intuitionistic fuzzy optimization model: On the basis of intuitionistic fuzzy sets, an intuitionistic fuzzy optimization, the crisp transformation and the solution procedure is described by Nayak and Pal [15, 16].

According to IFO theory, we are to maximize the degree of acceptance of the IF objective(s) and constraints and to minimize the degree of rejection of IF objective(s) and constraints as

$$\left.\begin{array}{c} \max\limits_{x \in \Re^n}\{\mu_k(x)\}; \\ \min\limits_{x}\{\nu_k(x)\}; \\ \mu_k(x), \nu_k(x) \geq 0; \\ \mu_k(x) \geq \nu_k(x); \\ 0 \leq \mu_k(x) + \nu_k(x) \leq 1; \end{array}\right\} k = 1, 2, \ldots, p+q$$

where $\mu_k(x)$ denotes the degree of acceptance of x from the k^{th} IFS and $\nu_k(x)$ denotes the degree of rejection of x from the k^{th} IFS. According to Atanassov property of IFS, the conjunction of intuitionistic fuzzy objective(s) and constraints in a space of alternatives U is defined as

$$A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle : x \in U\}, \tag{2}$$

which is defined as the *intuitionistic fuzzy decision set* (IFDS), where A denotes the integrated intuitionistic fuzzy objective/ goals and B denotes integrated intuitionistic fuzzy constraint set and they can be written as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\} = \{\langle x, \min_{i=1}^{p} \mu_i(x), \max_{i=1}^{p} \nu_i(x)\rangle : x \in U\} \tag{3}$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in U\} = \{\langle x, \min_{j=1}^{q} \mu_j(x), \max_{j=1}^{q} \nu_j(x)\rangle : x \in U\}. \tag{4}$$

Let the intuitionistic fuzzy decision set (2) be denoted by C, then min-aggregator is used for conjunction and max operator for disjunction

$$C = A \cap B = \{\langle x, \mu_C(x), \nu_C(x)\}\rangle | x \in U\}, \tag{5}$$

where, $$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\} = \min_{k=1}^{p+q} \mu_k(x) \tag{6}$$

and $$\nu_C(x) = \max\{\nu_A(x), \nu_B(x)\} = \max_{k=1}^{p+q} \nu_k(x), \tag{7}$$

where $\mu_C(x)$ denotes the degree of acceptance of IFDS and $\nu_C(x)$ denotes the degree of rejection of IFDS. Therefore,

$$\mu_C(x) \leq \mu_k(x), \nu_C(x) \geq \nu_k(x); \;\; 1 \leq k \leq p+q. \tag{8}$$

The formula can be transformed to the following system

$$\begin{array}{cc} \max \alpha, \quad \min \beta & \\ \alpha \leq \mu_k(x); & k = 1, 2, \ldots, p+q \\ \beta \geq \nu_k(x); & k = 1, 2, \ldots, p+q \\ \alpha \geq \beta; & \text{and } \alpha + \beta \leq 1; \alpha, \beta \geq 0 \end{array}$$

where α denotes the minimal acceptable degree of objective(s) and constraints and β denotes the maximal degree of rejection of objective(s) and constraints. The IFO model can be changed into the following certainty (non-fuzzy) optimization model as:

$$\left.\begin{array}{cc} \max(\alpha - \beta) & \\ \alpha \leq \mu_k(x); & k = 1, 2, \ldots, p+q \\ \beta \geq \nu_k(x); & k = 1, 2, \ldots, p+q \\ \alpha \geq \beta; & \text{and } \alpha + \beta \leq 1; \alpha, \beta \geq 0 \end{array}\right\} \tag{9}$$

which can be easily solved by some simplex methods.

3 Interval number

Let $\Re$ be the set of all real numbers. An interval, Moore [17], may be expressed as

$$\overline{a} = [a_L, a_R] = \{x : a_L \leq x \leq a_R, a_L \in \Re, a_R \in \Re\}, \tag{10}$$

where a_L and a_R are called the lower and upper limits of the interval $\overline{a}$, respectively.If $a_L = a_R$ then $\overline{a} = [a_L, a_R]$ is reduced to a real number a, where $a = a_L = a_R$. Alternatively an interval $\overline{a}$ can be expressed in mean-width or center-radius form as $\overline{a} = \langle m(\overline{a}), w(\overline{a})\rangle$, where $m(\overline{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\overline{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\overline{a}$. The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

Basic interval arithmetic: Let $\overline{a} = [a_L, a_R] = \langle m(\overline{a}), w(\overline{a})\rangle$ and $\overline{b} = [b_L, b_R] = \langle m(\overline{b}), w(\overline{b})\rangle \in I(\Re)$, then

$$\overline{a} + \overline{b} = [a_L + b_L, a_R + b_R]; \;\; \overline{a} + \overline{b} = \langle m(\overline{a}) + m(\overline{b}), w(\overline{a}) + w(\overline{b})\rangle. \tag{11}$$

The multiplication of an interval by a real number $c \neq 0$ is defined as

$$c\overline{a} = [ca_L, ca_R]; \quad \text{if } c \geq 0 \text{ and } c\overline{a} = [ca_R, ca_L]; \quad \text{if } c < 0.$$
$$c\overline{a} = c\langle m(\overline{a}), w(\overline{a})\rangle = \langle cm(\overline{a}), |c|w(\overline{a})\rangle. \tag{12}$$

The difference of these two interval numbers is

$$\overline{a}-\overline{b}=[a_L-b_R,a_R-b_L]. \tag{13}$$

The product of these two distinct interval numbers is given by

$$\overline{a}.\overline{b}=[\min\{a_L.b_L,a_L.b_R,a_R.b_L,a_R.b_R\},\max\{a_L.b_L,a_L.b_R,a_R.b_L,a_R.b_R\}]\,. \tag{14}$$

The division of these two interval numbers with $0\notin B$ is given by

$$\overline{a}/\overline{b}=\left[\min\left\{\frac{a_L}{b_L},\frac{a_L}{b_R},\frac{a_R}{b_L},\frac{a_R}{b_R}\right\},\max\left\{\frac{a_L}{b_L},\frac{a_L}{b_R},\frac{a_R}{b_L},\frac{a_R}{b_R}\right\}\right]. \tag{15}$$

Comparison between interval numbers:

A brief comparison on different interval orders is given in [1, 14]. Let $\overline{a}=[a_L,a_R]=\langle m(\overline{a}),w(\overline{a})\rangle,\overline{b}=[b_L,b_R]=\langle m(\overline{b}),w(\overline{b})\rangle$ be two interval numbers within $I(\Re)$.

Definition 5 For $m(\overline{a})\leq m(\overline{b})$ and $w(\overline{a})+w(\overline{b})\neq 0$, an acceptability index to the premise $\overline{a}\prec\overline{b}$ is defined as follows [14]:

$$\Psi(\overline{a}\prec\overline{b})=\frac{m(\overline{b})-m(\overline{a})}{w(\overline{a})+w(\overline{b})}, \tag{16}$$

which is the value judgement or satisfaction degree of the decision makers (DM) that the interval $\overline{a}$ is not superior to $\overline{b}$ ($\overline{b}$ is not inferior to $\overline{a}$) in terms of value.

Thus, the max operator ”$\vee$” for two intervals $\overline{a}$ and $\overline{b}$ is defined as follows [14]:

$$\overline{a}\vee\overline{b}=\begin{cases}\overline{b}, & \text{if } \Psi(\overline{a}\leq\overline{b})>0\\ \overline{a}, & \text{if } \Psi(\overline{a}\leq\overline{b})=0 \text{ and } w(\overline{a})<w(\overline{b}) \text{ and DM is pessimistic}\\ \overline{b}, & \text{if } \Psi(\overline{a}\leq\overline{b})=0 \text{ and } w(\overline{a})<w(\overline{b}) \text{ and DM is optimistic.}\end{cases} \tag{17}$$

Similarly, the min operator ”$\wedge$” for two intervals $\overline{a}$ and $\overline{b}$ is defined as follows [14]:

$$\overline{a}\wedge\overline{b}=\begin{cases}\overline{b}, & \text{if } \Psi(\overline{b}\leq\overline{a})>0\\ \overline{a}, & \text{if } \Psi(\overline{b}\leq\overline{a})=0 \text{ and } w(\overline{a})>w(\overline{b}) \text{ and DM is pessimistic}\\ \overline{b}, & \text{if } \Psi(\overline{b}\leq\widetilde{a})=0 \text{ and } w(\overline{a})>w(\overline{b}) \text{ and DM is optimistic.}\end{cases} \tag{18}$$

In the sequent discussions, the max operator ”$\vee$” in equation (17) and the min operator ”$\wedge$” in equation (18) are meant to be in the sense of equation (16) unless specially stated.

Optimization in interval environment: Now we defined a general objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize }\left\{\overline{Z}(x)=\overline{A}_1x_1+\overline{A}_2x_2+\ldots+\overline{A}_nx_n;\;\; x\in S\right\}, \tag{19}$$

where S is a feasible region of x and A_i is an interval number. Let the interval coefficient. Since each interval coefficients A_i are interval number then $\overline{Z}(x)$ is of the form $[Z_L(x),Z_R(x)]=\langle Z_C(x),Z_W(x)\rangle$. The solution set of (19) can be obtained as the Pareto optimal solutions of the multiobjective problem [8]

$$\text{Minimize }\{Z_R(x),Z_C(x);\;\; x\in S\} \tag{20}$$

3.1 Nearest interval approximation

In this section, we approximate a fuzzy number by a crisp number according to Grzegorzewski [13]. For two arbitrary fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ with α - cuts $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$ respectively, distance between $\widetilde{A}$ and $\widetilde{B}$ is given by

$$d(\widetilde{A}, \widetilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha} \tag{21}$$

. For a given fuzzy number $\widetilde{A}$ we will try to find a closed interval $C_d(\widetilde{A})$ which is the nearest to the fuzzy number $\widetilde{A}$ with respect to metric d. Let $(C_d(\widetilde{A}))_\alpha = C_d(\widetilde{A}) = [C_L, C_R], \forall\alpha \in (0, 1]$. Now we are to minimize

$$d(\widetilde{A}, C_d(\widetilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L)^2 d\alpha + \int_0^1 (A_R(\alpha) - C_R)^2 d\alpha} \tag{22}$$

with respect to C_L and C_R. In order to minimize $d(\widetilde{A}, C_d(\widetilde{A}))$ it suffices to minimize the function $D(C_L, C_R) = d^2(\widetilde{A}, C_d(\widetilde{A}))$.
Thus we have to find partial derivatives and then to solve $\frac{\partial D(C_L,C_R)}{\partial C_L} = 0$ and $\frac{\partial D(C_L,C_R)}{\partial C_R} = 0$ we have

$$C_L = \int_0^1 (A_L(\alpha) d\alpha \text{ and } C_R = \int_0^1 (A_R(\alpha) d\alpha \tag{23}$$

Moreover, since

$$\begin{vmatrix} \frac{\partial^2 D(C_L,C_R)}{\partial C_L^2} & \frac{\partial^2 D(C_L,C_R)}{\partial C_L \partial C_R} \\ \frac{\partial^2 D(C_L,C_R)}{\partial C_L \partial C_R} & \frac{\partial^2 D(C_L,C_R)}{\partial C_R^2} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0 \tag{24}$$

and $\frac{\partial^2 D(C_L,C_R)}{\partial C_L^2} = 2 > 0$, then C_L and C_R given by (23), actually, minimize $D(C_L, C_R)$ and simultaneously minimize $d(\widetilde{A}, C_d(\widetilde{A})$. Therefore, the interval

$$C_d(\widetilde{A}) = \left[\int_0^1 (A_L(\alpha) d\alpha, \int_0^1 (A_R(\alpha) d\alpha\right] \tag{25}$$

is the nearest interval approximation of fuzzy number $\widetilde{A}$ with respect to metric d. Let $\widetilde{A} = (a_1, a_2, a_3)$ be a fuzzy number. The α - level interval of $\widetilde{A}$ is defined as $[A_L(\alpha), A_R(\alpha)]$. When $\widetilde{A}$ is a triangular fuzzy number(TFN) with the following membership function

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & ; \quad \text{if } a_1 \le x < a_2 \\ \frac{a_3-x}{a_3-a_2} & ; \quad \text{if } a_2 < x < a_3 \\ 0 \quad ; & \quad \text{otherwise.} \end{cases} \tag{26}$$

Then for $\alpha \in (0, 1]$, we get the α-level interval of $\widetilde{A}$ as

$$[A_L(\alpha), A_R(\alpha)] = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha]$$

and hence by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 [a_1 + (a_2 - a_1)\alpha]d\alpha = \frac{a_1 + a_2}{2},$$
$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 [a_3 - (a_3 - a_2)\alpha]d\alpha = \frac{a_2 + a_3}{2}.$$

Therefore, the interval number considering $\widetilde{A}$ as a TFN is $\left[\frac{a_1+a_2}{2}, \frac{a_3+a_2}{2}\right]$.

4 Model formulation

To develop the proposed EOQ model, the following notations and assumptions are used throughout the paper.

(i) t_1 the time of the inventory cycle when on hand inventory reaches to zero;

(ii) $T - t_1$ is the duration of the inventory cycle when stock out occurs;

(iii) $\widetilde{Q}$ lot size per cycle;

(iv) $\widetilde{S}$ the order level to which the inventory is planned in the beginning of each scheduling period;

(v) The demand rate $\widetilde{D}$ per unit time is imprecise in nature i.e., $\widetilde{D} = (d - \triangle_{d1}, d, d + \triangle_{d2})$;

(vi) The inventory carrying cost or holding cost per unit per unit time, the shortage cost per unit item per unit time, the ordering or setup cost per cycle are imprecise in nature i.e., $\widetilde{C}_1 = (C_1 - \triangle_{C11}, C_1, C_1 + \triangle_{C12}), \widetilde{C}_2 = (C_1 - \triangle_{C21}, C_2, C_2 + \triangle_{C22})$ and $\widetilde{C}_3 = (C_3 - \triangle_{C31}, C_3, C_3 + \triangle_{C32})$.

Assumptions: We have the following assumptions:

(i) Production rate or replenishment rate is infinite.

(ii) Lead time is zero.

(iii) The inventory planning horizon is infinite and the inventory system involves only one item and one stocking point.

A typical behavior of the EOQ purchasing inventory model with uniform demand and with shortage is depicted in Figure 1. In this model, we can easily observe that the inventory carrying cost C_1 as well as shortages cost C_2 will be involved only when $0 \leq S \leq Q$. In the above figure the area of $\triangle BCE$ represents the failure to meet the demand and the area of $\triangle AOB$ represents the inventory. Since Q is the lot size sufficient to meet the demand for time T, but ($< Q$ i.e. Ordering cost + Carrying S) amount of stock is planned in order to meet the demand for time t_1 shortage of amount $Q - S$ will arise for the entire remaining period $T - t_1$.

In this model, each production cycle time T consists of two parts t_1 and $T - t_1$ where,

(i) t_1 is the period during which the stock S decreases at the rate of D units per unit time and reaches to zero and

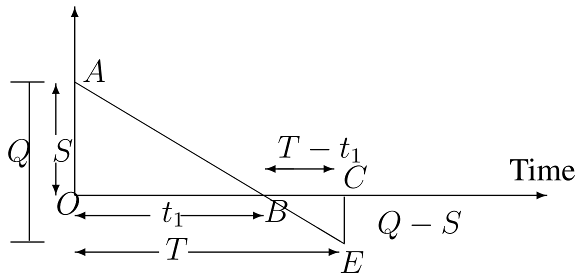

Figure 1: EOQ purchasing inventory model with shortage

(ii) $T - t_1$ is the period during which shortages are occurred.

Therefore, the total average cost $C(Q, S)$ is given by

$$\begin{aligned} C(Q,S) &= \text{Ordering cost + Carrying cost + Shortage cost} \\ &= \frac{C_3.D}{Q} + \frac{1}{2}.\frac{C_1.S^2}{Q} + \frac{1}{2}.\frac{C_2.(Q-S)^2}{Q} \end{aligned} \tag{27}$$

The purpose of the EOQ model is to find the optimal order quantity of inventory items at each time such that the total average cost is minimal. Thus by using calculus, we optimize $C(Q, S)$ and we get optimum values of Q, S and $C(Q, S)$.

In the crisp case the optimum values of Q and S for which the total average cost $C(Q, S)$ in (27) is minimum can be derived as

$$S^* = \sqrt{\frac{2.C_2.C_3.D}{C_1(C_1+C_2)}}, Q^* = \sqrt{\frac{2.C_3(C_1+C_2).D}{C_1.C_2}}.$$

4.1 Fuzzy EOQ model

We assume that the demand rate, holding cost, shortage cost, and set up cost are fuzzy numbers. Then the equation (27) reduces to

$$\widetilde{C}(Q,S) = \frac{\widetilde{C}_3.\widetilde{D}}{Q} + \frac{1}{2}.\frac{\widetilde{C}_1.S^2}{Q} + \frac{1}{2}.\frac{\widetilde{C}_2.(Q-S)^2}{Q}; \text{ where } 0 \leq S \leq Q \tag{28}$$

Now, we represent this fuzzy EOQ model to a deterministic form such that it can be easily tackled. Following Grzegorzewski [13], the fuzzy numbers are transformed into interval numbers as

$$\left.\begin{aligned} \widetilde{D} &= (d - \Delta_{d1}, d, d + \Delta_{d2}) \equiv [d_L, d_R] \\ \widetilde{C_1} &= (C_1 - \Delta_{C11}, C_1, C_1 + \Delta_{C12}) \equiv [C_{1L}, C_{1R}] \\ \widetilde{C_2} &= (C_2 - \Delta_{C21}, C_2, C_2 + \Delta_{C22}) \equiv [C_{2L}, C_{2R}] \\ \widetilde{C_3} &= (C_3 - \Delta_{C31}, C_3, C_3 + \Delta_{C32}) \equiv [C_{3L}, C_{3R}]. \end{aligned}\right\} \tag{29}$$

Using (29) the expression (28) becomes

$$\widetilde{C}(Q,S) = [f_L, f_R], \tag{30}$$

$$\text{where,} \quad f_L = \frac{C_{3L}.d_L}{Q} + \frac{1}{2}.\frac{C_{1L}.S^2}{Q} + \frac{1}{2}.\frac{C_{2L}.(Q-S)^2}{Q} \tag{31}$$

$$\text{and} \quad f_R = \frac{C_{3R}.d_R}{Q} + \frac{1}{2}.\frac{C_{1R}.S^2}{Q} + \frac{1}{2}.\frac{C_{2R}.(Q-S)^2}{Q} \tag{32}$$

Addition and other composition rules (seen in the section 3 in this paper) on interval numbers are used in these equations. Hence, the proposed model can be stated as

$$\text{Minimize } \{f_L(S,Q), f_R(S,Q)\}. \tag{33}$$

Generally, the multi-optimization problem (33), in the case of minimization problem, is formulated in a conservative sense from (20) as

$$\text{Minimize } \{f_C(S,Q), f_R(S,Q)\}\,. \tag{34}$$
$$\text{subject to} \qquad 0 \le S \le Q, \text{ where } f_C = \frac{f_L + f_R}{2}$$

5 IF programming technique for solution

To solve multi-objective minimization problem given by (34), we have used the following IF programming technique.

For each of the objective functions $f_C(S,Q), f_R(S,Q)$, we first find the lower bounds L_C, L_R (best values) and the upper bounds U_C, U_R (worst values), where L_C, L_R are the aspired level achievement and U_C, U_R are the highest acceptable level achievement for the objectives $f_C(S,Q)$, $f_R(S,Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance, or leeway, for objective $f_k(S,Q), k = C, R.$ Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of intuitionistic fuzzy programming technique are as follows

Step 1: Solve the multi-objective cost function as a single objective cost function using one objective at a time and ignoring all others.

Step 2: From the results of Step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From Step 2, we find for each objective, the best L_k and worst U_k values corresponding to the set of solutions. The initial fuzzy model of (28) can then be stated as, in terms of the aspiration levels for each objective, as follows: Find S and Q satisfying $f_k \widetilde{<} L_k$, $k = C, R$, subject to the non negatively conditions.

Step 4: Define a membership function (μ_{f_k}) and a non membership function (ν_{f_k}) for each objective function. A linear membership function is defined by as

$$\mu_{f_k} = \begin{cases} 1, & \text{if } f_k \le L_k \\ 1 - \frac{f_k - L_k}{d_k}, & \text{if } L_k \le f_k \le U_k \\ 0, & \text{if } f_k \ge U_k. \end{cases} \tag{35}$$

A linear non-membership function is defined by as

$$\nu_{f_k} = \begin{cases} 0, & \text{if } f_k \le L_k \\ \frac{f_k - L_k}{d_k}, & \text{if } L_k \le f_k \le U_k \\ 1, & \text{if } f_k \ge U_k. \end{cases} , \tag{36}$$

where $d_k = U_k - L_k$ is the tolerance of k^{th} objective function $f_k(S,Q)$.

Step 5: Find an equivalent crisp model by using membership and non-membership function for the initial fuzzy model.

Step 6: Solve the crisp model by appropriate mathematical programming algorithm. The solution obtained in Step 6 will be the optimal compromise solution of the multi-objective minimization problem given by (34).

If we use the linear membership and non-membership function as defined in (35) and (36), then an equivalent crisp (non-fuzzy) model for the intuitionistic fuzzy model can be formulated from (9) as

$$\left.\begin{array}{ll} \max(\alpha-\beta) & \\ \alpha \leq \mu_{f_k}; & k=1,2 \\ \beta \geq \nu_{f_k}; & k=1,2 \\ \alpha \geq \beta; & \text{and } \alpha+\beta \leq 1; \alpha, \beta \geq 0 \end{array}\right\} \tag{37}$$

which can be written in the form

$$\left.\begin{array}{c} \max(\alpha-\beta) \\ \alpha \leq 1-\frac{f_R-L_R}{U_R-L_R}; \\ \alpha \leq 1-\frac{f_C-L_C}{U_C-L_C}; \\ \beta \geq \frac{f_R-L_R}{U_R-L_R} \\ \beta \geq \frac{f_C-L_C}{U_C-L_C}; \\ 0 \leq S \leq Q; \ \ \alpha \geq \beta; \\ \text{and } \alpha+\beta \leq 1; \alpha, \beta \geq 0 \end{array}\right\} \tag{38}$$

5.1 Computational results

We consider inventory system with the following values of the parameter: $C_1 = 1.3$, $C_2 = 6$, $C_3 = 500$, $D = 19000$, $\triangle_{d_1} = 2000$, $\triangle_{d_2} = 2000$, $\triangle_{C_{11}} = 0.2$, $\triangle_{C_{12}} = 0.2$, $\triangle_{C_{21}} = 2$, $\triangle_{C_{22}} = 2$, $\triangle_{C_{31}} = 200$, $\triangle_{C_{32}} = 200$.

Considering the above fuzzy parameters as triangular fuzzy numbers (TFN), the nearest interval approximation according to Grzegorzewski [13] are $\widetilde{D} = [d_L, d_R] = [18000, 20000]$, $\widetilde{C}_1 = [C_{1L}, C_{1R}] = [1.2, 1.4]$, $\widetilde{C}_2 = [C_{2L}, C_{2R}] = [5, 7]$, $\widetilde{C}_3 = [C_{3L}, C_{3R}] = [400, 600]$.
Minimizing $f_R(S, Q)$, we get $S^R = 3779.6447$ and $Q^R = 4535.5737$. With these values of S^R and Q^R, the values of the objective functions f_R and f_C, denoted by f'_R and f'_C, are $f'_R = 5291.5026$ and $f'_C = 4541.8731$ respectively. Similarly minimizing $f_C(S, Q)$, we obtained $S^C = 3483.5330$ and $Q^C = 4238.4146$. With these values of S^C and Q^C, the values of the objective functions f_R and f_C, denoted by f''_R and f''_C, are $f''_R = 5305.9848$ and $f''_C = 4529.3564$ respectively. Then we calculate $L_R = min(f'_R, f''_R) = 5291.5026$, $U_R = max(f'_R, f''_R) = 5305.9848$, $L_C = min(f'_C, f''_C) = 4529.3564$, $U_C = max(f'_C, f''_C) = 4541.8731$. Using the equation (38), we formulate the following problem:

$$\left.\begin{array}{c} \max(\alpha-\beta) \\ 14.482161\alpha Q \leq 5305.984783Q - 1200000 - 0.7S^2 - 3.5(Q-S)^2; \\ 12.516723\alpha Q \leq 4541.873084Q - 960000 - 0.65S^2 - 3(Q-S)^2; \\ 14.482161\beta Q \geq 1200000 + 0.7S^2 + 3.5(Q-S)^2 - 5291.502622Q; \\ 12.516723\beta Q \geq 960000 + 0.65S^2 + 3(Q-S)^2 - 4529.356361Q; \\ 0 \leq S \leq Q; \ \ \alpha \geq \beta; \\ \text{and } \alpha+\beta \leq 1; \alpha, \beta \geq 0 \end{array}\right\} \tag{39}$$

where L_R, U_R, L_C and U_C are given above. The Pareto optimal solution of the problem is obtained as follows:

$$\begin{array}{l} f_L^* = 3769.8416, f_C^* = 4532.4780, f_R^* = 5295.1144, \\ S^* = 3629.225, Q^* = 4385.157(\alpha = 0.7506033, \beta = .2493967) \end{array}$$

5.2 Sensitivity analysis

Based on the numerical example considered above, we now study sensitivity of $\widetilde{S}^*$, $\widetilde{Q}^*$, f_L^*, f_C^* and f_R^* to changes in the values of the system parameters $\widetilde{C}_1$, $\widetilde{C}_2$, $\widetilde{C}_3$ and $\widetilde{D}$. The sensitivity analysis is performed by changing mid value of each parameters by $+50\%$, $+25\%$, -25% and -50%; taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown the following Table 1.

Table1: Effect of changes in the various parameters of the inventory model

Mid value of the parameter	% change	%Change in S^*	Q^*	f_L^*	f_C^*	f_R^*
$m(\widetilde{C}_1)$	$+50$	-21.0704	-14.5081	$+18.4864$	$+17.3903$	$+16.6099$
	$+25$	-12.0290	-8.3750	$+10.0875$	$+9.4113$	$+8.9299$
	-25	$+17.2473$	$+12.3720$	-12.5121	-11.4212	-10.6446
	-50	$+45.3200$	$+33.2239$	-28.9133	-25.9490	-23.8385
$m(\widetilde{C}_2)$	$+50$	$+6.8352$	$+0.7964$	$+4.4140$	$+3.3123$	$+2.5279$
	$+25$	$+1.6654$	$+1.6356$	$+2.2861$	$+1.8280$	$+1.5019$
	-25	$+2.50$	$+2.5746$	-3.7333	-2.8303	-2.1874
	-50	-6.9004	$+7.1070$	-10.8398	-7.8066	-5.6471
$m(\widetilde{C}_3)$	$+50$	$+20.0571$	$+19.9977$	$+26.7219$	$+17.7948$	$+18.9933$
	$+25$	$+10.3469$	$+10.2923$	$+14.0926$	$+11.6499$	$+9.9108$
	-25	-12.2150	-12.2584	-16.3410	-13.2067	-10.9753
	-50	-25.9930	-26.0296	-36.1618	-28.7678	-23.5036
$m(\widetilde{D})$	$+50$	$+21.4307$	$+21.3706$	$+23.3410$	$+22.2366$	$+21.4504$
	$+25$	$+10.7024$	$+10.7180$	$+12.2125$	$+11.6575$	$+11.2624$
	-25	-13.1791	-13.2220	-14.0663	-13.2449	-12.6601
	-50	-28.3354	-28.3709	-30.8503	-28.9000	-27.5115

From the Table 1, it is seen that

(i) $\widetilde{S}^*$ is fairly sensitive while $\widetilde{Q}^*$, f_L^*, f_C^* and f_R^* are moderately sensitive to changes in the value of the carrying cost $\widetilde{C}_1$.

(ii) Each of $\widetilde{S}^*$, $\widetilde{Q}^*$, f_L^*, f_C^* and f_R^* are not much sensitive to changes in the value of the shortage cost $\widetilde{C}_2$.

(iii) Each of $\widetilde{S}^*$, $\widetilde{Q}^*$, f_L^*, f_C^* and f_R^* are moderately sensitive to changes in the value of the setup cost $\widetilde{C}_3$ and $\widetilde{D}$.

6 Conclusion

In this paper, we have presented an inventory model with shortage, where carrying cost, shortage cost, ordering or setup cost and demand are assumed as fuzzy numbers instead of crisp or probabilistic in nature to make the inventory model more realistic. At first, we convert these fuzzy numbers in to interval numbers and then using intuitionistic fuzzy optimization model a solution procedure is given. In this approach, the degree of acceptance and the degree of rejection are introduced together. These cannot be simply consider as a complement of each other and the sum of their value is less than or equal to 1. A numerical example illustrates the proposed methods. Lastly, to study the effect of the determined quantities on changes of different parameters, a sensitivity analysis is also presented.

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References

[1] A. Sengupta and T. K. Pal, On comparing interval numbers, *European Journal of Operational Research*, 127(1) (2000) 28 - 43.

[2] Atanassov, K. Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87–96.

[3] Atanassov, K. *Intuitionistic fuzzy sets: Theory and Applications*, Physica-Verlag, 1999.

[4] Chung, K. J. An algorithm for an inventory model with inventory-level-dependent demand rate, *Computational Operation Research*, 30 (2003) 1311–1317.

[5] Park, K. S. Fuzzy set theoretic interpretation of economic order quantity, *IEEE Transactions on Systems, Man and Cybernetics, SMC*, 17 (1987) 1082–1084.

[6] Lin D. C., J.S. Yao. Fuzzy economic production for production inventory, *Fuzzy Sets and Systems*, 111 (2000) 465–495.

[7] Haris, F. How many parts to make at once, Factory, *The Magazine of Management,* 10(2)(1913), 135–136, 152.

[8] Ishibuchi, H., H. Tanaka. Multiobjective programming in optimization of the interval objective function, *European Journal of Operational Research*, 48 (1990) 219–225.

[9] Zimmermann, H.-J. Fuzzy Set Theory and its Application, Kluwer Academic Publishers, Boston, 1991.

[10] Zadeh, L. A. Fuzzy sets, *Information and Control*, 8 (1965) 338–352.

[11] Vujosevic, M., D. Petrovic, R. Petrovic. EOQ formula when inventory cost is fuzzy, *International Journal of Production Economics*, 45 (1996) 499–504.

[12] Gen, M., Y. Tsujimura, D. Zheng. An application of fuzzy set theory to inventory contol models, *Computers and Industrial Engineering*, 33 (1997) 553–556.

[13] Grzegorzewski, P. Nearest interval approximation of a fuzzy number, *Fuzzy Sets and Systems*, 130 (2002) 321–330.

[14] Nayak, P. K., M. Pal. The Bi-matrix games with interval pay-offs and its Nash Equlibrium strategy *The Journal of Fuzzy Mathematics*, 17(2)(2009) 421–435.

[15] Nayak, P. K., M. Pal. Bi-matrix games with intuitionistic fuzzy goals *Iranian Journal of Fuzzy Systems*, 1(7)(2010) 65–79.

[16] Nayak, P. K., M. Pal. Intuitionistic fuzzy optimization technique for Nash equilibrium solution of multi-objective bi-matrix games, to appear in *Journal of Uncertain Systems*.

[17] Moore, R.E. *Method and Application of Interval Analysis,* SIAM, Philadelphia, 1979.

[18] Bellman, R.E., L.A.Zadeh, Decision making under fuzzy enviornment, *Management Science*, 17(1970) 209–215.

[19] Steuer, R.E. Algorithm for linear programming problems with interval objective function coefficients, *Mathematics of operation research*, 6(1981) 333–348.

[20] Tang, S. Interval number and fuzzy number linear programming, *Fuzzy Sets and Systems*, 66(1994) 301–306.

[21] Chakrabortty, S., M. Pal, P.K. Nayak. Multisection technique to solve interval-valued purchasing inventory models without shortage, *Journal of Information and Computing Science,* 5(3)(2010) 173–182.

[22] Chakrabortty, S., M. Pal, P.K. Nayak. Solution of interval-valued manufacturing inventory model with shortages, *International Journal of Engineering and Physical Sciences,* 2010, 4:2 96–101.