**Notes on Intuitionistic Fuzzy Sets**
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# A new operator over intuitionistic fuzzy sets

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**Abstract:** A new operation is introduced over the intuitionistic fuzzy sets. Some of its properties are studied. It is a basis for introducing a new operator from the "weight-center" topological operator type over an intuitionistic fuzzy set.
**Keywords:** Intuitionistic fuzzy operation, Intuitionistic fuzzy operator, Weight-center operator.
**2010 Mathematics Subject Classification:** 03E72.

## 1 Introduction

In [1] some operations were introduced over Intuitionistic Fuzzy Sets (IFSs, see [1, 2]) and their properties were studied. In [2] it was mentioned that a part of these operations had not been used for any real purposes, and in the second book, they were omitted.

In the present research, a new operation is introduced and we hope that it will find its real applications.

## 2 Main results

First, following [1, 2], we mention that if the set $E$ is fixed, then the IFS $A$ in $E$ is defined by:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Now, for two IFSs $A$ and $B$ such that for each $x \in E$:

$$\mu_A(x) + \nu_A(x) + \mu_B(x) + \nu_B(x) > 0, \tag{1}$$

we define

$$A \triangle B = \left\{ \left\langle x, \frac{\mu_A(x) + \mu_B(x)}{\mu_A(x) + \nu_A(x) + \mu_B(x) + \nu_B(x)}, \frac{\nu_A(x) + \nu_B(x)}{\mu_A(x) + \nu_A(x) + \mu_B(x) + \nu_B(x)} \right\rangle | x \in E \right\}.$$

Let us assume that if the condition (1) is not satisfied for some $y \in E$, then

$$\left\langle y, \frac{\mu_A(x) + \mu_B(x)}{\mu_A(x) + \nu_A(x) + \mu_B(x) + \nu_B(x)}, \frac{\nu_A(x) + \nu_B(x)}{\mu_A(x) + \nu_A(x) + \mu_B(x) + \nu_B(x)} \right\rangle = \langle y, 0, 0 \rangle.$$

We can check that operation $\triangle$ is commutative, but not associative.

For the case of Intuitionistic Fuzzy Pairs (IFPs, see [3]), it has the form

$$\langle a, b \rangle \triangle \langle c, d \rangle = \left\langle \frac{a+c}{a+b+c+d}, \frac{b+d}{a+b+c+d} \right\rangle,$$

where $a, b, c, d \in [0, 1]$ and $a + b \leq 1, c + d \leq 1$ such that $a + b + c + d > 0$. For the case, when $a + b + c + d = 0$ we can assume as above that

$$\langle 0, 0 \rangle \triangle \langle 0, 0 \rangle = \langle 0, 0 \rangle.$$

For this (simpler) case, checking that operation $\triangle$ is not associative is easier.

We check that

$$\langle a, b \rangle \triangle \langle 1, 0 \rangle = \left\langle \frac{a+1}{a+b+1}, \frac{b}{a+b+1} \right\rangle,$$

$$\langle a, b \rangle \triangle \langle 0, 0 \rangle = \left\langle \frac{a}{a+b}, \frac{b}{a+b} \right\rangle,$$

$$\langle a, b \rangle \triangle \langle 0, 1 \rangle = \left\langle \frac{a}{a+b+1}, \frac{b+1}{a+b+1} \right\rangle,$$

$$\langle a, b \rangle \triangle \langle a, b \rangle = \left\langle \frac{a}{a+b}, \frac{b}{a+b} \right\rangle.$$

Having in mind the well-known IFS-triangular interpretation from Figure 1, we will show step-by-step the way for receiving a point

$$\left\langle \frac{a+c}{a+b+c+d}, \frac{b+d}{a+b+c+d} \right\rangle,$$

when we have points $\langle a, b \rangle$ and $\langle c, d \rangle$.

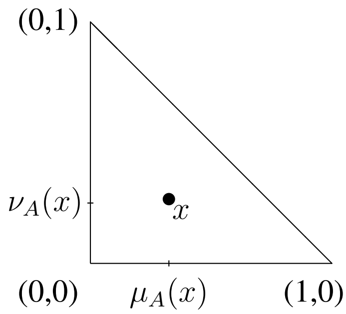


Figure 1. Geometric interpretation of the element $x$ ofthe IFS $A$.

In Figure 2, we draw section with length $a+c+b+d$ (see Figure 2). There are different cases for the length of this section in comparison with 1, but the procedure for all they is similar.

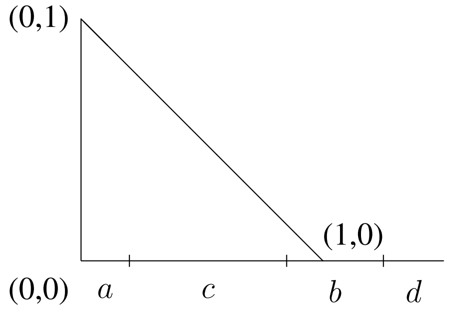


Figure 2. First step of the algorithm for determining the element $x$ from the geometric interpretation of the element $x$ of the IFS $A \triangle B$.

We fit together the point with coordinates $\langle a+b+c+d, 0\rangle$ with the point with coordinates $\langle 0, 1\rangle$ (see Figure 3), constructing a line. After this, we construct a line from point $\langle a+c, 0\rangle$ that is parallel with the previous one. It cut the ordinate in point $P$. It is calculated easy that the coordinates of point $P$ are

$$\left\langle 0, \frac{a+c}{a+b+c+d}\right\rangle.$$

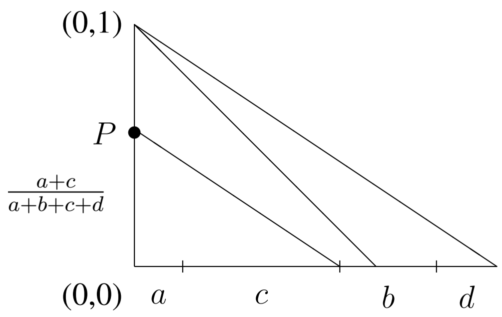


Figure 3. Second step of the algorithm for determining the element $x$ from the geometric interpretation of the element $x$ of the IFS $A \triangle B$.

We construct a line from point $P$ that is parallel to the hypothenuse of the IFS-interpretation triangle. The line cut the absciss in point $Q$ (see Figure 4). Its coordinates are

$$\left\langle \frac{a+c}{a+b+c+d}, 0\right\rangle.$$

Finally, we construct a perpendicular from point $Q$ to the hypothenuse of the IFS-interpretation triangle. The perpendicular cut the hypothenuse in point $R$ (see Figure 5). Its coordinates are

$$\left\langle \frac{a+c}{a+b+c+d}, \frac{b+d}{a+b+c+d}\right\rangle.$$

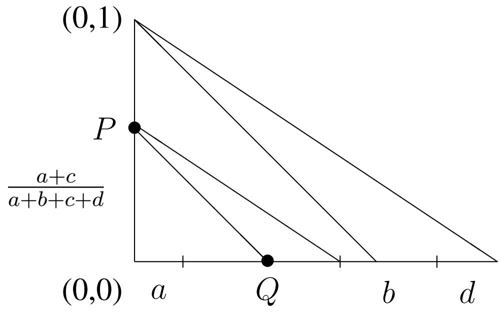


Figure 4. Third step of the algorithm for determining the element $x$ from the geometric interpretation of the element $x$ of the IFS $A \triangle B$.

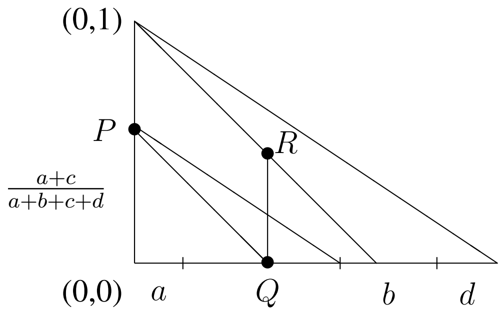


Figure 5. Fourth step of the algorithm for determining the element $x$ from the geometric interpretation of the element $x$ of the IFS $A \triangle B$.

Therefore, point $R$ represents the IFP that is a result of $\langle a, b\rangle \triangle \langle c, d\rangle$.

We can extend operation $\triangle$ from binary to $n$-ary form for $n$ IFPs $\langle a_1, b_1\rangle, \langle a_2, b_2\rangle, ..., \langle a_n, b_n\rangle$, as follows:

$$\triangle(\langle a_1, b_1\rangle, \langle a_2, b_2\rangle, ..., \langle a_n, b_n\rangle) = \left\langle \frac{\sum\limits_{i=1}^{n} a_i}{\sum\limits_{i=1}^{n}(a_i + b_i)}, \frac{\sum\limits_{i=1}^{n} b_i}{\sum\limits_{i=1}^{n}(a_i + b_i)} \right\rangle.$$

Now, we return to the IFS-form of the operation $\triangle$. When $E$ is a finite set, we define the operator

$$\bigtriangleup A = \left\{ \left\langle x, \frac{\sum\limits_{x \in E} \mu_A(x)}{\sum\limits_{x \in E} (\mu_A(x) + \nu_A(x))}, \frac{\sum\limits_{x \in E} \nu_A(x)}{\sum\limits_{x \in E} (\mu_A(x) + \nu_A(x))} \right\rangle | x \in E \right\}.$$

We can see that

$$\bigtriangleup\bigtriangleup A = \bigtriangleup A.$$

# 3 Conclusion

In [4] a procedure for de-i-fuzzification is described. It juxtaposes to each IFP $\langle \mu_A(x), \nu_A(x) \rangle$ related to element $x \in E$ the IFP

$$\left\langle \frac{\mu_A(x)}{\mu_A(x) + \nu_A(x)}, \frac{\nu_A(x)}{\mu_A(x) + \nu_A(x)} \right\rangle.$$

Therefore, we can represent the results of this procedure by one of both formulas

$$A \mathbin{\ooalign{$\triangle$\cr\hidewidth$\circ$\hidewidth\cr}} A$$

or

$$A \mathbin{\ooalign{$\triangle$\cr\hidewidth$\circ$\hidewidth\cr}} O^*,$$

where

$$O^* = \{\langle x, 0, 1 \rangle | x \in E\}.$$

# References


[1] Atanassov, K. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer, Heidelberg.

[2] Atanassov, K. (2012). *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin.

[3] Atanassov, K., Szmidt, E. & Kacprzyk, J. (2013). On Intuitionistic Fuzzy Pairs, *Notes on Intuitionistic Fuzzy Sets*, 19 (3), 1–13.

[4] Atanassova, V. & Sotirov, S. (2012). A new formula for de-i-fuzzification of intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 18 (3), 49–51.