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On Intuitionistic Fuzzy Trees and Their Index Matrix Interpretation

Panagiotis Chountas[1], Anthony Shannon[2], Parvathi Rangasamy[3] and Krassimir Atanassov[4]

[1] ECS - University of Westminster, London, HA1 3TP, UK
e-mail: *chountp@wmin.ac.uk*
[2] Warrane College, University of New South Wales, Kensington, 1465, Australia
e-mail: *t.shannon@warrane.unsw.edu.au*
[3] Department of Mathematics, Vellalar College for Women,
Erode - 638 009, Tamilnadu, India
e-mail: *paarvathis@rediffmail.com*
[4] Centre of Biomedical Engineering, Bulgarian Academy of Sciences
P.O.Box 12, Sofia-1113, Bulgaria
International University College, Dobrich, Bulgaria
e-mail: *krat@bas.bg*

1 Introduction

The concept of Intuitionistic Fuzzy Graph (IFG) was introduced in 1994 in [12]. It was an object of some subsequent extensions (see [4, 6, 13]), representations (see [2, 3, 5]) and applications (see [5]). In [8] we discussed an intuitionistic fuzzy version of the special particular case of a graph – the tree, called an *Intuitionistic Fuzzy Tree (IFTree).*

Below we will give the definition of an IFTree, will introduce its index matrix interpretation and will give an example for an application of the IFTrees.

2 Definition and properties of intuitionistic fuzzy trees

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

Let the oriented graph $G = (V, A)$ be given, where V is a set of vertices and A is a set of arcs. Every graph arc connects one or two graph vertices (see, e.g., [9]).

Following [12], we shall note that the set

$$A^* = \{\langle\langle v, w\rangle, \mu_A(v, w), \nu_A(v, w)\rangle \mid \langle v, w\rangle \in V \times V\}$$

is called an IFG if the functions $\mu_A : V \times V \to [0, 1]$ and $\nu_A : V \times V \to [0, 1]$ define the respective degrees of membership and non-membership of the element $\langle v, w\rangle \in V \times V$ and for all $\langle v, w\rangle \in V \times V$:

$$0 \leq \mu_A(v, w) + \nu_A(v, w) \leq 1.$$

The above definition can be transformed directly to the case of IFTree, but we will extend the new object.

Let us have a (fixed) set of vertices $\mathcal{V}$. An IFTree T (over $\mathcal{V}$) will be the ordered pair $T = (V^*, A^*)$, where

$$V \subset \mathcal{V},$$
$$V^* = \{\langle v, \mu_V(v), \nu_V(v)\rangle | v \in V\},$$
$$A \subset V \times V,$$
$$A^* = \{\langle g, \mu_A(g), \nu_A(g)\rangle | (\exists v, w \in V)(g = \langle v, w\rangle \in A)\},$$

where
$\mu_V(v)$ and $\nu_V(v)$ are degrees of membership and non-membership of the element $v \in \mathcal{V}$ to V and

$$0 \leq \mu_V(v) + \nu(v) \leq 1.$$

$\mu_A(g)$ and $\nu_A(g)$ are degrees of membership and non-membership of the element $g = \langle v, w\rangle \in V \times V$ to A and

$$0 \leq \mu_A(g) + \nu_A(g) \leq 1.$$

The IFTree $T = (V^*, A^*)$ is:
a) *weak well constructed (WWC-IFTree)* if

$$(\forall v, w \in V)((\exists g \in A)(g = \langle v, w\rangle) \to (\mu_V(v) \geq \mu_V(w) \& \nu_V(v) \leq \nu_V(w));$$

b) *strong well constructed (SWC-IFTree)* if

$$(\forall v, w \in V)((\exists g \in A)(g = \langle v, w\rangle)$$
$$\to (\mu_V(v) \geq \max(\mu_V(w), \mu_A(g)) \& \nu_V(v) \leq \min(\nu_V(w), \nu_A(g)));$$

c) *average well constructed (AWC-IFTree)* if

$$(\forall v, w \in V)((\exists g \in A)(g = \langle v, w\rangle) \to (\mu_V(v) \geq \frac{\mu_V(w) + \mu_A(g)}{2} \& \nu_V(v) \leq \frac{\nu_V(w) + \nu_A(g)}{2}).$$

Let two IFTrees $T_1 = (V_1^*, G_1^*)$ and $T_2 = (V_2^*, G_2^*)$ be given. We define:

$$T_1 \cup T_2 = (V_1^*, A_1^*) \cup (V_2^*, A_2^*) = (V_1^* \cup V_2^*, A_1^* \cup A_2^*),$$

$$T_1 \cap T_2 = (V_1^*, A_1^*) \cap (V_2^*, A_2^*) = (V_1^* \cap V_2^*, A_1^* \cap A_2^*).$$

Let

$$\mathcal{P}(X) = \{Y | Y \subset X\},$$

and let for $T = (V^*, A^*)$

$$T_{full} = (E(V), E(A)),$$
$$T_{empty} = (O(V), O(A)),$$

where

$$E(V) = \{\langle v, 1, 0\rangle | v \in \mathcal{V}\},$$
$$O(V) = \{\langle v, 0, 1\rangle | v \in \mathcal{V}\},$$
$$E(A) = \{\langle g, 1, 0\rangle | (\exists v, w \in V)(g = \langle v, w\rangle) \in \mathcal{V} \times \mathcal{V}\},$$
$$O(A) = \{\langle g, 0, 1\rangle | (\exists v, w \in V)(g = \langle v, w\rangle) \in \mathcal{V} \times \mathcal{V}\}.$$

Theorem: $(\mathcal{P}(\mathcal{V}), \cup, T_{empty})$ and $(\mathcal{P}(\mathcal{V}), \cap, T_{full})$ are commutative monoids.

3 Index matrix interpretation of the intuitionistic fuzzy trees

Following [1] the basic definitions and properties related to IMs will be given.

Let I be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. By an IM with index sets K and L $(K, L \subset I)$ we will mean the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array}$$

(or briefly: $[K, L, \{a_{k_i,l_j}\}]$), where $K = \{k_1, k_2, ..., k_m\}, L = \{l_1, l_2, ..., l_n\}$, for $1 \leq i \leq m$, and for $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$ – the set of the real numbers.

For two IMs different operations and relations are defined in [1, 7]. Here we will give only one of them.

Let $G = (V, A)$ be a given IFTree. We can construct its standard incidence matrix. After this, we can change the elements of the matrix with their degrees of membership and non-membership. Finally, numbering the rows and columts of the matrix with the identifiers of the IFTree vertices, we will obtain an IM.

For example, if we have the IFTree from Fig. 1, we can construct the IM that corresponds to its incidence matrix:

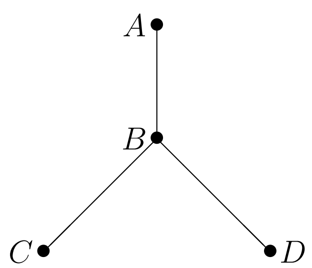

Fig. 1

$$[\{A, B, C, D\}, \{A, B, C, D\},$$

	A	B	C	D
A	$\langle\mu(A,A),\nu(A,A)\rangle$	$\langle\mu(A,B),\nu(A,B)\rangle$	$\langle\mu(A,C),\nu(A,C)\rangle$	$\langle\mu(A,D),\nu(A,D)\rangle$
B	$\langle\mu(B,A),\nu(B,A)\rangle$	$\langle\mu(B,B),\nu(B,B)\rangle$	$\langle\mu(B,C),\nu(B,C)\rangle$	$\langle\mu(B,D),\nu(B,D)\rangle$
C	$\langle\mu(C,A),\nu(C,A)\rangle$	$\langle\mu(C,B),\nu(C,B)\rangle$	$\langle\mu(C,C),\nu(C,C)\rangle$	$\langle\mu(C,D),\nu(C,D)\rangle$
D	$\langle\mu(D,A),\nu(D,A)\rangle$	$\langle\mu(D,B),\nu(D,B)\rangle$	$\langle\mu(D,C),\nu(D,C)\rangle$	$\langle\mu(D,D),\nu(D,D)\rangle$

Having in mind that arcs AA, AC, AD, BB, CC, CD and DD do not exist, we can modify the above IM to the form:

$$[\{A, B, C, D\}, \{A, B, C, D\},$$

	A	B	C	D
A	$\langle 0,1\rangle$	$\langle\mu(A,B),\nu(A,B)\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
B	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle\mu(B,C),\nu(B,C)\rangle$	$\langle\mu(B,D),\nu(B,D)\rangle$
C	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
D	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$

Now, we see, that all elements of the column indeced with A and all elements of the rows indeced with C and D are $\langle 0, 1\rangle$. Thereforfe, we can omit these two rows and the column and we will obtain the essentiall simple IM:

$$[\{A, B, C, D\}, \{A, B, C, D\},$$

	B	C	D
A	$\langle\mu(A,B),\nu(A,B)\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
B	$\langle 0,1\rangle$	$\langle\mu(B,C),\nu(B,C)\rangle$	$\langle\mu(B,D),\nu(B,D)\rangle$

Finally, having in mind that already there is not a column with index A and rows with indeces C and D, we obyain as a final form of the IM:

$$[\{A, B\}, \{B, C, D\},$$

	B	C	D
A	$\langle\mu(A,B),\nu(A,B)\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
B	$\langle 0,1\rangle$	$\langle\mu(B,C),\nu(B,C)\rangle$	$\langle\mu(B,D),\nu(B,D)\rangle$

4 Conclusion

In this paper, the definition of IFTree is outlined. Also, index matrix interpretation of the intuitionistic fuzzy trees is discussed. The authors further proposed to build intuitionistic fuzzy trees accordingly in various application domains, and their salient features can be assessed, compared with other existing tree models and algorithms.

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