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A new modal operator over intuitionistic fuzzy sets. Part 2

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Abstract: A new operator from modal type is introduced over the intuitionistic fuzzy sets. Some of its properties are studied. It is an extension of the series of modal type of operators, defined during last years.
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1 Introduction

In a series of papers collected in [2], a new type of intuitionistic fuzzy modal operators is introduced and some of their properties are studied. In the present paper, a new operator from modal type is introduced and some of its basic properties are studied. In the Conclusion, Open Problems are formulated.

2 Preliminary Results

Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Different relations and operations are introduced over the IFSs. Some of them (see, e.g. [1, 2]) are the following

$$\begin{array}{lcl}
A \subseteq B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\
A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)), \\
\neg A & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\
A @ B & = & \{\left\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2} \right\rangle | x \in E\}, \\
\Box A & = & \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}, \\
\Diamond A & = & \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.
\end{array}$$

In [2] some types of modal operators are described. The most general form of the second type of modal operators was introduced in [3]. It has the form

$$\otimes_{\alpha,\beta,\gamma,\delta} A = \{\langle x, \alpha.\mu_A(x) + \beta.\nu_A(x), \gamma.\mu_A(x) + \delta.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\alpha + \beta \leq 1, \gamma + \delta \leq 1$.

3 Main Results

Here, we introduce the following new operator from modal type, that is an extension of the operator $\otimes_{\alpha,\beta,\gamma,\delta}$. It has the form

$$\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A = \{\langle x, \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma, \delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ and

$$\max(\alpha + \delta, \beta + \varepsilon) + \gamma + \zeta \leq 1. \quad (1)$$

From (1) we see immediately that

$$\max(\alpha, \beta) + \gamma \leq \max(\alpha + \delta, \beta + \varepsilon) + \gamma + \zeta \leq 1 \tag{2}$$

and

$$\max(\delta, \varepsilon) + \zeta \leq \max(\alpha + \delta, \beta + \varepsilon) + \gamma + \zeta \leq 1. \tag{3}$$

It is easy to see that

$$\oplus_{1,0,0,0,1,0} A = A,$$

$$\oplus_{0,1,0,1,0,0} A = \neg A$$

and

$$\otimes_{\alpha,\beta,\gamma,\delta} A = \oplus_{\alpha,\gamma,0,\beta,\delta,0} A.$$

Therefore, this operator gives the possibility to express the operation identity, the operation "classical negation" and the operator $\otimes_{\alpha,\beta,\gamma,\delta}$. In this way, by varying the values of the variables $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$ in the $[0; 1]$ range, we can obtain the whole continuity of sets existing between a given set A and its classical negation $\neg A$.

Let us study the basic properties of the new operator.

First, we check that the new set is an IFS. Really, from (2) and (3) we obtain:

$$0 \leq \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma \leq \max(\alpha, \beta).(\mu_A(x) + \nu_A(x)) + \gamma$$

$$\leq \max(\alpha, \beta) + \gamma \leq 1,$$

$$0 \leq \delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta \leq \max(\delta, \varepsilon).(\mu_A(x) + \nu_A(x)) + \zeta$$

$$\leq \max(\delta, \varepsilon) + \zeta \leq 1$$

and from (1):

$$0 \leq \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma + \delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta$$

$$= (\alpha + \delta).\mu_A(x) + (\beta + \varepsilon).\nu_A(x) + \gamma + \zeta$$

$$\leq \max(\alpha + \delta, \beta + \varepsilon).(\mu_A(x) + \nu_A(x)) + \gamma + \zeta \leq 1.$$

Theorem 1. For every IFS A and for every six real numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ such that (1) is valid,

$$\neg \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \neg A = \oplus_{\delta,\gamma,\beta,\alpha} A.$$

Proof. We obtain sequentially that

$$\neg \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \neg A$$

$$= \neg \otimes_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}$$

$$= \neg\{\langle x, \alpha.\nu_A(x) + \beta.\mu_A(x) + \gamma, \delta.\nu_A(x) + \varepsilon.\mu_A(x) + \zeta\rangle | x \in E\}$$

$$= \{\langle x, \delta.\nu_A(x) + \varepsilon.\mu_A(x) + \zeta, \alpha.\nu_A(x) + \beta.\mu_A(x) + \gamma\rangle | x \in E\}$$

$$= \oplus_{\delta,\varepsilon,\zeta,\alpha,\beta,\gamma} A.$$

This completes the proof. □

Theorem 2. For every two IFSs A and B and for every six real numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ such that (1) is valid, it holds that

$$\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A@B) = \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A @ \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} B.$$

Proof. We obtain sequentially

$$\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A@B)$$

$$= \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \left(\{\langle x, \frac{\mu_A(x) + \mu_B(x)}{2}, \frac{\nu_A(x) + \nu_B(x)}{2}\rangle | x \in E\} \right)$$

$$= \{\langle x, \alpha \left(\frac{\mu_A(x) + \mu_B(x)}{2} \right) + \beta \left(\frac{\nu_A(x) + \nu_B(x)}{2} \right) + \gamma,$$

$$\delta \left(\frac{\mu_A(x) + \mu_B(x)}{2} \right) + \varepsilon \left(\frac{\nu_A(x) + \nu_B(x)}{2} \right) + \zeta\rangle | x \in E\}$$

$$= \{\langle x, \frac{\alpha\mu_A(x) + \alpha\mu_B(x)}{2} + \frac{\beta\nu_A(x) + \beta\nu_B(x)}{2} + \gamma,$$

$$\frac{\delta\mu_A(x) + \delta\mu_B(x)}{2} + \frac{\varepsilon\nu_A(x) + \varepsilon\nu_B(x)}{2} + \zeta\rangle | x \in E\}$$

$$= \{\langle x, \frac{\alpha\mu_A(x) + \beta\nu_A(x) + \gamma}{2} + \frac{\alpha\mu_B(x) + \beta\nu_B(x) + \gamma}{2},$$

$$\frac{\delta\mu_A(x) + \varepsilon\nu_A(x) + \zeta}{2} + \frac{\delta\mu_B(x) + \varepsilon\nu_B(x) + \zeta}{2}\rangle | x \in E\}$$

$$= \{\langle x, \alpha\mu_A(x) + \beta\nu_A(x) + \gamma, \delta\mu_A(x) + \varepsilon\nu_A(x) + \zeta\rangle | x \in E\}$$

$$@\{\langle x, \alpha\mu_B(x) + \beta\nu_B(x) + \gamma, \delta\mu_B(x) + \varepsilon\nu_B(x) + \zeta\rangle | x \in E\}$$

$$= \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A @ \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} B. \qquad \square$$

Theorem 3. For every IFS A and for every six real numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ such that (1) is valid, it holds that

(a) $\square \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A \subseteq \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \square A$,
(b) $\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \lozenge A \subseteq \lozenge \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A$.

Proof. First, we obtain

$$\square \oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A$$

$$= \square \{\langle x, \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma, \delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta\rangle | x \in E\}$$

$$= \{\langle x, \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma, 1 - (\alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma)\rangle | x \in E\}$$

and

$$\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} \square A$$

$$= \{\langle x, \alpha\mu_A(x) + \beta(1 - \mu_A(x)) + \gamma, \delta\mu_A(x) + \varepsilon(1 - \mu_A(x)) + \zeta\rangle | x \in E\}.$$

Immediately, we see that

$$\alpha\mu_A(x) + \beta(1 - \mu_A(x)) + \gamma \geq \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma$$

and

$$1 - (\alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma) - (\delta\mu_A(x) + \varepsilon(1 - \mu_A(x)) + \zeta)$$

$$= 1 - (\alpha + \delta - \varepsilon).\mu_A(x) - \beta.\nu_A(x) - \gamma - \zeta$$

$$\geq 1 - (\alpha + \delta - \varepsilon).\mu_A(x) - \beta.(1 - \mu_A(x)) - \gamma - \zeta$$

$$= 1 - (\alpha + \delta - \beta - \varepsilon).\mu_A(x) - \beta - \gamma - \zeta$$

$$\geq 1 - (\alpha + \delta - \beta - \varepsilon) - \beta - \gamma - \zeta$$

$$\geq 1 - \alpha - \delta + \varepsilon - \gamma - \zeta$$

from (1)

$$\geq \max(\alpha + \delta, \beta + \varepsilon) - \alpha - \delta + \varepsilon \geq \varepsilon \geq 0. \qquad \square$$

Theorem 4. For every IFS A, for every six real numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ such that (1) is valid, and for every six real numbers $a, b, c, d, e, f \in [0, 1]$ such that $\max(a + d, b + e) + c + f \leq 1$ is valid, it holds that

$$\oplus_{a,b,c,d,e,f}(\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A)) = \oplus_{a\alpha+b\delta, a\beta+b\varepsilon, a\gamma+b\zeta+c, d\alpha+e\delta, d\beta+e\varepsilon, d\gamma+e\zeta+f}(A).$$

Proof. Let A be an IFS and let the 12 real numbers are given. Then

$$\oplus_{a,b,c,d,e,f}(\oplus_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A))$$
$$= \oplus_{a,b,c,d,e,f}\{\langle x, \alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma, \delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta\rangle | x \in E\}$$
$$= \{\langle x, a(\alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma) + b(\delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta) + c,$$
$$d(\alpha.\mu_A(x) + \beta.\nu_A(x) + \gamma) + e(\delta.\mu_A(x) + \varepsilon.\nu_A(x) + \zeta) + f\rangle | x \in E\}$$
$$= \{\langle x, (a\alpha + b\delta).\mu_A(x) + (a\beta + b\varepsilon).\nu_A(x) + (a\gamma + b\zeta + c),$$
$$(d\alpha + e\delta).\mu_A(x) + (d\beta + e\varepsilon).\nu_A(x) + (d\gamma + e\zeta + f)\rangle | x \in E\}$$
$$= \oplus_{a\alpha+b\delta, a\beta+b\varepsilon, a\gamma+b\zeta+c, d\alpha+e\delta, d\beta+e\varepsilon, d\gamma+e\zeta+f}(A). \qquad \square$$

Finally, we illustrate the relations between different modal type of operators with Fig. 1.

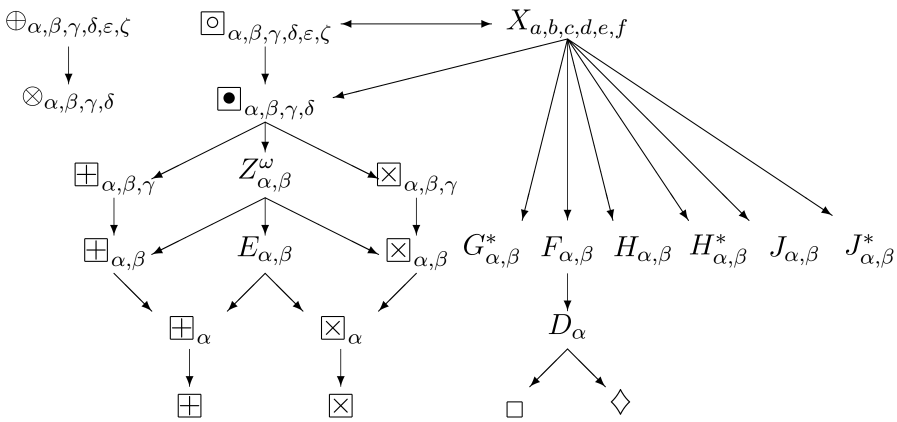

Fig. 1.

4 Conclusion

In the present paper, a new modal operator is introduced. It is extension of operator $\otimes_{\alpha,\beta,\gamma,\delta}$ and it is different from the rest modal operators, defined over IFSs. It arises some open problems, as the following ones.

Open Problem 1: Can operator $\otimes_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ be represented by the extended modal operators?

Open Problem 2: Can operator $\otimes_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ be used for representation of some type of modal operators?

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