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Modifications of the weight-center operator, defined over intuitionistic fuzzy sets. Part 3

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Abstract: A third modification of the weight-center operator, defined over an intuitionistic fuzzy set is introduced. Its basic properties are shown. The connection with the previous modifications of this operator are discussed.
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1 Introduction

In the present, third, part of the research, a new modification of the weight-center operator W, defined over intuitionistic fuzzy sets in [2], will be introduced.

Initially, we give some basic definitions, related to the Intuitionistic Fuzzy Sets (IFSs), following [1].

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For every two IFSs A and B a lot of operations, relations and operators are defined (see, e.g. [1]), the most important of which, related to the present research, are:

$$\begin{array}{lcl}
A \subseteq B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\
A \subset_\Box B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x)), \\
A \subset_\Diamond B & iff & (\forall x \in E)(\nu_A(x) \geq \nu_B(x)), \\
A = B & iff & (\forall x \in E)(\mu_A(x) = \mu_B(x) \ \& \ \nu_A(x) = \nu_B(x)), \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\
C(A) & = & \{\langle x, K, L\rangle | x \in E\}, \\
I(A) & = & \{\langle x, k, l\rangle | x \in E\}, \\
C_\nu(A) & = & \{\langle x, \mu_A(x), L\rangle | x \in E\}, \\
I_\mu(A) & = & \{\langle x, k, \nu_A(x)\rangle | x \in E\},
\end{array}$$

where

$$K = \sup_{y \in E} \mu_A(y),$$
$$L = \inf_{y \in E} \nu_A(y),$$
$$k = \inf_{y \in E} \mu_A(y),$$
$$l = \sup_{y \in E} \nu_A(y).$$

In [2] we introduced the following operator, defined for IFSs over a finite universe E:

$$W(A) = \{\langle x, \frac{\sum_{y \in E} \mu_A(y)}{card(E)}, \frac{\sum_{y \in E} \nu_A(y)}{card(E)}\rangle | x \in E\}, \quad (1)$$

where $card(E)$ is the number of the elements of a finite universe E.

In [1] the operators $H_{\alpha,\beta}$ and $J_{\alpha,\beta}$ are defined by

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\}.$$

Obviously, for every IFS X over universe E,

$$H_{0,0}(X) = \{\langle x, 0, \nu_X(x)\rangle | x \in E\},$$

$$J_{0,0}(X) = \{\langle x, \mu_X(x), 0\rangle | x \in E\}.$$

Let the IFS $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$. Therefore,

$$\sum_{y \in E} \mu_B(y) > 0,$$

$$\sum_{y \in E} \nu_B(y) > 0,$$

$$\sum_{y \in E} (\mu_B(y) + \nu_B(y)) > 0.$$

In [5, 6] we modified this operator to the forms

$$W_B^1(A) = \{\langle x, \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{card(E) \sum\limits_{y \in E} \mu_B(y)}, \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{card(E) \sum\limits_{y \in E} \nu_B(y)} \rangle | x \in E\} \tag{2}$$

and

$$W_B^2(A) = \{\langle x, \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))}, \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))} \rangle | x \in E\}, \tag{3}$$

respectively, where $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$.

It is inspired by papers of Ricardo Alberto Marques Pereira and Rita Almeida Ribeiro [4], and Vania Peneva and Ivan Popchev [3].

2 Third modification of the weight-center operator

Let

$$||X|| = \frac{\sum\limits_{y \in E} (\mu_X(y) + \nu_X(y))}{card(E)}$$

be a norm of X.

Let A and B be two IFSs over the finite universe E, so that $||A|| \leq ||B||$.

Now, we introduce *"the third modified weight-center operator"* over IFSs A and B over the finite universe E.

Let everywhere below, $B \neq H_{0,0}(B)$, $B \neq J_{0,0}(B)$ and $||A|| \leq ||B||$. Therefore,

$$\sum_{y \in E} (\mu_B(y) + \nu_B(y)) > 0$$

and

$$\sum_{y \in E} (\mu_A(y) + \nu_A(y)) \leq \sum_{y \in E} (\mu_B(y) + \nu_B(y)).$$

The new operator has the form

$$W_B^3(A) = \{\langle x, \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{\sum\limits_{y \in E} (\mu_B(y) + \nu_B(y))}, \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y \in E} (\mu_B(y) + \nu_B(y))} \rangle | x \in E\}, \tag{4}$$

First, we see that the definition is correct, i.e., $W^3_B(A)$ is an IFS. Really, for every $x \in E$,

$$0 \le \frac{(\sum\limits_{y\in E} \mu_A(y)).\mu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \le \frac{\sum\limits_{y\in E} \mu_A(y)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \le 1,$$

$$0 \le \frac{(\sum\limits_{y\in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \le \frac{\sum\limits_{y\in E} \nu_A(y)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \le 1$$

and

$$\frac{(\sum\limits_{y\in E} \mu_A(y)).\mu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} + \frac{(\sum\limits_{y\in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))}$$

$$= \frac{(\sum\limits_{y\in E} \mu_A(y)).\mu_B(x) + (\sum\limits_{y\in E} \nu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))}$$

$$\le \frac{\sum\limits_{y\in E} \mu_A(y) + \sum\limits_{y\in E} \nu_A(y)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))} \le 1.$$

Theorem 1: For every two IFSs A and B over the finite universe E, so that $B \neq H_{0,0}(B)$, $B \neq J_{0,0}(B)$ and $||A|| \le ||B||$:

(a) $\overline{W^3_B(\overline{A})} = W^3_{\overline{B}}(A)$,

(b) $I(W^3_B(A)) = W^3_B(I(A))$,

(c) $C(W^3_B(A)) = W^3_B(C(A))$,

(d) $I_\mu(W^3_B(A)) = W^3_B(I_\mu(A))$,

(c) $C_\nu(W^3_B(A)) = W^3_B(C_\nu(A))$.

Proof: Let us check the validity of (a) for given IFSs A and $B \neq U^*$ over universe E.

$$\overline{W^3_B(\overline{A})} = \overline{W^3_B(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\})}$$

$$= \overline{\{\langle x, \frac{(\sum\limits_{y\in E} \nu_A(y)).\mu_B(x)}{\sum\limits_{y\in E}(\nu_B(y)+\mu_B(y))}, \frac{(\sum\limits_{y\in E} \mu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\nu_B(y)+\mu_B(y))}\rangle | x \in E\}}$$

$$= \{\langle x, \frac{(\sum\limits_{y\in E} \mu_A(y)).\nu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))}, \frac{(\sum\limits_{y\in E} \nu_A(y)).\mu_B(x)}{\sum\limits_{y\in E}(\mu_B(y)+\nu_B(y))}\rangle | x \in E\} = W^3_{\overline{B}}(A).$$

(b) – (e) are proved analogously.

From (2)–(4), the validity of the following assertions is checked.

Theorem 2: For every two IFSs A and $B \neq U^*$ over the finite universe E:

$$W^1_B(A)A \subset_{\Box} W^2_B(A) \subset_{\Box} W^3_B(A),$$

$$W^1_B(A)A \subset_{\Diamond} W^2_B(A) \subset_{\Diamond} W^3_B(A).$$

3 Conclusion

In the next authors' research, we will introduce another new modification of operator W_B and will study some of its properties.

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