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On Intuitionistic Fuzzy Pairs of n-th Type

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Abstract: The concept of intuitionistic fuzzy pair of n-th type is introduced and studied. Some of the definitions of relations, operations and operators over intuitionistic fuzzy sets are considered and studied in the terms of these pairs.
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1 Introduction

The research over Intuitionistic Fuzzy Sets (IFSs) started in 1983 and from the beginning, the concept of Intuitionistic Fuzzy Pair (IFP) starts be used, but even 30 years later, detailed description of the IFPs was given in authors' paper [4].

In 1989, the concept of an Intuitionistic Fuzzy Set of Second Type (IFS2T) was introduced and its geometrical interpretation was discussed in [1]. Its more general form is the concept of the Intuitionistic Fuzzy Set of n-th Type (IFSnT). These new concepts are object of active research from the beginning of the present century - see [6, 7]. Two PhD theses – of Parvathi Rangasamy, India [9] and Peter Vassilev, Bulgaria [8] were defended. These concepts are discussed in some books, e.g. [2, 5]. But since 2013 some authors incorrectly re-defined the concept of IFS2T under the name "Pythagorean Fuzzy Set" (PFS) in spite of the protests of the first author. What is surprising is that these authors had not made the effort to study the properties of the IFS2T (at least these, discussed in [2]).

In the present paper, the authors introduce the concepts of Intuitionistic Fuzzy Pair of Second Type (IFP2T) and Intuitionistic Fuzzy Pair of n-th Type (IFPnT) by analogy with IFPs and with IFS2Ts and IFSnTs.

Immediately, we like to mention one very important question. The authors writing on PFSs assert that they are extension/generalization of the IFSs. This is not valid. The fact is that IFSnTs are extension/generalization as of IFS ($n = 1$), as well as of IFS2Ts ($n = 2$), but obviously, IFSs and IFS2Ts are two independent types of sets and it is incorrect to assert that IFS2Ts are extension/generalization of IFSs.

Here, we give a formal definition of an IFP2T and collect definitions of all operations, relations and operators.

2 Geometrical Interpretations of an IFP2T

The IFP2T is an object with the form $x = \langle a, b\rangle$, where $a, b \in [0, 1]$ and $a^2 + b^2 \leq 1$, that is used as an evaluation of some object or process and which components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. The geometrical interpretation of the IFP2Ts are the same as of the IFS2Ts. It is shown on Fig. 1.

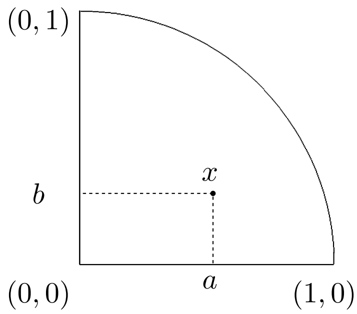

Fig. 1

For the needs of the discussion below, we define the notion of Intuitionistic Fuzzy Tautological Pair (IFTP) by:

$$x \text{ is an IFTP if and only if } a \geq b,$$

while p is a Tautological Pair (TP) iff $a = 1$ and $b = 0$.

Obviously, these definitions are valid simultaneously as for the IFPs, as well as for the IFP2Ts.

If

$$\pi(x) = \pi(\langle a, b \rangle) = \sqrt{1 - a^2 - b^2}$$

then $\pi(x)$ is the degree of non-determinacy of the IFP2T x.

In case of ordinary fuzzy sets, $\pi(x) = 0$ for every IFP2T x.

If $\langle a, b \rangle$ is an IFP, then $\langle a, b \rangle$ is an IFP2T. Indeed, from $a + b \leq$ it follows that

$$a^2 + b^2 \leq a + b \leq 1.$$

Here we will define over the IFP2Ts relations, operations, and operators from different types.

3 Relations over IFPs

Let us have two IFPs $x = \langle a, b \rangle$ and $y = \langle c, d \rangle$. We define the relations

$$\begin{array}{ccc}
x <_\Box y & \text{iff} & a < c \\
x <_\Diamond y & \text{iff} & b > d \\
x < y & \text{iff} & a < c \text{ and } b > d \\
x \leq_\Box y & \text{iff} & a \leq c \\
x \leq_\Diamond y & \text{iff} & b \geq d \\
x \leq y & \text{iff} & a \leq c \text{ and } b \geq d \\
x >_\Box y & \text{iff} & a > c
\end{array}$$

$$\begin{array}{ccc}
x >_\Diamond y & \text{iff} & b < d \\
x > y & \text{iff} & a > c \text{ and } b < d \\
x \geq_\Box y & \text{iff} & a \geq c \\
x \geq_\Diamond y & \text{iff} & b \leq d \\
x \geq y & \text{iff} & a \geq c \text{ and } b \leq d \\
x =_\Box y & \text{iff} & a = c \\
x =_\Diamond y & \text{iff} & b = d \\
x = y & \text{iff} & a = c \text{ and } b = d.
\end{array}$$

It is important to mention that all relations between IFP2Ts coincide with their respective, defined over IFPs.

4 Operations over IFP2Ts

In some definitions below, we use functions sg and $\overline{\text{sg}}$ defined by,

$$\text{sg}(x) = \begin{cases} 1, & \text{if } x > 0 \\ \\ 0, & \text{if } x \leq 0 \end{cases},$$

$$\overline{\text{sg}}(x) = \begin{cases} 0, & \text{if } x > 0 \\ \\ 1, & \text{if } x \leq 0 \end{cases}.$$

Let us have two IFP2Ts $x = \langle a, b \rangle$ and $y = \langle c, d \rangle$.

First, we see that the first definitions of the operations "conjunction" and "disjunction" between IFP2Ts coincide with their respective, defined over IFPs:

$$\begin{aligned} x \&_1 y &= \langle \min(a, c), \max(b, d) \rangle \\ x \vee_1 y &= \langle \max(a, c)), \min(b, d) \rangle \\ x \&_2 y &= \langle a + c - a.c, b.d \rangle \\ x \vee_2 y &= \langle a.c, b + d - b.d \rangle. \end{aligned}$$

Open Problem 1. Now, there are a lot of new operations "conjunction" and "disjunction" between IFPs, that must obtain suitable IFP2T-forms.

Second, we define the first five operations "implication" and "negation" over IFP2Ts:

$$\begin{aligned} x \to_1 y &= \langle \max(b, \min(a, c)), \min(a, d) \rangle, \\ x \to_2 y &= \langle \overline{\text{sg}}(a - c), d.\text{sg}(a - c) \rangle, \\ x \to_3 y &= \langle 1 - (1 - c).\text{sg}(a - c)), d.\text{sg}(a - c) \rangle, \\ x \to_4 y &= \langle \max(b, c), \min(a, d) \rangle, \\ x \to_5 y &= \langle \min(1, b + c), \max(0, a + d - 1) \rangle. \end{aligned}$$

$$\begin{aligned} \neg_1 x &= \langle b, a \rangle, \\ \neg_2 x &= \langle \overline{\text{sg}}(a), \text{sg}(a) \rangle, \\ \neg_3 x &= \langle b, a.b + a^2 \rangle, \\ \neg_4 &= \langle x, b, \sqrt{1 - b^2} \rangle, \\ \neg_5 &= \langle x, \overline{\text{sg}}(1 - b), \text{sg}(1 - b) \rangle. \end{aligned}$$

Immediately, we see that the first four implications and negations coincide with the older one (for IFPs), while the fifth implications and negations are different.

We must mention that negation $\neg_1$ corresponds to implications $\rightarrow_1, \rightarrow_4$, $\rightarrow_5$, and negation $\neg_2$ – to implications $\rightarrow_2$ and $\rightarrow_3$.

Open Problem 2. Now, there are 189 different implications and 54 different negations between IFPs, that must obtain suitable IFP2T-forms.

5 Operators over IFPs

There are some types of operators over IFPs and a part of them can be defined over IFP2Ts. Here, we discuss some of these operators. The first group of operators are the modal ones.

Let as above, $x = \langle a, b\rangle$ be an IFP and let $\alpha, \beta \in [0, 1]$. Then the standard modal operators defined over x have the forms:

$$\Box x = \langle a, \sqrt{1 - a^2}\rangle,$$

$$\Diamond x = \langle \sqrt{1 - b^2}, b\rangle.$$

The geometrical interpretations of these operators are given on Figs. 2 and 3.

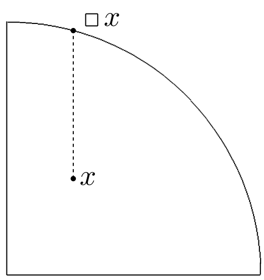

Fig 2

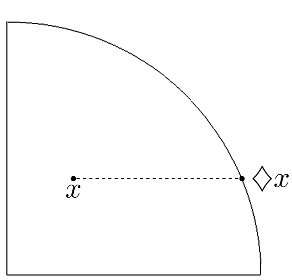

Fig. 3

Open Problem 3. To find suitable forms of the extended modal operators, defined over IFPs (cf. [4]).

The second type of operators is from another (similar to modal) type. The first four of them are:

$$\boxplus x = \langle \frac{a}{2}, \frac{b+1}{2} \rangle$$
$$\boxtimes x = \langle \frac{a+1}{2}, \frac{b}{2} \rangle$$
$$\boxplus_\alpha x = \langle \alpha.a, \alpha.b + 1 - \alpha \rangle$$
$$\boxtimes_\alpha x = \langle \alpha.a + 1 - \alpha, \alpha.b \rangle,$$

where $\alpha \in [0, 1]$.

We can see that these definition are correct. For example, from $a^2 + b^2 \leq 1$ it follows that

$$\begin{aligned} &(\alpha.a + 1 - \alpha)^2 + (\alpha.b)^2 \\ &= \alpha^2 a^2 + 2\alpha(1 - \alpha)a + (1 - \alpha)^2 + \alpha^2 b^2 \\ &= \alpha^2(a^2 + b^2) + 2\alpha(1 - \alpha)a + (1 - \alpha)^2 \\ &\leq \alpha^2 + 2\alpha(1 - \alpha)a + (1 - \alpha)^2 = 1. \end{aligned}$$

Open Problem 4. To find suitable forms of the extended modal operators of the second type, defined over IFPs (cf. [4]).

The third type of operators is from level type. They are

$$P_{\alpha,\beta} x = \langle \max(\alpha, a), \min(\beta, b) \rangle$$
$$Q_{\alpha,\beta} x = \langle \min(\alpha, a), \max(\beta, b) \rangle,$$

for $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

They coincide with the level operators, defined over IFPs.

6 Conclusion

In the present paper, we transform the definitions of the basic relations, operations and operators to the concept of an IFP2T. In future, we will give definitions new operations and operators over IFPs.

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