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Evaluation of credit risk in SMEs financing using index matrices and intuitionistic fuzzy estimations

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Abstract: In the paper will be present an methodology for assessment the credit risk in Small and medium-sized enterprises (SMEs) using theories of the Index Matrices and Intuitionistic Fuzzy Sets. In this way the procedure for assessment of the credit will be unified and optimized.

Keywords: Credit risk assessment, Index matrix, Intuitionistic fuzzy sets, Intuitionistic fuzzy index matrix.

AMS Classification: 03E72.

1 The complexity of multi-layered service level requirements

The aim of this paper is to present an algorithm for assessment the credit risk. For this aim, we will use the theories of the index matrices (IMs see [1, 3, 5]) and intuitionistic fuzzy sets (IFS, see [2, 4]).

The application of fuzzy sets theory is a common practice in variety of problems for multi-criteria selection and ranking. In [8] are discussed several groups of fuzzy algorithms and its application according to the available input criteria and weight coefficients information represented by crisp or fuzzy numbers. In [6], the theory of the intuitionistic fuzzy set is used for estimating the degree of bank liquidity. In [9], is constructed a generalized net model of the methodology for analysis of the creditworthiness and evaluation of credit risk in small and medium-sized enterprises financing. The model describes the most important steps of the process of evaluation of a business bank credit proposal. Here, the authors present the methodology for determination of the intuitionistic fuzzy assessments (see [2, 4]) for degree of the acceptance and degree of non-acceptance of the credit risk.

In world practice, assessment of the credit capacity of the borrower, the credit risk accordingly, is done on the grounds of complex analysis done by using a system of indexes. One of the methodologies applied in the international bank practice as a criterion of assessment is referred as kind of the loan, loan term, nature of the collateral, objective, borrower's reputation, place of the loan in the credit portfolio, etc (see Table 1.).

Table 1. Criteria of risk assessment of given credit

	Index
1.	History and property of the borrower
2.	Relationships with banks, state authorities and institutions
3.	Quality of management
4.	Scope of activity and capability of realization of the credit project
5.	Market positions
6.	General risk
7.	Branch risk
8.	Kind of credit
9.	Credit amount
10.	Term of repayment of the credit
11.	Credit collateral
12.	Financial status of the borrower
13.	Sources for repayment of the credit
14.	Term of delay of callable sums under the credit
15.	Place of the credit in the credit portfolio according to Regulation 9 of Bulgarian National Bank, [7]

The criteria show that the credit risk analysis depends on fifteen qualitative and quantitative indexes, characterizing borrower's status, the prospects of his development and the external factors influence. The general credit risk assessment is an aggregate of the assessments of the said fifteen indexes. The final result of that process should be a motivated opinion for taking decision for the future behaviour of the bank towards the requested or already granted credit, as well as the periodical risk assessment of the granted credits.

For the purposes of the present method, it is necessary each one of the identified fifteen criteria to be evaluated individually and classified in one of the following five risk levels: optimal, low risk, average risk, high risk, extreme risk.

Besides the credit exposure classification, also numerical evaluation of the aforementioned risk levels must be performed by awarding various weights to the indexes.

Based on the selected weights of the indexes, for each credit exposure actual points are awarded and for the lowest risk group (credits with optimal risk) the number of points represents the double amount of the weight. The number derived that way serves as an interval for the determination of the points of the remaining 4 groups of credit exposures. Depending on the derived sum, the credit risk is the result based on the awarded points.

According to certain specialists, the awarded weights should be within the range 0.5 – 2.5 and to be in conformity with the importance of the relevant index, but also with certain subjective factors affecting its evaluation. Based on the specifics, each bank could select other weights of the indexes.

2 Algorithmization of evaluation methods and procedures

We use c_i criteria of risk assessment of the credit. The criterion c_i has weight t_i, $i = 1, \ldots, 15$. Each criterion has to be evaluated individually and classified in one of the five risk levels r_j, $j = 1, \ldots, 5$ (for optimal risk, low risk, average risk, high risk, extreme risk respectively).

So we use the following sets:

- Set of criteria $C_i = \{c_1, c_2, \ldots, c_{15}\}$,
- Set of weights of the criteria $T_i = \{t_1, t_2, \ldots, t_{15}\}$,
- Set of risk levels $R_j = \{r_1, r_2, \ldots, r_5\}$.

2.1. Determination of the intuitionistic fuzzy index matrix for credit exposures

Step A1. Constructing of the index matrix T_C with weight coefficients corresponding to each criterion:

$$T_C = \begin{array}{c|cccc} & c_1 & c_2 & \ldots & c_{15} \\ \hline t & t_1 & t_2 & \ldots & t_{15} \end{array}.$$

Step A2. Constructing of the index matrix with the points awarded to the credit exposures. Via the weights of the criteria we determine the points for the lowest risk group (credits with optimal risk). Its values are double the amount of the weight coefficients (in column r_1). Using these values we determine the points of the remaining 4 groups of credit exposures.

$$C^r = \begin{array}{c|ccccc} & r_1 & \ldots & r_j & \ldots & r_5 \\ \hline c_1 & \alpha_{1,1} & \ldots & \alpha_{1,j} & \ldots & \alpha_{1,5} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ c_i & \alpha_{2,1} & \ldots & \alpha_{i,j} & \vdots & \alpha_{2,5} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{15} & \alpha_{15,1} & \ldots & \alpha_{15,j} & \ldots & \alpha_{15,5} \end{array},$$

where

$$\alpha_{i,j} = 2.j.t_i, \tag{1}$$

for $i = 1, \ldots, 15$, $j = 1, \ldots, 5$.

More precisely

$$C^r = \begin{array}{c|ccccc} & r_1 & \ldots & r_j & \ldots & r_5 \\ \hline c_1 & 2.t_1 & \ldots & 2.j.t_1 & \ldots & 2.5.t_1 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_i & 2.t_i & \ldots & 2.j.t_i & \ldots & 2.5.t_i \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{15} & 2.t_{15} & \ldots & 2.j.t_{15} & \ldots & 2.5.t_{15} \end{array}.$$

Step A3. Determination of the intuitionistic fuzzy index matrix C_{IFS} with intuitionistic fuzzy evaluations using the risk levels.

According to the five risk levels (optimal risk, low risk, average risk, high risk, extreme risk) we introduce five degrees of the intuitionistic fuzzy evaluations:

1) For optimal risk: $\langle \mu_1, \nu_1 \rangle$;
2) For low risk: $\langle \mu_2, \nu_2 \rangle$;
3) For average risk: $\langle \mu_3, \nu_3 \rangle$;
4) For high risk: $\langle \mu_4, \nu_4 \rangle$;
5) For high risk: $\langle \mu_5, \nu_5 \rangle$.

$$C_{IFS} = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & \langle \mu_1, \nu_1 \rangle & \langle \mu_2, \nu_2 \rangle & \langle \mu_3, \nu_3 \rangle & \langle \mu_4, \nu_4 \rangle & \langle \mu_5, \nu_5 \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_i & \langle \mu_1, \nu_1 \rangle & \langle \mu_2, \nu_2 \rangle & \langle \mu_3, \nu_3 \rangle & \langle \mu_4, \nu_4 \rangle & \langle \mu_5, \nu_5 \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{15} & \langle \mu_1, \nu_1 \rangle & \langle \mu_2, \nu_2 \rangle & \langle \mu_3, \nu_3 \rangle & \langle \mu_4, \nu_4 \rangle & \langle \mu_5, \nu_5 \rangle \end{array},$$

where

$$\langle \mu_j, \nu_j \rangle = \left\langle \frac{\alpha_{i,j} - \alpha_{i,1}}{\alpha_{i,5} - \alpha_{i,1}},\ 1 - \frac{\alpha_{i,j} - \alpha_{i,1}}{\alpha_{i,5} - \alpha_{i,1}} \right\rangle, \quad (2)$$

for $i = 1, \ldots, 15$, $j = 1, \ldots, 5$.

More precisely

$$C_{IFS} = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & \langle 1,0 \rangle & \langle 0.75,0.25 \rangle & \langle 0.5,0.5 \rangle & \langle 0.25,0.75 \rangle & \langle 0,1 \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_i & \langle 1,0 \rangle & \langle 0.75,0.25 \rangle & \langle 0.5,0.5 \rangle & \langle 0.25,0.75 \rangle & \langle 0,1 \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{15} & \langle 1,0 \rangle & \langle 0.75,0.25 \rangle & \langle 0.5,0.5 \rangle & \langle 0.25,0.75 \rangle & \langle 0,1 \rangle \end{array}.$$

Step A4. Constructing of the intuitionistic fuzzy index matrix with intuitionistic fuzzy evaluations awarded to the credit exposures.

$$C_{IFS}^{r} = \begin{array}{c|ccccc} & r_1 & \cdots & r_j & \cdots & r_5 \\ \hline c_1 & \beta_{1,1} & \cdots & \beta_{1,j} & \cdots & \beta_{1,5} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ c_i & \beta_{2,1} & \cdots & \beta_{i,j} & \vdots & \beta_{2,5} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{15} & \beta_{15,1} & \cdots & \beta_{15,j} & \cdots & \beta_{15,5} \end{array},$$

where

$$\beta_{i,j} = \langle \varepsilon_{i,j}, \delta_{i,j} \rangle = \left\langle \frac{t_i . \mu_{i,j}}{\max\limits_{1 \le i \le 15}(t_i)},\ \frac{t_i . \nu_{i,j}}{\max\limits_{1 \le i \le 15}(t_i)} \right\rangle, \quad (3)$$

for $i = 1, \ldots, 15$, $j = 1, \ldots, 5$.

2.2. Assessment of the risk for current credit

Step B1. Constructing of the index matrix with risk levels corresponding to each criterion for current credit:

$$C_{cu} = \begin{array}{c|ccccc} & r_1 & \cdots & r_j & \cdots & r_5 \\ \hline c_1 & \gamma_{1,1} & \cdots & \gamma_{1,j} & \cdots & \gamma_{1,5} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ c_i & \gamma_{2,1} & \cdots & \gamma_{i,j} & \vdots & \gamma_{2,5} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{15} & \gamma_{15,1} & \cdots & \gamma_{15,j} & \cdots & \gamma_{15,5} \end{array},$$

where $\gamma_{i,j} = \{0, 1\}$, for $i = 1, \ldots, 15$, $j = 1, \ldots, 5$.

For example, if the risk level according criterion c_2 is “optimal risk”, we will have $\gamma_{2,1} = 1$ and $\gamma_{2,2} = \gamma_{2,3} = \gamma_{2,4} = \gamma_{2,5} = 0$, i.e. in the second row (for criterion c_2) we will have only one value “1”. When there is no information for some criterion, then in its row there will be only values “0”.

Step B2. Multiplication of the elements of the intuitionistic fuzzy index matrix C^r_{IFS} with values of the index matrices C_{cu}:

$$C^r_{IFS,cu} = \begin{array}{c|ccccc} & r_1 & \cdots & r_j & \cdots & r_5 \\ \hline c_1 & \beta_{1,1}.\gamma_{1,1} & \cdots & \beta_{1,j}.\gamma_{1,j} & \cdots & \beta_{1,5}.\gamma_{1,5} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_i & \beta_{i,1}.\gamma_{i,1} & \cdots & \beta_{i,j}.\gamma_{i,j} & \cdots & \beta_{i,5}.\gamma_{i,5} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{15} & \beta_{15,1}.\gamma_{15,1} & \cdots & \beta_{15,j}.\gamma_{15,j} & \cdots & \beta_{15,5}.\gamma_{15,5} \end{array},$$

$$C^r_{IFS,cu} = \begin{array}{c|ccccc} & r_1 & \cdots & r_j & \cdots & r_5 \\ \hline c_1 & \left\langle \frac{t_1.\mu_{1,1}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{1,1}, \frac{t_1.\nu_{1,1}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{1,1} \right\rangle & \cdots & \left\langle \frac{t_1.\mu_{1,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{1,j}, \frac{t_1.\nu_{1,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{1,j} \right\rangle & \cdots & \left\langle \frac{t_1.\mu_{1,5}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{1,5}, \frac{t_1.\nu_{1,5}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{1,5} \right\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_i & \left\langle \frac{t_i.\mu_{i,1}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,1}, \frac{t_i.\nu_{i,1}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,1} \right\rangle & \cdots & \left\langle \frac{t_i.\mu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j}, \frac{t_i.\nu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j} \right\rangle & \cdots & \left\langle \frac{t_i.\mu_{i,5}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,5}, \frac{t_i.\nu_{i,5}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,5} \right\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{15} & \left\langle \frac{t_{15}.\mu_{15,1}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{15,1}, \frac{t_{15}.\nu_{15,1}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{15,1} \right\rangle & \cdots & \left\langle \frac{t_{15}.\mu_{15,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{15,j}, \frac{t_{15}.\nu_{15,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{15,j} \right\rangle & \cdots & \left\langle \frac{t_{15}.\mu_{15,5}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{15,5}, \frac{t_{15}.\nu_{15,5}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{15,5} \right\rangle \end{array},$$

for $i = 1, \ldots, 15$, $j = 1, \ldots, 5$.

Step B3. Determining the degrees of the acceptance μ_P and non-acceptance ν_P of the credit risk for the current credit:

$$\mu_P = \sum_{i=1}^{15} \sum_{j=1}^{5} \frac{t_i.\mu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j}\ , \quad \nu_P = \sum_{i=1}^{15} \sum_{j=1}^{5} \frac{t_i.\nu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j}\ .$$

The degree of uncertainty π_P is for the cases when some of the criteria have no evaluations and:

$$\pi_P = 1 - \mu_P - \nu_P.$$

Step B4. Assessment the credit risk using intuitionistic fuzzy estimations $\langle \mu_P, \nu_P \rangle$:

1) Optimal risk credit: $0.8 < \mu_P \leq 1$ and $0 \leq \nu_P < 0.2$, $\mu_P + \nu_P \leq 1$;
2) Low risk credit: $0.6 < \mu_P \leq 0.8$ and $0.2 \leq \nu_P < 0.4$, $\mu_P + \nu_P \leq 1$;
3) Average risk credit: $0.4 < \mu_P \leq 0.6$ and $0.4 \leq \nu_P < 0.6$, $\mu_P + \nu_P \leq 1$;
4) High risk credit: $0.2 < \mu_P \leq 0.4$ and $0.6 \leq \nu_P < 0.8$, $\mu_P + \nu_P \leq 1$;
5) Extreme risk credit: $0 \leq \mu_P \leq 0.2$ and $0.8 \leq \nu_P \leq 1$, $\mu_P + \nu_P \leq 1$.

3 Example

In the following example, let us present the process of determination of the intuitionistic fuzzy index matrix for credit exposures.

Step A1.

$$T_C = \begin{array}{c|ccccccccccccccc} & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ \hline t & 0.5 & 1 & 0.5 & 1.5 & 1.5 & 0.5 & 1 & 2 & 0.5 & 1 & 1 & 2 & 2 & 1 & 2.5 \end{array}.$$

Step A2.

$$C^r = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & 1 & 2 & 3 & 4 & 5 \\ c_2 & 2 & 4 & 6 & 8 & 10 \\ c_3 & 1 & 2 & 3 & 4 & 5 \\ c_4 & 3 & 6 & 9 & 12 & 15 \\ c_5 & 3 & 6 & 9 & 12 & 15 \\ c_6 & 1 & 2 & 3 & 4 & 5 \\ c_7 & 2 & 4 & 6 & 8 & 10 \\ c_8 & 4 & 8 & 12 & 16 & 20 \\ c_9 & 1 & 2 & 3 & 4 & 5 \\ c_{10} & 2 & 4 & 6 & 8 & 10 \\ c_{11} & 2 & 4 & 6 & 8 & 10 \\ c_{12} & 4 & 8 & 12 & 16 & 20 \\ c_{13} & 4 & 8 & 12 & 16 & 20 \\ c_{14} & 2 & 4 & 6 & 8 & 10 \\ c_{15} & 5 & 10 & 15 & 20 & 25 \end{array}.$$

Step A3.

$C_{IFS} =$

	r_1	r_2	r_3	r_4	r_5
c_1	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_2	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_3	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_4	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_5	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_6	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_7	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_8	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_9	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_{10}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_{11}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_{12}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_{13}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_{14}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$
c_{15}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$

.

Step A4.

$C_{IFS} =$

	r_1	r_2	r_3	r_4	r_5
c_1	$\langle 0.2,0\rangle$	$\langle 0.15,0.05\rangle$	$\langle 0.1,0.1\rangle$	$\langle 0.05,0.15\rangle$	$\langle 0,0.2\rangle$
c_2	$\langle 0.4,0\rangle$	$\langle 0.3,0.1\rangle$	$\langle 0.2,0.2\rangle$	$\langle 0.1,0.3\rangle$	$\langle 0,0.4\rangle$
c_3	$\langle 0.2,0\rangle$	$\langle 0.15,0.05\rangle$	$\langle 0.1,0.1\rangle$	$\langle 0.05,0.15\rangle$	$\langle 0,0.2\rangle$
c_4	$\langle 0.6,0\rangle$	$\langle 0.45,0.15\rangle$	$\langle 0.3,0.3\rangle$	$\langle 0.15,0.45\rangle$	$\langle 0,0.6\rangle$
c_5	$\langle 0.6,0\rangle$	$\langle 0.45,0.15\rangle$	$\langle 0.3,0.3\rangle$	$\langle 0.15,0.45\rangle$	$\langle 0,0.6\rangle$
c_6	$\langle 0.2,0\rangle$	$\langle 0.15,0.05\rangle$	$\langle 0.1,0.1\rangle$	$\langle 0.05,0.15\rangle$	$\langle 0,0.2\rangle$
c_7	$\langle 0.4,0\rangle$	$\langle 0.3,0.1\rangle$	$\langle 0.2,0.2\rangle$	$\langle 0.1,0.3\rangle$	$\langle 0,0.4\rangle$
c_8	$\langle 0.8,0\rangle$	$\langle 0.6,0.2\rangle$	$\langle 0.4,0.4\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0,0.8\rangle$
c_9	$\langle 0.2,0\rangle$	$\langle 0.15,0.05\rangle$	$\langle 0.1,0.1\rangle$	$\langle 0.05,0.15\rangle$	$\langle 0,0.2\rangle$
c_{10}	$\langle 0.4,0\rangle$	$\langle 0.3,0.1\rangle$	$\langle 0.2,0.2\rangle$	$\langle 0.1,0.3\rangle$	$\langle 0,4\rangle$
c_{11}	$\langle 0.4,0\rangle$	$\langle 0.3,0.1\rangle$	$\langle 0.2,0.2\rangle$	$\langle 0.1,0.3\rangle$	$\langle 0,4\rangle$
c_{12}	$\langle 0.8,0\rangle$	$\langle 0.6,0.2\rangle$	$\langle 0.4,0.4\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0,0.8\rangle$
c_{13}	$\langle 0.8,0\rangle$	$\langle 0.6,0.2\rangle$	$\langle 0.4,0.4\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0,0.8\rangle$
c_{14}	$\langle 0.4,0\rangle$	$\langle 0.3,0.1\rangle$	$\langle 0.2,0.2\rangle$	$\langle 0.1,0.3\rangle$	$\langle 0,0.4\rangle$
c_{15}	$\langle 1,0\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.25,0.75\rangle$	$\langle 0,1\rangle$

.

Let us also present the assessment of the risk for current credit.

Step B1. – Case 1

$$C_{cu} = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & 1 & 0 & 0 & 0 & 0 \\ c_2 & 1 & 0 & 0 & 0 & 0 \\ c_3 & 0 & 1 & 0 & 0 & 0 \\ c_4 & 0 & 1 & 0 & 0 & 0 \\ c_5 & 1 & 0 & 0 & 0 & 0 \\ c_6 & 0 & 1 & 0 & 0 & 0 \\ c_7 & 1 & 0 & 0 & 0 & 0 \\ c_8 & 0 & 1 & 0 & 0 & 0 \\ c_9 & 1 & 0 & 0 & 0 & 0 \\ c_{10} & 1 & 0 & 0 & 0 & 0 \\ c_{11} & 1 & 0 & 0 & 0 & 0 \\ c_{12} & 0 & 1 & 0 & 0 & 0 \\ c_{13} & 0 & 1 & 0 & 0 & 0 \\ c_{14} & 1 & 0 & 0 & 0 & 0 \\ c_{15} & 0 & 1 & 0 & 0 & 0 \end{array}.$$

Step B2. – Case 1

$$C^{r}_{IFS,cu} = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & \langle 0.2,0 \rangle & 0 & 0 & 0 & 0 \\ c_2 & \langle 0.4,0 \rangle & 0 & 0 & 0 & 0 \\ c_3 & 0 & \langle 0.15,0.05 \rangle & 0 & 0 & 0 \\ c_4 & 0 & \langle 0.45,0.15 \rangle & 0 & 0 & 0 \\ c_5 & \langle 0.6,0 \rangle & 0 & 0 & 0 & 0 \\ c_6 & 0 & \langle 0.15,0.05 \rangle & 0 & 0 & 0 \\ c_7 & \langle 0.4,0 \rangle & 0 & 0 & 0 & 0 \\ c_8 & 0 & \langle 0.6,0.2 \rangle & 0 & 0 & 0 \\ c_9 & \langle 0.2,0 \rangle & 0 & 0 & 0 & 0 \\ c_{10} & \langle 0.4,0 \rangle & 0 & 0 & 0 & 0 \\ c_{11} & \langle 0.4,0 \rangle & 0 & 0 & 0 & 0 \\ c_{12} & 0 & \langle 0.6,0.2 \rangle & 0 & 0 & 0 \\ c_{13} & 0 & \langle 0.6,0.2 \rangle & 0 & 0 & 0 \\ c_{14} & \langle 0.4,0 \rangle & 0 & 0 & 0 & 0 \\ c_{15} & 0 & \langle 0.75,0.25 \rangle & 0 & 0 & 0 \end{array}.$$

Step B3. – Case 1

$$\mu_P = \sum_{i=1}^{15} \sum_{j=1}^{5} \frac{t_i . \mu_{i,j}}{\max\limits_{1 \le i \le 15} (t_i)} . \gamma_{i,j} = 0.85,$$

$$\nu_P = \sum_{i=1}^{15}\sum_{j=1}^{5}\frac{t_i.\nu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j} = 0.15,$$

$$\pi_P = 1 - \mu_P - \nu_P = 0.$$

Step B4. – Case 1

$0.8 < \mu_P \le 1$ and $0 \le \nu_P < 0.2$, $\mu_P + \nu_P \le 1 \Rightarrow$ This is an optimal risk credit.

Step B1. – Case 2

$$C_{cu} = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & 0 & 0 & 1 & 0 & 0 \\ c_2 & 0 & 0 & 1 & 0 & 0 \\ c_3 & 0 & 0 & 1 & 0 & 0 \\ c_4 & 0 & 0 & 0 & 1 & 0 \\ c_5 & 0 & 0 & 1 & 0 & 0 \\ c_6 & 0 & 0 & 0 & 1 & 0 \\ c_7 & 0 & 0 & 0 & 0 & 1 \\ c_8 & 0 & 0 & 0 & 1 & 0 \\ c_9 & 0 & 0 & 0 & 1 & 0 \\ c_{10} & 0 & 0 & 1 & 0 & 0 \\ c_{11} & 0 & 0 & 1 & 0 & 0 \\ c_{12} & 0 & 0 & 0 & 1 & 0 \\ c_{13} & 0 & 0 & 1 & 0 & 0 \\ c_{14} & 0 & 0 & 0 & 1 & 0 \\ c_{15} & 0 & 0 & 0 & 1 & 0 \end{array}.$$

Step B2. – Case 2

$$C^{r}_{IFS,cu} = \begin{array}{c|ccccc} & r_1 & r_2 & r_3 & r_4 & r_5 \\ \hline c_1 & 0 & 0 & \langle 0.1,0.1\rangle & 0 & 0 \\ c_2 & 0 & 0 & \langle 0.2,0.2\rangle & 0 & 0 \\ c_3 & 0 & 0 & \langle 0.1,0.1\rangle & 0 & 0 \\ c_4 & 0 & 0 & 0 & \langle 0.15,0.45\rangle & 0 \\ c_5 & 0 & 0 & \langle 0.3,0.3\rangle & 0 & 0 \\ c_6 & 0 & 0 & 0 & \langle 0.05,0.15\rangle & 0 \\ c_7 & 0 & 0 & 0 & 0 & \langle 0,0.4\rangle \\ c_8 & 0 & 0 & 0 & \langle 0.2,0.6\rangle & 0 \\ c_9 & 0 & 0 & 0 & \langle 0.05,0.15\rangle & 0 \\ c_{10} & 0 & 0 & \langle 0.2,0.2\rangle & 0 & 0 \\ c_{11} & 0 & 0 & \langle 0.2,0.2\rangle & 0 & 0 \\ c_{12} & 0 & 0 & 0 & \langle 0.2,0.6\rangle & 0 \\ c_{13} & 0 & 0 & \langle 0.4,0.4\rangle & 0 & 0 \\ c_{14} & 0 & 0 & 0 & \langle 0.1,0.3\rangle & 0 \\ c_{15} & 0 & 0 & 0 & \langle 0.25,0.75\rangle & 0 \end{array}.$$

Step B3. – Case 2

$$\mu_P = \sum_{i=1}^{15}\sum_{j=1}^{5}\frac{t_i.\mu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j} = 0.34,$$

$$\nu_P = \sum_{i=1}^{15}\sum_{j=1}^{5}\frac{t_i.\nu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j} = 0.66,$$

$$\pi_P = 1 - \mu_P - \nu_P = 0.$$

Step B4. – Case 2

$0.2 < \mu_P \le 0.4$ and $0.6 \le \nu_P < 0.8$, $\mu_P + \nu_P \le 1 \Rightarrow$ This is a high risk credit.

Case 3. We have no answer for the criterion c_{13}.

Step B3. – Case 3

$$\mu_P = \sum_{i=1}^{15}\sum_{j=1}^{5}\frac{t_i.\mu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j} = 0.28,$$

$$\nu_P = \sum_{i=1}^{15}\sum_{j=1}^{5}\frac{t_i.\nu_{i,j}}{\max\limits_{1\le i\le 15}(t_i)}.\gamma_{i,j} = 0.61,$$

$$\pi_P = 1 - \mu_P - \nu_P = 0.11.$$

Step B4. – Case 3

$0.2 < \mu_P \le 0.4$ and $0.6 \le \nu_P < 0.8$, $\mu_P + \nu_P \le 1 \Rightarrow$ This is a high risk credit.

4 Conclusions

In the present paper, the methodology for evaluation of credit risk in firm financing using the theories of the index matrices and intuitionistic fuzzy sets is proposed. For credit risk assessment we used fifteen specified criteria and defined their degrees of acceptance and of non-acceptance. Thus, decision of the bank concerning the loan request can be made.

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