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A new formula for de-i-fuzzification of intuitionistic fuzzy sets

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Abstract: A new procedure for de-i-fuzzification of intuitionistic fuzzy sets is described.

Keywords: Intuitionistic fuzzy sets, Defuzzification, De-i-fuzzification.

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1 Introduction

The idea and techniques for *defuzzification* have been thoroughly developed, facilitating decision makers who operate with fuzzy data to transform these data to crisp ones. Some of the most widely used defuzzification techniques have been discussed in [3].

Similarly to the process of defuzzification, which results in assigning a crisp (Boolean) value to the members of a fuzzy set (FS), we can also talk about *de-i-fuzzification,* which results in assigning a standard fuzzy value to the members of an intuitionistic fuzzy set (IFS, [1]). In this process of 'flattening' of the IFS to a standard FS, the third degree of uncertainty is reduced to 0, while its initial value is redistributed between the degrees of membership and non-membership, which now sum up to 1.

2 Main result

In [2], where the process of de-i-fuzzification has been considered for the first time, the most intuitive case is discussed where the value of the degree of uncertainty is equally shared between the degrees of membership and non-membership.

Shortly, its sense is the following: a given IFS is transformed to a fuzzy set by the transformation:

$$D_{0.5}(A) = \{\langle x, \mu'_A(x), \nu'_A(x)\rangle \mid x \in E\} = \{\langle x, \tfrac{1}{2}(1 + \mu_A(x) - \nu_A(x)), \tfrac{1}{2}(1 - \mu_A(x) + \nu_A(x))\rangle \mid x \in E\},$$

where $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\}$ is an IFS for which E is a fixed universe and the functions $\mu_A : E \to [0;1]$ and $\nu_A : E \to [0;1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$.

A modal operator D_α where $\alpha \in [0;1]$ is defined (see [1]) by

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1-\alpha).\pi_A(x)\rangle \mid x \in E\}.$$

As shown in [1], operator D_α simultaneously generalizes the two simplest modal operators, defined over IFSs, namely 'necessity', denoted by $\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle \mid x \in E\}$ and 'possibility', denoted by $\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle \mid x \in E\}$.

It is easily seen that in the extreme values of α $D_0(x) = \Box x$ and $D_1(x) = \Diamond x$. The geometrical interpretation of operator D_α is illustrated on Fig. 1.

Therefore, the procedure of de-i-fuzzification, discussed in [2], transforms a given point x to the middle of the section between $\Box x$ and $\Diamond x$, as shown on Fig. 2.

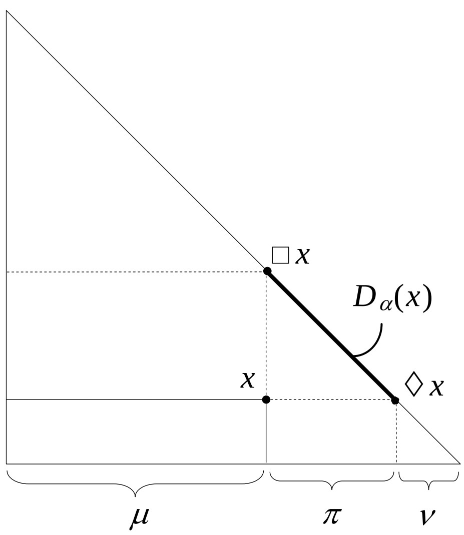

Figure 1: Operator $D_\alpha(x)$, $\alpha \in [0;1]$.

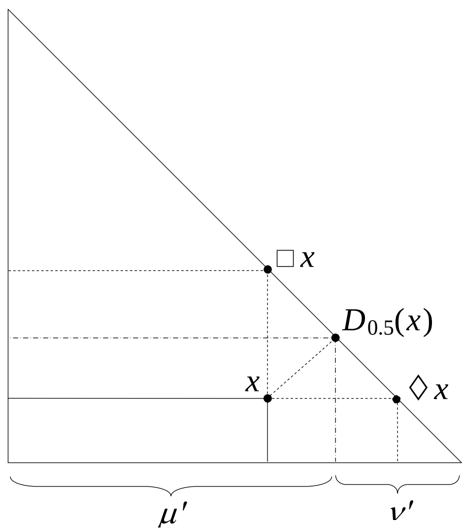

Figure 2: De-i-fuzzification as proposed in [2].

Now, we are going to consider another variant of de-i-fuzzification, in which the degree of uncertainty is proportionally distributed among the degrees of membership and non-membership, with respect to their values. In other words, we transform the degrees of membership and non-membership of point x to mapping x^* by $\langle \mu_A(x), \nu_A(x)\rangle \to \langle \mu_A{}^*(x),\ \nu_A{}^*(x)\rangle$, where

$$\langle \mu_A{}^*(x),\ \nu_A{}^*(x)\rangle = \left\langle \mu_A(x) + \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}.\pi_A(x),\ \nu_A(x) + \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)}.\pi_A(x)\right\rangle.$$

Having in mind the equality $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for every $x \in E$, we obtain that its de-i-fuzzified mapping x^* has coordinates ($\mu_A{}^*(x)$, $\nu_A{}^*(x)$), calculated as follows:

$$\mu_A{}^*(x) = \mu_A(x) + \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}.\pi_A(x) = \mu_A(x)\left(1+\frac{\pi_A(x)}{\mu_A(x)+\nu_A(x)}\right)$$

$$= \mu_A(x).\frac{\mu_A(x)+\nu_A(x)+\pi_A(x)}{\mu_A(x)+\nu_A(x)} = \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}$$

and

$$\nu_A{}^*(x) = \nu_A(x) + \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)}.\pi_A(x) = \nu_A(x)\left(1+\frac{\pi_A(x)}{\mu_A(x)+\nu_A(x)}\right)$$

$$= \nu_A(x).\frac{\mu_A(x)+\nu_A(x)+\pi_A(x)}{\mu_A(x)+\nu_A(x)} = \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)}.$$

Therefore,

$$\langle \mu_A(x), \nu_A(x)\rangle \to \left\langle \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}, \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)} \right\rangle.$$

Obviously, this new point x^* belongs to the hypotenuse of the interpretation triangle and represents its intersection point with the line connecting the (0; 0) point and point x. The geometrical interpretation of the new transformation is shown on Fig. 3.

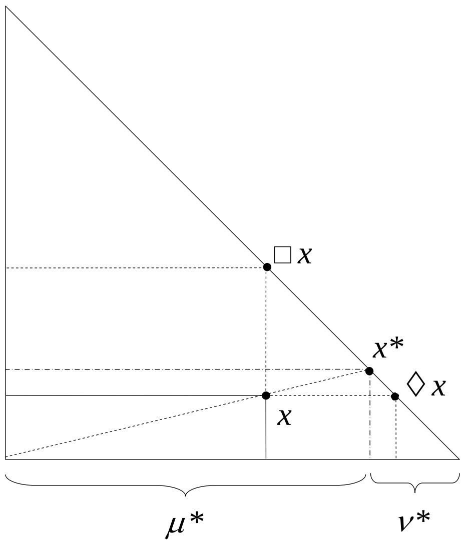

Figure 3: The mapping x^* as produced by the proposed de-i-fuzzification approach.

Acknowledgements

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