NIFS 12 (2006), 1, 45–55

ROUGH SETS ON INTUITIONISTIC FUZZY APPROXIMATION SPACES

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Abstract: The notion of intuitionistic fuzzy approximation space is introduced. Rough sets on such spaces are defined and some of their properties are studied.

Key words: Rough sets, intuitionistic fuzzy approximation space, intuitionistic fuzzy proximity relation, (α, β)-discernible, (α, β)-rough , and (α, β)-similar.

1. Introduction.

Rough sets were introduced by Pawlak ([3]) as an extension of crisp sets to model imperfect knowledge. In fact, the idea of rough set consists of the approximation of a set by a pair of sets called the lower and upper approximation of this set.

As observed by Pawlak, our perception of knowledge about universe depends upon our capability to classify objects. It is wellknown that every classification or partition of a set induces an equivalence relation on it and the converse also holds. So, the definition of Rough set depends upon equivalence relations defined over a set.

However, as it has been found in practice, pure classification of objects, that is classification which is a partition is less useful or rarely available. So, attempts have been made to relax the requirement of equivalence relations to define rough sets by taking a fuzzy approximation space which depends upon the concept of fuzzy proximity relation ([2]), which are more abundant than equivalence relations.

In this article, we define intuitionistic fuzzy proximity relations (IF-proximity relation) and use it to introduce the notion of intuitionistic fuzzy approximation space (IF-approximation space). Then we define rough sets on IF-approximation space and study their properties.

2. Rough sets on Fuzzy Approximation Space.

We unfold the background of this article in the current section by presenting the definitions, notations and results on rough sets on fuzzy approximation space.

Definition 2.1. Let U be an universe. We define a *fuzzy relation* on U as a fuzzy subset of U x U.

Definition 2.2. A fuzzy relation R on U is said to be a *fuzzy proximity relation* if

(2.1) $\mu_R(x,x)=1$ for all $x\in U$, and

(2.2) $\mu_R(x,y)=\mu_R(y,x)$ for all $x,\ y\in U$.

Definition 2.3. Let R be a fuzzy proximity relation on U. Then, for any $\alpha\in[0,1]$, then the elements of R_α, the α-cut of R, are said to be α-*similar with respect to R.*

If x and y are α-similar with respect to R then we write $x\ R_\alpha\ y$.

Definition 2.4. Two elements x and y in U are said to be *α-identical with respect to R*, denoted by $'x\ R(\alpha)\ y'$ if either x is *α-similar* to y or x *is transitively α-similar to y* with respect to R, that is, there exists a sequence of elements $u_1, u_2, \ldots, u_n$ in U such that $x R_\alpha\ u_1, u_1\ R_\alpha\ u_2, \ldots u_n\ R_\alpha\ y$.

The relation R(α), for each fixed $\alpha\in[0,1]$ is an equivalence relation on U.

Definition 2.5. The pair (U, R) is called a *fuzzy approximation space.*

For any $\alpha\in[0,1]$, we denote by R_α^* the set of all equivalence classes of R(α). Also, we call $(U,\ R(\alpha))$, the *generated approximation space associated with R and α.*

Definition 2.6. Let $X\subseteq U$. Then the *rough set of X in* $(U,\ R(\alpha))$ is denoted by $(\underline{X_\alpha}, \overline{X_\alpha})$, where

(2.3) $\underline{X_\alpha}=\bigcup\{Y: Y\in R_\alpha^*\ \ and\ \ Y\subseteq X\}$

and

(2.4) $\overline{X_\alpha}=\bigcup\{Y: Y\in R_\alpha^*\ \ and\ \ Y\cap X\neq\phi\}$.

Here, $\underline{X_\alpha}$ is called the *α-lower approximation of X* and $\overline{X_\alpha}$ is called the *α-upper approximation of X.*

Definition 2.7. Let X ⊆ U. Then for any $\alpha\in[0,1]$, we say that X is *α-discernible* if and only if $\underline{X_\alpha}=\overline{X_\alpha}$ and X is said to be α-*rough* if and only if $\underline{X_\alpha}\neq\overline{X_\alpha}$.

Many properties on α-lower approximation and α-upper approximation, similar to those for lower and upper approximation of rough sets have been established by Dey ([2]). We shall be presenting extensions of these results in the context of IF-approximation spaces and prove some of them in the next section.

3. *Rough Sets on IF-approximation Spaces.*

As an extension of fuzzy sets introduced by Zadeh ([6]), intuitionistic fuzzy sets were introduced by Attanasov ([1]). Fuzzy relations and intuitionistic fuzzy relations are extension of the concept of crisp relations.

We first define the following concepts leading to the introduction of rough sets on IF-approximation space. We use the standard notations 'μ' for member ship and 'ν' for non membership functions associated with an intuitionistic fuzzy set.

Definition 3.1. An *intuitionistic fuzzy relation* on U is an intuitionistic fuzzy subset of U x U.

Definition 3.2. An *intuitionistic fuzzy relation R on U is said to be an intuitionistic fuzzy proximity relation (IF-proximity relation)* if

(3.1) $$\mu_R(x, x) = 1 \ \ and \ \ \nu_R(x,x) = 0 \ \ for \ \ all \ \ x \in U$$

and

(3.2) $$\mu_R(x, y) = \mu_R(y, x), \ \nu_R(x, y) = \nu_R(y, x) \ \ for \ \ all \ \ x, y \in U .$$

We write $J = \{(\alpha, \beta) / \alpha, \beta, \in [0, 1] \ \ and \ \ 0 \le \alpha + \beta \le 1\}$.

Definition 3.3. Let R be an IF-proximity relation on U. Then for any $(\alpha, \beta) \in J$, the (α, β)*-cut* '$R_{\alpha,\beta}$' of R is given by

(3.3) $$R_{\alpha, \beta} = \{(x,y) / \mu_R(x,y) \ge \alpha \ \ and \ \ \nu_R(x, y) \le \beta\}.$$

Definition 3.4. Let R be an IF-proximity relation on U. We say that two elements x and y are *(α, β)-similar with respect to R* if $(x,y) \in R_{\alpha,\beta}$ and write $x R_{\alpha,\beta} \ y$.

Definition 3.5. Let R be an IF-proximity relation on U. We say that two elements x and y are *(α,β)-identical with respect to R* for $(\alpha, \beta) \in J$, written as $x \ R(\alpha, \beta) y$ if and only if $x \ R_{\alpha,\beta} \ y$ or there exists a sequence of elements $u_1, u_2, \ldots u_n$ in U such that $x \ R_{\alpha,\beta} \ u_1, u_1 \ R_{\alpha,\beta} \ u_2, \ \ldots u_n \ R_{\alpha,\beta} \ y$ (we say x is *transitively (α,β)-similar* to y with respect to R this case).

Note 3.1. It is easy to see that for any $(\alpha, \beta) \in J, \ R(\alpha, \beta)$ is an equivalence relation on U.

We use $R^*_{\alpha,\beta}$ to denote the set of equivalence classes generated by the equivalence relation R(α,β) for any (α, β) ∈ J. By $[x]_{\alpha,\beta}$ we denote the R(α,β)-equivalence class of an element x in U.

An *intuitionistic fuzzy approximation space* (IF-approximation space) is a pair (U, R).

Note 3.2. An IF-approximation space (U, R) generate usual approximation space $\left(U,\ R(\alpha,\beta)\right)$ of Pawlak for every $(\alpha,\beta)\in J$.

Theorem 3.1. If $\alpha_1 \geq \alpha_2$ *and* $\beta_1 \leq \beta_2$ then

(3.4) $$R_{\alpha_1,\ \beta_1} \subseteq R_{\alpha_2,\ \beta_2},$$

(3.5) $$R(\alpha_1,\ \beta_1) \subseteq R(\alpha_2,\ \beta_2) \text{ and}$$

(3.6) $$R^*_{\alpha_1,\ \beta_1} \subseteq R^*_{\alpha_2,\ \beta_2},$$

in the sense that every equivalence class in $R^*_{\alpha_1,\beta_1}$ is contained in some equivalence class in $R^*_{\alpha_2,\beta_2}$.

Proof. We have by hypothesis,

$$\begin{aligned}(x,y)\in R_{\alpha_1,\ \beta_1} &\Rightarrow \mu_R(x,y)\geq \alpha_1 \ \text{ and }\ \nu_R(x,y)\leq \beta_1\\ &\Rightarrow \mu_R(x,y)\geq \alpha_2 \ \text{ and }\ \nu_R(x,y)\leq \beta_2\\ &\Rightarrow (x,y)\in R_{\alpha_2,\ \beta_2}.\end{aligned}$$

This proves (3.4).

Again $(x,y)\in R\,(\alpha_1,\ \beta_1) \Rightarrow x\ R_{\alpha_1,\ \beta_1}\ y$ *or* $x\ R_{\alpha_1,\ \beta_1}\ u_1, u_1\ R_{\alpha_1,\ \beta_1}\ u_2,\ \ldots,\ u_n\ R_{\alpha_1,\ \beta_1}\ y$. In the first case by (3.4) $x\ R_{\alpha_2,\ \beta_2}\ y$. So, $(x,y)\in R\,(\alpha_2,\ \beta_2)$. In the second case $x\ R_{\alpha_2,\ \beta_2}\ u_1, u_1\ R_{\alpha_2,\ \beta_2}\ u_2\ ,\ldots u_n\ R_{\alpha_2,\ \beta_2}\ y$. So, $(x,y)\in R(\alpha_2,\ \beta_2)$.

This proves (3.5).

Next, let $[x]_{\alpha_1,\ \beta_1}\in R^*_{\alpha_1,\ \beta_1}$. Then for any $y\in[x]_{\alpha_1,\ \beta_1}$, $y\ R_{\alpha_1,\ \beta_1}\ x$. So by (3.4) $y\ R_{\alpha_2,\ \beta_2}\ x$. Hence $y\in[x]_{\alpha_2,\ \beta_2}$. Thus we get $[x]_{\alpha_1,\ \beta_1}\subseteq[x]_{\alpha_2,\ \beta_2}$. This proves (3.6).

Theorem 3.2 . Let R and S be two IF-proximity relations on U. Then for any pair $(\alpha,\beta)\in J$,

(3.7) $$(R\cup S)(\alpha,\ \beta)\subseteq R(\alpha,\ \beta)\cup S(\alpha,\beta)$$

and

(3.8) $$(R\cap S)(\alpha,\beta)\supseteq R(\alpha,\ \beta)\cap S(\alpha,\ \beta)$$

Proof. Let $(x,y)\in(R\cup S)(\alpha,\beta)$. Then we have $x(R\cup S)(\alpha,\ \beta)\ y$. This implies $x(R\cup S)_{\alpha,\beta}\ y$ or there exists a sequence of elements $u_1,\ u_2,\ \ldots,\ u_n$ such that

$$x(R\cup S)_{\alpha,\ \beta}\ u_1, u_1(R\cup S)_{\alpha,\beta}\ u_2,\ \ldots,\ u_n(R\cup S)_{\alpha,\beta}\ y.$$

The second case being similar, we consider only the first case

$$x(R \cup S)_{\alpha,\beta}\ y \Rightarrow \mu_{R\cup S}(x,y) \geq \alpha \ \ and \ \ \nu_{R\cup S}(x,y) \leq \beta$$

$$\Rightarrow \max\{\mu_R(x,y),\ \mu_S(x,y)\} \geq \alpha \ \ and \ \ \max\{\nu_R(x,y),\ \nu_S(x,y)\} \leq \beta$$

$$\Rightarrow \mu_R(x,y) \geq \alpha \ or \ \mu_S(x,y) \geq \alpha \ \ and \ \nu_R(x,y),\ \nu_S(x,y) \leq \beta$$

$$\Rightarrow \{\mu_R(x,y) \geq \alpha \ \ and \ \ \nu_R(x,y) \leq \beta\} \ \ or \ \ \{\mu_S(x,y) \geq \alpha \ and \ \ \nu_S(x,y) \leq \beta\}$$

$$\Rightarrow x\ R(\alpha,\ \beta)\ y \ or \ x\ S(\alpha,\ \beta)\ y$$

$$\Rightarrow x\ R(\alpha,\ \beta) \cup S(\alpha,\ \beta)\ y$$

$$\Rightarrow (x,y) \in R(\alpha,\beta) \cup S(\alpha,\ \beta).$$

This proves (3.7). Proof of (3.8) is similar.

Theorem 3.3. If R and S are two IF-Proximity relations on U then for any pair $(\alpha,\beta) \in J$,

(3.9) $$(R \cup S)^*_{\alpha,\beta} \subseteq R^*_{\alpha,\beta} \cup S^*_{\alpha,\beta}$$

and

(3.10) $$(R \cap S)^*_{\alpha,\beta} \supseteq R^*_{\alpha,\beta} \cap S^*_{\alpha,\beta}.$$

Proof. Let $[x] \in (R \cup S)^*_{\alpha,\beta}$. Then by (3.7), for any y,

$$y \in [x] \Rightarrow (x,y) \in (R \cup S)(\alpha,\beta)$$

$$\Rightarrow (x,y) \in R(\alpha,\beta) \cup S(\alpha,\beta)$$

$$\Rightarrow (x,y) \in R(\alpha,\beta) \ or \ (x,y) \in S(\alpha,\beta)$$

$$\Rightarrow [x] \in R^*_{\alpha,\beta} \ \ or \ \ [x] \in S^*_{\alpha,\beta}$$

$$\Rightarrow [x] \in R^*_{\alpha,\beta} \cup S^*_{\alpha,\beta}.$$

This proves (3.9). Proof of (3.10) follows similarly.

Consider on IF-approximation space (U,R) and $X \subseteq U$. Also, we take any pair $(\alpha,\beta) \in J$.

Definition 3.6. The rough set on X in the generalised approximation space $(U, R(\alpha,\beta))$ is denoted by

(3.11) $\left(\underline{R}_{\alpha,\beta}\ X,\ \overline{R}_{\alpha,\beta}\ X\right)$ *or* $\left(\underline{X}_{\alpha,\beta},\ \overline{X}_{\alpha,\beta}\right)$ in short, where

(3.12) $\underline{X}_{\alpha,\beta} = \bigcup\left\{Y : Y \in R^{*}_{\alpha,\beta}\ \ and\ \ Y \subseteq X\right\}$ and

$$\overline{X}_{\alpha,\beta} = \bigcup\left\{Y : Y \in R^{*}_{\alpha,\beta}\ \ and\ \ Y \cap X \neq \phi\right\}.$$

Definition 3.7. Let X be a rough set in the generalised approximation space $\left(U,\ R(\alpha,\beta)\right)$. Then we define

(3.13) $BNR_{\alpha,\beta}\left(X\right) = \overline{X}_{\alpha,\beta} - \underline{X}_{\alpha,\beta}$,

called the $\left(\alpha,\ \beta\right)$*-boundary of X with respect to R.*

Definition 3.8. Let X be a rough set in the generalised approximation space $\left(U,\ R(\alpha,\beta)\right)$. Then we say

(3.14) X is $\left(\alpha,\beta\right)$*-discernible with respect to R* if and only if $\underline{X}_{\alpha,\beta} = \overline{X}_{\alpha,\beta}$, and

(3.15) X is $\left(\alpha,\beta\right)$*-rough with respect to R* if and only if $\underline{X}_{\alpha,\beta} \neq \overline{X}_{\alpha,\beta}$.

Example. Let $U = \left\{x_1,\ x_2,\ x_3,\ x_4\ ,x_5\right\}$. We define an IF-proximity relation R on U is given by

R	x_1	x_2	x_3	x_4	x_5
x_1	(1,0)	(.8,.1)	(.6,.3)	(.5,.3)	(.2,.6)
x_2	(.8,.1)	(1,0)	(.7,.2)	(.6,.3)	(.4,.5)
x_3	(.6,.3)	(.7,.2)	(1,0)	(.9,.1)	(.6,.3)
x_4	(.5,.3)	(.6,.3)	(.9,.1)	(1,0)	(.4,.5)
x_5	(.2,.6)	(.4,.5)	(.6,.3)	(.4,.5)	(1,0)

Here, for $\alpha \in \left(\cdot 7, \cdot 8\right]$ *and* $\beta \in \left(\cdot 1, \cdot 2\right]$,

$$R^{*}_{\alpha,\beta} = \left\{\left\{x_1,\ x_2\right\},\ \left\{x_3,\ x_4\right\},\ \left\{x_5\right\}\right\}$$

and for $\left(\cdot 9,\ \cdot 1\right]$ *and* $\beta \in \left(0,\ \cdot 1\right]$

$$R^{*}_{\alpha,\beta} = \left\{\left\{x_1\right\},\ \left\{x_2\right\},\ \left\{x_3\right\},\left\{x_4\right\},\ \left\{x_5\right\}\right\}.$$

Let us consider two subsets $X_1 = \left\{x_1,\ x_3\right\}$ *and* $X_2 = \left\{x_1,\ x_4,\ x_5\right\}$ of U. Now, for $\alpha \in \left(\cdot 7,\ \cdot 8\right]$ *and* $\beta \in \left(\cdot 1,\ \cdot 2\right]$

$$\underline{X}_{1,\alpha,\beta} = \phi\ \ and\ \ \overline{X}_{1,\alpha,\beta} = \left\{x_1,\ x_2,\ x_3,\ x_4\right\},$$

so that X_1 is $\left(\alpha,\beta\right)$-rough. On the other hand for $\alpha \in \left(\cdot 9,\ \cdot 1\right]$ *and* $\beta \in \left(0, \cdot 1\right]$,

$$\underline{X}_{2,\alpha,\beta} = \left\{x_2,\ x_4,\ x_5\right\}\ \ and\ \ \overline{X}_{2,\alpha,\beta} = \left\{x_2,\ x_4,\ x_5\right\},$$

so that X_2 in $(\alpha,\ \beta)$-discernible.

The following theorem establishes properties of $(\alpha,\ \beta)$-lower approximation and $(\alpha,\ \beta)$-upper approximation sets of a rough set. We shall provide proofs for only two cases. The proofs of other cases are similar.

Theorem 3.4. Let X and Y be rough sets in the generalised approximation space $(U,\ R(\alpha,\beta)),\ (\alpha,\beta)\in J$. Then

(3.16) $$\underline{X}_{\alpha,\ \beta}\subseteq X\subseteq \overline{X}_{\alpha,\ \beta}.$$

(3.17) $$\underline{\phi}_{\alpha,\ \beta}=\overline{\phi}_{\alpha,\beta}=\phi,\ \underline{U}_{\alpha,\beta}=\overline{U}_{\alpha,\beta}=U.$$

(3.18) $$\overline{(X\cup Y)}_{\alpha,\beta}=\overline{X}_{\alpha,\ \beta}\cup \overline{Y}_{\alpha,\beta}.$$

(3.19) $$\underline{(X\cap Y)}_{\alpha,\beta}=\underline{X}_{\alpha,\ \beta}\cap \underline{Y}_{\alpha,\beta}$$

(3.20) $$X\subseteq Y\Rightarrow \underline{X}_{\alpha,\beta}\subseteq \underline{Y}_{\alpha,\beta}.$$

(3.21) $$X\subseteq Y\Rightarrow \overline{X}_{\alpha,\beta}\subseteq \overline{Y}_{\alpha,\ \beta}.$$

(3.22) $$\underline{(X\cup Y)}_{\alpha,\ \beta}\supseteq \underline{X}_{\alpha,\beta}\cup \underline{Y}_{\alpha,\ \beta}$$

(3.23) $$\left(\overline{X\cap Y}\right)_{\alpha,\beta}\subseteq \overline{X}_{\alpha,\ \beta}\cap \overline{Y}_{\alpha,\beta}$$

Proof. $x\in \underline{X}_{\alpha,\beta}\Rightarrow x\in[y]\in R^*_{\alpha,\beta}\ \ and\ \ [y]\subseteq X.$

$$\Rightarrow x\in X.$$

Again, $x\in X\Rightarrow x\ R(\alpha,\ \beta)\ x$ for any $(\alpha,\ \beta)\in J$ by (3.1)

$\Rightarrow x\in[x]\in R^*_{\alpha,\beta}$, such that $[x]\cap X\neq\phi$

$$\Rightarrow x\in \overline{X}_{\alpha,\beta}.$$

This proves (3.16).

Next, $x\in \overline{(X\cup Y)}_{\alpha,\ \beta}\Rightarrow$ there exists an equivalence class [z] with respect to $R(\alpha,\beta)$ such that $x\in[z]\ \ and\ \ [z]\cap(X\cup Y)\neq\phi.$

$$\Rightarrow x\in[z]\ and\ [z]\cap X\neq\phi\ \ or\ \ [z]\cap Y\neq\phi$$

$$\Rightarrow x\in \overline{X}_{\alpha,\beta}\ \ or\ \ x\in \overline{Y}_{\alpha,\beta}$$

(3.24) $\Rightarrow x \in \overline{X}_{\alpha,\beta} \bigcup \overline{Y}_{\alpha,\beta}$

Also, $x \in \overline{X}_{\alpha,\beta} \bigcup \overline{Y}_{\alpha,\beta} \Rightarrow x \in \overline{X}_{\alpha,\beta}$ *or* $x \in \overline{Y}_{\alpha,\beta}$

$\Rightarrow \exists [y]$ such that $x \in [y]$ *and* $[y] \bigcap X \neq \phi$ or $\exists [z]$ such that $x \in [z]$ *and* $[z] \bigcap X \neq \phi$.

$\Rightarrow$ In any case there exists an equivalence class containing x which has nonempty intersection with $X \bigcup Y$.

(3.25) $\Rightarrow x \in \overline{(X \bigcup Y)}_{\alpha,\beta}$.

We get (3.18) from (3.24) and (3.25).

Theorem 3.5. If $\alpha_1 \geq \alpha_2$ *and* $\beta_1 \leq \beta_2$ then

(3.26) $\underline{X}_{\alpha_1,\beta_2} \subseteq \underline{X}_{\alpha_1,\beta_1}$

(3.27) $\overline{X}_{\alpha_2,\beta_2} \subseteq \overline{X}_{\alpha_1,\beta_1}$.

Proof. Let $x \in \underline{X}_{\alpha_2,\beta_2}$. Then $[x]_{\alpha_2,\beta_2} \subseteq X$.

Now,

$$y \in [x]_{\alpha_1,\beta_1} \Rightarrow \mu_R(x,y) \geq \alpha_1 \ \text{and} \ \nu_R(x,y) \leq \beta_1$$

$$\Rightarrow \mu_R(x,y) \geq \alpha_2 \ \text{and} \ \nu_R(x,y) \leq \beta_2$$

$$\Rightarrow y \in [x]_{\alpha_2,\beta_2}.$$

So, $[x]_{\alpha_1,\beta_1} \subseteq X$. Hence $x \in \underline{X}_{\alpha_1,\beta_1}$. This proves (3.26).

Again $x \in \overline{X}_{\alpha_1,\beta_1} \Rightarrow [x]_{\alpha_1,\beta_1} \bigcap X \neq \phi$

$\Rightarrow [x]_{\alpha_2,\beta_2} \bigcap X \neq \phi$, as above

$\Rightarrow x \in \overline{X}_{\alpha_2,\beta_2}$.

This completes the proof.

Theorem 3.6. Let R and S be two IF-proximity relations on U and α,β be chosen level values with $(\alpha,\beta) \in J$. Then

(3.28) $\underline{(R \bigcup S)}_{\alpha,\beta} X \subseteq \underline{R}_{\alpha,\beta} X \, U \, \underline{S}_{\alpha,\beta} X$,

(3.29) $\overline{(R \bigcup S)}_{\alpha,\beta} X \supseteq \overline{R}_{\alpha,\beta} X \bigcup \overline{S}_{\alpha,\beta} X$,

(3.30) $\underline{(R \bigcap S)}_{\alpha,\beta} X \supseteq \underline{R}_{\alpha,\beta} X \bigcap \underline{S}_{\alpha,\beta} X$

and

(3.31) $$\overline{(R \cap S)}_{\alpha,\beta}\ X \subseteq \overline{R}_{\alpha,\beta}\ X \cap \overline{S}_{\alpha,\beta}\ X\ .$$

Proof. We prove only (3.28). Rest of the proofs are similar.

4. Kinds of rough sets on IF-approximation spaces.

Pawlak ([4]) has introduced the concept of kinds of rough sets depending upon the characteristics of their lower and upper approximation sets. This was extended to the setting of rough sets on fuzzy approximation spaces by Tripathy ([5]), where properties on union and intersection of rough sets of same kind have been studied.

In this section we extend the concept further by defining kinds of rough sets on IF-approximation spaces. Also, we provide physical interpretation of each kind of such sets and state properties on union and intersection of these sets, which can be proved in the same lines as in ([5]).

Definition 4.1. (i) If $\underline{X}_{\alpha,\beta} \neq \phi$, $\overline{X}_{\alpha,\beta} \neq U$ then we say that X is *roughly* $R_{\alpha,\beta}$ - *definable.*

(ii) If $\underline{X}_{\alpha,\beta} = \phi$ *and* $\overline{X}_{\alpha,\beta} \neq U$ then we say that X is *internally* $R_{\alpha,\beta}$*-undefinable.*

(iii) If $\underline{X}_{\alpha,\beta} \neq \phi$ *and* $\overline{X}_{\alpha,\beta} = U$ then we say X is *extrenally* $R_{\alpha,\beta}$ - *undefinable..*

(iv) If $\underline{X}_{\alpha,\beta} = \phi$ *and* $\overline{X}_{\alpha,\beta} \neq U$ then we say that X is *totally* $R_{\alpha,\beta}$ - *undefinable.*

For any subset X of U, we denote by - X the complement of X in U.

Physical interpretations.

(i) A set X is roughly $R_{\alpha,\beta}$-definable means that we are able to decide for some elements of U whether they are (α,β)-similar/transitively (α,β)-similar to some elements of X or –X with respect to R.

(ii) A set X is roughly $R_{\alpha,\beta}$-undefinable means that we are able to decide whether some elements of U are (α,β)-similar/transitively (α,β)-similar to some elements of –X but we are unable to indicate this property for any element of X with respect to R.

(iii) A set X is externally $R_{\alpha,\beta}$-definable means that we are able to decide whether some elements of U are (α,β)-similar/transitively (α,β)-similar to some elements of X, but we are unable to indicate this property for any element of –X with respect to R.

(iv) A set X is totally $R_{\alpha,\beta}$-undefinable means that we are unable to decide for any element of U whether it is (α,β)-similar/transitively $(\alpha,\ \beta)$-similar to some element of X or –X with respect to R.

The following theorem is an extension of the corresponding theorem in [5]. The proof requires the use of results of section 3 above which are extensions of the corresponding results used in ([5]). It may be noted that in ([5]) examples have been provided to show that in general the results can not be sharpened.

Theorem 4.1.

(i) If X and Y are internally $R_{\alpha,\beta}$-undefinable then $X \cap Y$ is internally $R_{\alpha,\beta}$-undefinable.

(ii) If X and Y are internally $R_{\alpha,\beta}$-undefinable then $X \cup Y$ can be in one of the four classes.

(iii) If X and Y are roughly $R_{\alpha,\beta}$-definable then $X \cup Y$ can be roughly $R_{\alpha,\beta}$-definable or internally $R_{\alpha,\beta}$-undefinable.

(iv) If X and Y are roughly $R_{\alpha,\beta}$-definable then $X \cup Y$ may be roughly $R_{\alpha,\beta}$-definable or externally $R_{\alpha,\beta}$-undefinable.

(v) If X and Y are externally $R_{\alpha,\beta}$-undefinable then $X \cap Y$ can be in any one of the four classes.

(vi) If X and Y are externally $R_{\alpha,\beta}$-undefinable there $X \cup Y$ is externally $R_{\alpha,\beta}$-undefinable.

(vii) If X and Y are totally $R_{\alpha,\beta}$-undefinable then $X \cap Y$ can be internally $R_{\alpha,\beta}$-undefinable or totally $R_{\alpha,\beta}$-undefinable.

(viii) If X and Y are totally $R_{\alpha,\beta}$-undefinable then $X \cup Y$ can be externally $R_{\alpha,\beta}$-undefinable or totally $R_{\alpha,\beta}$-undefinable.

5. Conclusion.

In this article we introduced the concept of IF-approximation spaces and rough sets defined over them. Some properties of these notions have been established. Also, the concept of kinds of rough sets is extended to this general context. Some more results in the line of those in ([5]) and application of rough sets on IF-approximation spaces to the study of dependency of attributes are to come out in a subsequent paper.

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