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(t,s)-intuitionistic fuzzy subgroups

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Abstract: In this paper, we give concept of intuitionistic fuzzy point (IFP), and give two kinds of definitions of intuitionistic fuzzy subgroups (IFSG) by the use of t-norm and s-norm, and discuss their properties, we also introduce normal IFSG of a group G, and build intuitionistic fuzzy quotient qroup of a group.

Keywords: Intuitionistic fuzzy set, group, IFP, IFSG.

1. Preliminary

Definition 1.1[1]. Let a set E be fixed, An IFS A in E is an object having the form

$$A^* = \{<x, \mu_A(x), \upsilon_A(x)> | x \in E\}$$

where the function $\mu_A: E \to [0,1]$ and $\upsilon_A: E \to [0,1]$ define the degree of membership and degree of nonmembership of the element $x \in E$ to the set A which is subset of E, and for every $x \in E$: $0 \leqslant \mu_A(x) + \upsilon_A(x) \leqslant 1$.

Defintion 1.2. Let A be a IFS over E. If IFS B satisfies:

$$\mu_B(x') = \begin{cases} \lambda & x' = x \\ 0, & x' \neq x \end{cases}, \upsilon_B(x') = \begin{cases} \mu & x' = x \\ 1 & x' \neq x \end{cases}$$

then B is called as an intuitionistic fuzzy point (IFP), it is denoted as $x_{(\lambda,\mu)}$. IF $\mu_A(x) \geqslant \lambda, \upsilon_A(x) \leqslant \mu$, then we say that $x_{(\lambda,\mu)}$ belong to A, it is denoted as $x_{(\lambda,\mu)} \in A$.

Definition 1.3. A mapping $T: [0,1] \times [0.1] \to [0,1]$ is called a t-norm if for all $\lambda, \mu, \xi, \eta \in [0,1]$, we have

(1) $T(\lambda,0) = 0, T(\lambda,1) = \lambda$;

(2) $T(\lambda,\mu) = T(\mu,\lambda)$;

(3) $\lambda \leqslant \xi, \mu \leqslant \eta \Rightarrow T(\lambda,\mu) \leqslant T(\xi,\eta)$;

(4) $T(\lambda, T(\mu,\xi)) = T(T(\lambda,\mu),\xi)$.

Definition 1.4. A mapping $S: [0,1] \times [0,1] \to [0,1]$ is called a s-norm if for all $\lambda, \mu, \xi, \eta \in [0,1]$, we have

(1) $S(\lambda,0) = \lambda, S(\lambda,1) = 1$;

(2) $S(\lambda,\mu) = S(\mu,\lambda)$;

(3) $\lambda \leqslant \xi, \mu \leqslant \eta \Rightarrow S(\lambda,\mu) \leqslant S(\xi,\eta)$;

(4) $S(\lambda, S(\mu,\xi)) = S(S(\lambda,\mu),\xi)$.

Let T be a t-norm and $S(a,b)=1-T(1-a,1-b)$, $\forall a,b\in[0,1]$. Then S is a s-norm and is said to be a derived s-norm by T.

2. (t,s)—intuitionistic fuzzy subgroups

Definition 2.1 Let A be an IFS of group G. A is said to be an (t,s)-intuitionistic fuzzy subgroups of G((t,s)-IFSG)if for any $x,y\in G$ and $\lambda,\mu\ t,r\in[0,1]$, we have

(1)$x_{(\lambda,t)},y_{(\mu,r)}\in A \Rightarrow (xy)_{(T(\lambda,\mu),S(t,r))}\in A$; (2)$x_{(\lambda,t)}\in A \Rightarrow x^{-1}_{(\lambda,t)}\in A$.

where T is a t-norm, S is a derived s-norm by T, i.e $S(a,b)=1-T(1-a,1-b)$, $\forall a,b\in[0,1]$.

Theorem 2.1 Let G be a group and A be a IFS over G, then A is an (t,s)-IFSG of G, if and only if

(i) $\mu_A(xy)\geqslant T(\mu_A(x),\mu_A(y)),\upsilon_A(xy)\leqslant S(\upsilon_A(x),\upsilon_A(y)),\forall x,y\in G$;

(ii) $\mu_A(x^{-1})\geqslant\mu_A(x),\upsilon_A(x^{-1})\leqslant\upsilon_A(x),\forall x\in G$.

where T is a t-norm, S is a derived s-norm by T.

proof: "$\Rightarrow$" (i)For any $x,y\in G$, $x_{(\mu_A(x),\upsilon_A(x))}\in A$, $y_{(\mu_A(y),\upsilon_A(y))}\in A$.

Since A is a (t,s)-IFSG of G, so $(xy)_{(T(\mu_A(x),\mu_A(y)),S(\upsilon_A(x),\upsilon_A(y)))}\in A$ and consequently $\mu_A(xy)\geqslant T(\mu_A(x),\mu_A(y)),\upsilon_A(xy)\leqslant S(\upsilon_A(x),\upsilon_A(y))$.

(ii) is clear.

"$\Rightarrow$" (1) Let $x_{(\lambda,t)}\ y_{(\mu,r)}\in A$. Then $\mu_A(x)\geqslant\lambda,\mu_A(y)\geqslant\mu,\upsilon_A(x)\leqslant t,\upsilon_A(y)\leqslant r$ and consequently $\mu_A(xy)\geqslant T(\mu_A(x),\mu_A(y))\geqslant T(\lambda,\mu),\upsilon_A(xy)\leqslant S(\upsilon_A(x),\upsilon_A(y)\leqslant S(t,r)$. By $\lambda+t\leqslant 1,\mu+r\leqslant 1$, we have

$T(\lambda,\mu)+S(t,r)=T(\lambda,\mu)+(1-T(1-t,1-r))\leqslant T(1-t,1-r)+(1-T(1-t,1-r))=1$, therefore $(xy)_{(T(\lambda,\mu),S(t,r))}\in A$.

(2)is clear.

Theorem2.2, Let A be a IFS of a group G, e is identity of group G. IF

$$T(\mu_A(x),\mu_A(e))\geqslant\mu_A(x),S(\upsilon_A(x),\upsilon_A(e))\leqslant\upsilon_A(x),\ \forall x\in G.$$

then A is a (t,s)-IFSG of group G if and only if

$$\mu_A(xy^{-1})\geqslant T(\mu_A(x),\mu_A(y)),\upsilon_A(xy^{-1})\leqslant S(\upsilon_A(x),\upsilon_A(y)).$$

where T is a t-norm and S is a derived s-norm by T.

proof: "$\Rightarrow$"Let A be an IFS of G, then

$\mu_A(xy^{-1})\geqslant T(\mu_A(x),\mu_A(y^{-1}))\geqslant T(\mu_A(x),\mu_A(y)),\upsilon_A(xy^{-1})\leqslant S(\upsilon_A(x),\upsilon_A(y^{-1}))\leqslant S(\upsilon_A(x),\upsilon_A(y))$.

" " Since $\upsilon_A(xy^{-1})\leqslant S(\upsilon_A(e),\upsilon_A(y))\leqslant\upsilon_A(y)$, it follows that

$\upsilon_A(xy)=\upsilon_A(x(y^{-1})^{-1})\leqslant S(\upsilon_A(x),\upsilon_A(y^{-1}))\leqslant S(\upsilon_A(x),\upsilon_A(y))$,

$\mu_A(xy)\geqslant T(\mu_A(x),\mu_A(y))$ and $\mu_A(x^{-1})\geqslant\mu_A(x)$ are proved analogously.

So A is a (t,s)-IFSG of G.

Definition 2.2 Let X, Y be sets, $F:X\rightarrow Y$ is a mapping. A is an IFS of X, B is an IFS of Y. We define: $f^{-1}(B)$and $f(A)$ are IFSs of X and Y respectively. where $\mu_{f^{-1}(B)}(x)=\mu_B(f(x)),\upsilon_{f^{-1}}(B)(x)=\upsilon_B(f(x)),\mu_{f(A)}(y)=\sup\limits_{f(x)=y}\{\mu_A(x)\},\upsilon_{f(A)}(y)=\inf\limits_{f(x)=y}\{\upsilon_A(x)\}$and appoint

$\sup\{\lambda \mid \lambda \in \varnothing\}=0, \inf\{\lambda \mid \lambda \in \varnothing\}=1$.

Remark: $\mu_{f(A)}(y)+\upsilon_{f(A)}(y) \leqslant 1 \ (\forall\ y \in Y)$.

Definition 2.3 Let f be a mapping from set X to set Y, f is said to be accessible if for any IFS A of X, we have $\sup\limits_{f(x)=y}\{\mu_A(x)\}=\max\limits_{f(x)=y}\{\mu_A(x)\}$, $\inf\limits_{f(x)=y}\{\upsilon_A(x)\}=\min\limits_{f(x)=y}\{\upsilon_A(x)\}$.

Theorem 2.3 Let G and G' be groups. $F: G \to G'$ is a homomorphism,

(1) If A is a (t,s)-IFSG of G and f is accessible, then f(A) is a (t,s)-IFSG of G',

(2) Let B be a (t,s)-IFSG of G' then $f^{-1}(B)$ is a (t,s)-IFSG of G.

proof: (1) Let A be (t,s)-IFSG of G, since f is accessible, for any $y_1, y_2 \in G'$, there exist x_1^0, $x_2^0 \in G$ such that $\mu_{f(A)}(y_i)=\mu_A(x_i^0)$, and $f(x_i^0)=y_i (i=1,2)$.

$$\mu_{f(A)}(y_1y_2)=\sup_{f(x)=y_1y_2}\{\mu_A(x)\} \geqslant \sup_{f(x_i)=y_i}\{\mu_A(x_1x_2)\} \geqslant \sup_{f(x_i)=y_i}\{T(\mu_A(x_1),\mu_A(x_2))\}$$

$$\geqslant T(\mu_A(x_1^0),\mu_A(x_2^0))=T(\mu_{f(A)}(y_1),\mu_{f(A)}(y_2)) \ (i=1,2).$$

$\upsilon_{f(A)}(y_1y_2) \leqslant S(\upsilon_{f(A)}(y_1),\upsilon_{f(A)}(y_2))$ is proved analogously.

For any $y \in G'$, $\mu_{f(A)}(y^{-1})=\sup\limits_{f(x)=y^{-1}}\{\mu_A(x)\}=\sup\limits_{f(x^{-1})=y}\{\mu_A((x^{-1})^{-1}\} \geqslant \sup\limits_{f(x^{-1})=y}\{\mu_A(x^{-1}\}=\sup\limits_{f(x)=y}\{\mu_A(x)\}=\mu_{f(A)}(y)$. $\upsilon_{f(A)}(y^{-1}) \leqslant \upsilon_{f(A)}(y)$ is proved analogously.

Therefore f(A) is a (t,s)-IFSG of G'.

(2) is clear.

3. Normal (t,s)-intuitionistic fuzzy subgroups

Definition 3.1 Let A be an (t,s)-IFSG of group G. If $x_{(\lambda,\mu)} \in A$, then $(y^{-1}xy)_{(\lambda,\mu)} \in A$, $\forall\ x,y \in G$, then A is called as a normal (t,s)-IFSG of G. ((t,s)-NIFSG).

Definition 3.2 Let A ba an IFS of G, For $a \in G$ we define:

$aA=\{<x,\mu_{aA}(x),\upsilon_{aA}(x)> \mid x \in G\}$, $Aa=\{<x,\mu_{Aa}(x),\upsilon_{Aa}(x)> \mid x \in G\}$,

where $\mu_{aA}(x)=\mu_A(a^{-1}x)$, $\upsilon_{aA}(x)=\upsilon_A(a^{-1}x)$, $\mu_{Aa}(x)=\mu_A(xa^{-1})$, $\upsilon_{Aa}(x)=\upsilon_A(xa^{-1})$.

Clearly we have

Theorem 3.1 Let A be an (t,s)-IFSG of group G. Then the following conditiions are equivalent.

(1) A is a (t,s)-NIFSG of G;

(2) $\mu_A(x^{-1}yx) \geqslant \mu_A(y)$, $\upsilon_A(x^{-1}yx) \leqslant \upsilon_A(y)$, $\forall\ x,y \in G$;

(3) $\mu_A(xy)=\mu_A(yx)$, $\upsilon_A(xy)=\upsilon_A(yx)$, $\forall\ x,y \in G$;

(4) $xA=Ax$, $\forall\ x \in G$.

Theorem 3.2 Let G and G' be groups. $f: G \to G'$ is a group homomorphhism,

(1) If A ia a (t,s)-NIFSG of G and f is accessible, then f(A) is a (t,s)-NIFSG of G'.

(2) If B is a (t,s)-NIFSG of G', then $f'(B)^{-1}$ is a (t,s)-NIFSG of G.

proof: By theroem2, we know that f(A) and $f^{-1}(B)$ are (t,s)-IFSG.

(1) For any $y_1, y \in G'$

$$\mu_{f(A)}(y_1^{-1}yy_1)=\sup_{f(x)=y_1^{-1}yy_2}\mu_A(x) \geqslant \sup_{\substack{f(x)=y \\ f(x_1)=y_1}}\mu_A(x_1^{-1}xx_1) \geqslant \sup_{f(x)=y}\{\mu_A(x)\}=\mu_{f(A)}(y).$$

$\upsilon_{f(A)}(y_1^{-1}yy_1 \leqslant \upsilon_{f(A)}(y)$ is proved analogously.

Therefore f(A) is a (t,s)-NIFSG of G'.

(2)For any $x, y \in G$

$\mu_{f^{-1}(B)}(x^{-1}yx) = \mu_B(f(x^{-1}yx)) = \mu_B(f(x)^{-1}f(y)f(x)) \geqslant \mu_B(f(y)) = \mu_{f^{-1}(B)}(y)$

$\upsilon_{f^{-1}(B)}(x^{-1}yx) \leqslant \upsilon_{f^{-1}(B)}(y)$ is proved avalogously.

Therefor $f^{-1}(B)$ is a (t,s) -NIFSG of G.

Definition 3.3 Let A, B be IFS of G, we define:

$$AB = \{ < x, \mu_{AB}(x), \upsilon_{AB}(x) > | x \in G \}.$$

where $\mu_{AB}(x) = \sup\limits_{a \in G} T(\mu_A(a), \mu_B(a^{-1}x)), \upsilon_{AB}(x) = \inf\limits_{a \in G} S(\upsilon_A(a), \upsilon_B(a^{-1}x))$;

$$A^{-1} = \{ < x, \mu_A(x^{-1}), \upsilon_A(x^{-1}) > | x \in G \}.$$

Proposition: $\mu_{AB}(x) + \upsilon_{AB}(x) \leqslant 1$, for $\forall\ x \in G$.

Theorem 3.3 Let A be a (t,s)-NIFSG of group G, and for any $x \in G$.

$T(\mu_A(x), \mu_A(e)) \geqslant \mu_A(x), S(\upsilon_A(x), \upsilon_A(e)) \leqslant \upsilon_A(x)$,

then (1) $(xA)(yA) = (xy)A, (Ax)(Ay) = A(xy)$;

(2) $(xA)^{-1} = x^{-1}A$;

(3) $G/A = \{ xA \mid x \in G \}$ is a group (G/A is called (t,s)-intuitionistic fuzzy quotient group).

proof (1) For any $z \in G$

$\mu_{(xA)(yA)}(z) = \sup\limits_{a \in G} T(\mu_{xA}(a), \mu_{yA}(a^{-1}z)) \geqslant T(\mu_{xA}(x), \mu_{yA}(x^{-1}z))$

$= T(\mu_A(e), \mu_A(y^{-1}x^{-1}z)) \geqslant \mu_A(y^{-1}x^{-1}z) = \mu_A((xy)^{-1}z) = \mu_{(xy)A}(z)$.

On the other hand,

$\mu_{(xy)A}(z) = \mu_A((xy)^{-1}z) = \mu_A(y^{-1}x^{-1}ayy^{-1}a^{-1}z) \geqslant T(\mu_A(y^{-1}x^{-1}ay), \mu_A(y^{-1}a^{-1}z)) =$ $T(\mu_{yAy^{-1}}(x^{-1}a), \mu_A(y^{-1}a^{-1}z)). \geqslant T(\mu_A(x^{-1}a), \mu_A(y^{-1}a^{-1}z)) = T(\mu_{xA}(a), \mu_{yA}(a^{-1}z))$.

So $\mu_{(xy)A}(z) \geqslant \sup\limits_{a \in G} T(\mu_{xA}(a), \mu_{yA}(a^{-1}z)) = \mu_{(xA)(yA)}(z)$.

Therefore $\mu_{(xA)(yA)}(z) = \mu_{(xyA)}(z)$. $\upsilon_{(xA)(yA)}(z) = \upsilon_{xyA}(z)$ is proved analogously.

(2)and (3) are clear.

References

1. K. Atanassov. Intuitionistic fuzzy sets. Fuzzy sets and systems, 20(1986):87～96.
2. J. M. Anthony and H. Sherwood Fuzzy Groups Redefined. J. Math. Anal Appl, 69 (1979)124～130.