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Generalized nets in Image Processing and Pattern Recognition

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Abstract:
Generalized Nets (GNs) are extensions of Petri nets and their other extensions and modifications. A lot of research was carried out in the last 15 years to show the possibilities for the representdtion by GNs of different objects from the area of Artificial Intelligence. In a series of papers they were used for modelling in image processing. The possibilities of the GNs as a tool for modelling in this area are presented and an example of a GN-model of a writer identification system is given.

1 Generalized nets and Artificial Intelligence

In [5] a supposition was launched that Generalized Nets (GNs, see [4]) could be a universal tool for modeling of intellectual processes, i.e. processes that could be associated to the domain of AI. If this is true different investigation areas could be described using the same mathematical instrument, which imagine a mathematical tool that can play the role of a relatively universal language. Therefore, they will be comparable to each other, which will facilitate the transfer of ideas from one area to another, as well as their stronger formalization and further development.

There exists, however, another problem, more difficult than this one. The problem is how can we generalize and extend this description within the framework of the GN-description of each event, adding new (perhaps not yet existing but theoretically possible) elements in a way that allows the newly obtained process (object) to be described by a GN as well. If we can achieve this, then it will be clear that the GNs are not only capable of describing processes (objects), but serve to construct new, even not yet existing processes (objects).

Finally, a third problem arises, namely to search for the possible directions of a further development of the GN-methodology and new objects to be described by it.

Roughly speaking, this is the basic idea so far and further work has to be done in this area.

Let us point out at the most interesting from the already existing results. They are related to the areas of
(a) expert systems – e.g., in [5] a few GN-models are proposed, able to describe the class of all ESs;
(b) machine learning – e.g., in [7] GNs, describing different machine learning processes are suggested;
(c) pattern recognition – e.g., in [11, 9, 12, 13, 14, 15, 16] different applications of pattern recognition are presented in terms of GNs;
(d) robotics – e.g., in [8, 33, 34] GNs are describing different robotic and flexible manufacturing systems;
(e) neural networks – e.g., in [17, 18, 19, 20, 21] GNs are applied to the different types of neural networks representing their functioning and the results of their work);
(f) problems related to scheduling, assignment, optimization, decision making, etc. based on heuristics (see, e.g., [25]);
and others.

2 Generalized net description of image processing and pattern recognition

Image processing and pattern recognition are the most intensively developing areas of research during the last decades. This interest is provoked by the necessity for the development of intelligent devices and system for the automatic detection of objects in complex images and their description and identification. It will steadily increase due to the new challenges imposed by the increased people's mobility, terrorist and criminal actions and the requirements for reliable access-permit identification/verification systems.

The solution of every specific problem in that area could be described through the following scheme: image capture, image pre-pro cessing including noise reduction, contrast improvement and edge detection, feature extraction and decision-making. Many heuristic and theoretically founded algorithms have been developed dealing with the above-mentioned sub-problems. However, since different sources of errors could influence the image quality on the one hand, and lack of sufficient a priori information could hamper the proper recognition, on the other hand, algorithms based on the theory of the Generalized Nets and IFS seem to be helpful for the description of the whole process. A few attempts in this direction have already been made. They concerns the problems of handwriting analysis and identification [10, 12, 14, 30], face recognition [15, 31] and speaker identification [11, 29].

In the following paragraph a specific task will be described as an illustration of the possibility of the GN theory application to the recognition problems.

3 An example: an application of the GNs to the problem of writer identification

The identification problem concerns different areas of the social and economic relations in the state. One of its aspects refers to writer identification. Most frequently it appears in forensic investigations when the identity of handwritten document is to be established or when a signature is to be verified. In all cases of writer identification the objectiveness of the analysis and reliability of conclusion are of great importance, especially if the handwriting is deliberately changed. However, at the time being the handwriting investigation is carried out by qualified experts on the basis of their experience and subjective evaluation of the similarity between letters, strokes and writing style (except special cases of signature verification in banking). In this situation different experts may disagree as to who is the writer of a particular document.

This is because the handwriting investigation is a difficult and non- well studied problem. Despite that during the last decades a lot of work has been done towards the formulation of a general methodology and groups of general and specific features have been established [1, 22, 24], the expert's decision is based on his own visual sensing and experience. To increase the objectiveness and diminsh expert workload quantitative methods have to be developed . However, some of the features used by the experts are of qualitative nature and can not be easily presented quantitatively. The other problem concerns the segmentation of the handwritten text aimed at the separation of words, letters or combination of letters [2, 30, 32].

The first computer-based systems appeared in 80's: FISH (The Forensic Information System for Handwriting) in Germany, NIFO (Netherlands Institute for Forensic Examination and Research) in The Netherlands. In Bulgaria the systems EXPERT and PRESS have been developed.

The systems FISH and NIFO measure some general features and extract similar handwritings from a large data base. EXPERT is used for the measurement of specific features and comparison with a small number of handwritings. PRESS was used for the pressure evaluation.

The aim of this paper is to show that the philosophy of GNs could be applied to the comprehensive description of writer identification problem, thus adding a new scientific area under the hat of GNs.

The Generalized Net (GN, see, e.g., [4, 28]) based model described in this paper is a further step to the comprehensive presentation of the writer identification process (see also [12, 30]).

3.1 Description of a writer identification system

A general block-diagram of a computer-based handwriting investigation system is swhown in Fig. 1. The blocks are breafly described underneath.

3.1.1. Image enhancement

Since very often the handwritten materials are of poor quality, it is necessary to achieve some pre-processing before starting evaluating them. The goal of this pre-processing is two-fold: a) to improve image quality including contrast enhancement, random and structured noise reduction, and edge sharpening. In such a way the image will become more pleasing visually, on the one hand, and will offer better possibilities for the automatic analysis, on the other hand; b) to correct and analyze strokes and complex lines using morphological operations depending on the features that will be measured. This is especially important for the analysis of the specific handwriting features, where the skeleton of the characters is going to be used.

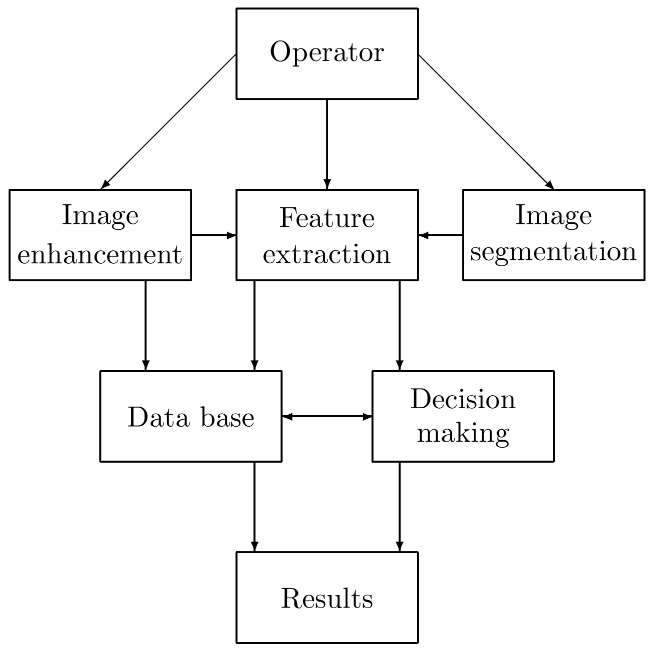

Figure 1: Block-diagram of the system

3.1.2. Text segmentation

This is a major problem in the automatic document analysis [13]. In handwriting analysis additional difficulties may arise due to the possibility of significant variation in placing rows, words and letters in the sheet. While, in general, the separation of rows could be easily achieved, special techniques are required for a proper detection of under-row and over-row strokes of some characters. Also, the medial axis of the row may consist of a few linear segments and its description is a problem that has to be solved, as well.

The segmentation of words in a row is the next step of the analysis. Different cases of concise writing or writing where letters are not connected between them are a challenge.

The most difficult problem concerns segmentation of letters and strokes. Except some special cases e.g. child's writing, their separation may be quite difficult even for a human being. For the solution of this problem a user-friendly man-machine dialog has to be developed.

3.1.3. Feature extraction

This is also a crucial problem that should be solved. While during the last decades a common methodology for the handwriting analysis has been set up, many of the suggested features are of qualitative character and are prone to different evaluation from different experts. Also, there are no strong recommendations as to what number of features is to be used for a reliable decision-making.

In this description we intend to use two types of features: continuous and categorical. Continuous features will refer to the graphometric parameters, like size of letters, distances between characteristic points, angles, ratios and so on. The categorical features will concern the so called *general* features. They will include the evaluation of common handwriting characteristics as degree of connection between letters (usually three degrees are accepted: low, moderate and high), slope (right, left, upright), motion (rectilinear, curvilinear, angular or arched, loop-like, oval, wavy or spiral), elaboration (presence of ornaments), direction of movement (clock-wise or counter-clock-wise), quantity of movement (average number of strokes used to draw separate letters) and like.

While many of the above-mentioned features can be easily imitated, there are features that are not seen and therefore difficult to falsify. In that respect a special attention will be paid to the distribution of pressure which is based on the writing habit and writer's experience. It will be analyzed in different ways in order to find the most reliable description. One of them will be based on the evaluation of the geometric parameters of areas of different pressure and their mutual disposition at different writing elements e.g. letters or separate strokes, signatures. A second one will be using the pressure change alongside the skeleton of the elements. After normalization the obtained functions will be used for the comparison between handwritings.

Except the described features which are reasonable and intuitively clear from the expert's point of view, other features that do not express a particular property of the handwriting will be investigated as well. These include topological features based on the measurement of general characteristics of particular characters. According to this approach characters are divided into specific segments that can be transformed piece- wise. Segments are determined automatically by topologically invariant points. Another approach is based on the presentation of the characters as modifications of ideal models. Thus a transformation between the model and the real character can be evaluated.

3.1.4. Decision-making

The overall estimation of the similarity between two handwritings will be obtained as a combination between two decision-making classifiers. One of them will deal with continuous features and the other will deal with categorical features.

The decision-making for the continuous features will be based on the evaluation of the similarity between particular elements from the handwritings under investigation. Since the overall estimation will be based first on the estimations of separate elements and second on groups of elements, multi-level classifiers have to be used. The first level will concern the comparison of basic elements like strokes, letters and signs of punctuation. At the output of these classifiers every element will be assigned a number that reveals the degree of similarity between the handwritings under investigation. Since a particular element may be detected in a few places in the text, an average similarity relative to this element will be calculated at the second stage. After the similarity is evaluated for all different elements, an overall evaluation will be obtained at the third level. One of the basic problems that has to be solved here concerns the weight factors of the elements, i.e. their classification power. Different types of decision rules will be used, including statistical, linear, heuristic, and rules based on NN. The categorical features are used mainly for the search of similar handwritings to a particular one from a large data-base of handwritings. Also, for the sake of one-to-one comparison mixed variables discriminant techniques could be used. A simple approach for the analysis of mixture of categorical and continuous data requires arbitrary scoring of all the categorical variables followed by the use of standard methods for multivariate continuous data, which in the case of classification means use of techniques such as linear or quadratic discriminant analysis. Following statistical approaches may be used: a non-parametric kernel approach, a semi-parametric approach through logistic discrimination, and a fully parametric method based on the location model.

3.1.5. Data base

It is anticipated that a hierarchical data-base of object oriented organization is designed

which will include both alpha- numerical data and images. Basic data consists of:
a) office data including case description, expert's name and other,
b) personal information about the writer when known,
c) image files of type BMP, TIF, JPG, DXF and others presenting the primary documents of expertise, and
d) feature values.

Data is to be organized in terms of the following classes of objects: *document* which includes general information from a) and b) and a reference to the document image file from c); *paragraph* which includes a description of a rectangular image region that is to be analyzed; *symbol* which includes the identifier of the character that has been analyzed, description of the corresponding image sub-region, list of specific points, list of features and their values. The classes following their inheritance will also include all the links between their objects required by the application system. To decrease the space, only one image copy will be presented for each document while the sub-images will be referred by the corresponding descriptors. Compression methods will be applied to the image files.

3.2 GN-model

In this section we shall construct a reduced Generalized Net (GN, see [4] shown on Figure 2. this GN is without temporal components, without transitions, places and tokens priorities and without places and arcs capacities, and for which the tokens keep all their history, that is, for every token α: $b(\alpha) = \infty$.

We shall describe the transition condition predicates and the tokens' characteristics not fully formally in order to ease the understanding of the actual formalism in use.

Initially, token α enters place l_1 with the initial characteristic:

"digital matrix of the text image"

Entering transition Z_1, token α can split into two or more tokens, if the original text has to be processed by different procedures. Each of the new tokens will be interpreted as an α-token. All of them will transfer independently in the next transition and all of them will unite in place l_6 generating again only one α-token.

$$Z_1 =< \{l_1\}, \{l_2, l_3, l_4, l_5\},$$

$$\begin{array}{c|cccc} & l_2 & l_3 & l_4 & l_5 \\ \hline l_1 & W_{1,2} & W_{1,3} & W_{1,4} & W_{1,5} \end{array}, \vee(l_1) >,$$

where
$W_{1,2} =$ "the noise reduction is necessary",
$W_{1,3} =$ "the contrast enhancement is necessary",
$W_{1,4} =$ "the background elimination is necessary",
$W_{1,5} = \neg W_{1,2} \& \neg W_{1,3} \& \neg W_{1,4}$.

The tokens obtain following characteristics:

"digital matrix of the de-noised image"

in place l_2,

"digital matrix of the sharpened image"

in place l_3,

"digital matrix of the extracted text"

in place l_4, and they do not obtain any characteristic in place l_5.

Let us denote the current characteristic of each of α-tokens by x^{α}_{cu} and its characteristic obtained before s steps – by x^{α}_{cu-s}.

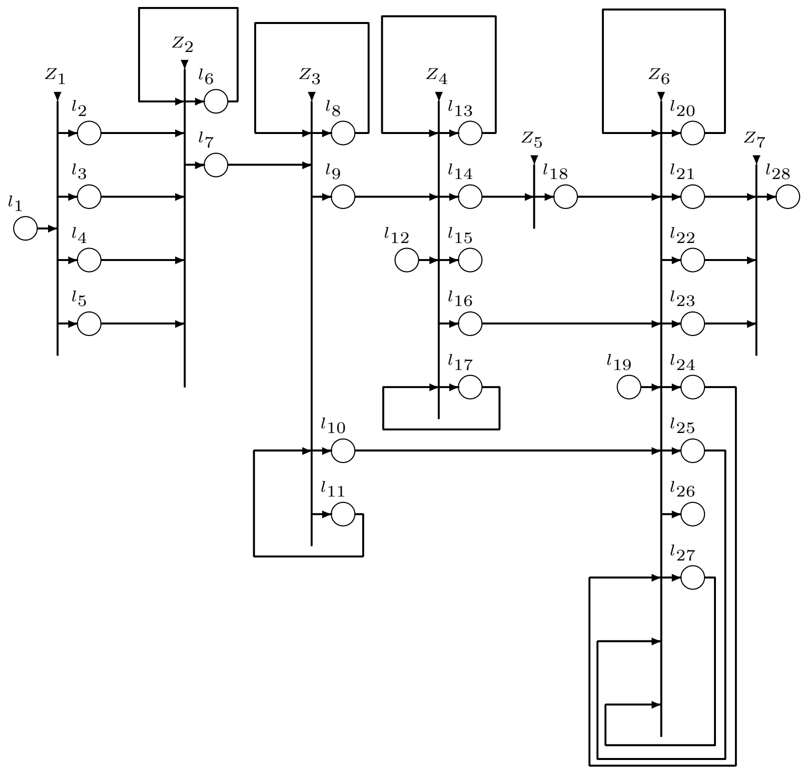

Figure 2: GN-model of the process of handwriting analysis

Transition Z_2 is described as

$$Z_2 =< \{l_2, l_3, l_4, l_5\}, \{l_6, l_7\},$$

	l_6	l_7
l_2	$true$	$false$
l_3	$true$	$false$
l_4	$true$	$false$
l_5	$true$	$false$
l_6	$false$	$true$

$$, \vee(l_2, l_3, l_4, l_5, l_6) > .$$

The tokens obtain characteristic

"enhanced image"

in place l_6, and they do not obtain any characteristic in place l_7.

Transition Z_3 describes the text segmentation. It has the form

$$Z_3 =< \{l_7, l_8, l_{11}\}, \{l_8, l_9, l_{10}, l_{11}\},$$

	l_8	l_9	l_{10}	l_{11}
l_7	$W_{7,8}$	$W_{7,9}$	$false$	$false$
l_8	$W_{8,8}$	$W_{8,9}$	$false$	$false$
l_{11}	$false$	$false$	$W_{11,10}$	$W_{11,11}$

,

$$\wedge(\vee(l_7, l_8), l_{11}) >,$$

In place l_{11} there is token β that transfers only in this place while processing α tokens in the transition, and it will enter place l_{10} when the last α-token enters place l_9.

Predicates $W_{7,8}, W_{7,9}, W_{8,8}, W_{8,9}, W_{11,10}$ and $W_{11,11}$ have the meaning:
$W_{7,8} =$ "the text contains more than one word",
$W_{7,9} =$ "the text contains exactly one word",
$W_{8,8} =$ "the text contains more than $s+1$ words, where s is the number of the cycles of the current token in place l_8",
$W_{8,9} = \neg W_{8,8}$,
$W_{11,10} =$ "there are not tokens in place l_8",
$W_{11,11} = \neg W_{11,10}$.

The tokens obtain the characteristics

"current word in the text"

in place l_8,

"distance between the current word, observing in place l_8 and its next word in the row; distance between the current row, where is placed the word, observing in place l_8 and its next row; declination of the text; height of the letters; other formal parameters determined by the used before the simulation"

in place l_{11}, and they do not obtain any characteristics in places l_9 and l_{10}.

Token β enters place l_{12} with an initial characteristic "user defined text and character features".

$$Z_4 =< \{l_9, l_{12}, l_{13}, l_{17}\}, \{l_{13}, l_{14}, l_{15}, l_{16}, l_{17}\},$$

	l_{13}	l_{14}	l_{15}	l_{16}	l_{17}
l_9	$W_{9,13}$	$W_{9,14}$	$false$	$false$	$false$
l_{12}	$false$	$false$	$true$	$false$	$false$
l_{13}	$W_{13,13}$	$W_{13,14}$	$false$	$false$	$false$
l_{17}	$false$	$false$	$false$	$W_{17,16}$	$W_{17,17}$

,

$$\wedge(\vee(l_9, l_{13}), l_{12}, l_{17}) >,$$

where
$W_{9,13} =$ "the word contains more than one letter",
$W_{9,14} =$ "the word contains exactly one letter",
$W_{13,13} =$ "the word contains more than $s+1$ letters, where s is the number of the cycles of the current token in place l_{13}",
$W_{13,14} = \neg W_{13,13}$,
$W_{17,16} =$ "there are not tokens in place l_{13}",
$W_{17,17} = \neg W_{17,16}$.

The tokens obtain the characteristics

"current letter in the word"

in place l_{13},

"values of the user-defined measurements of the parameters in the initial characteristic of token β"

in place l_{17}, and they do not obtain any characteristics in places l_{14} and l_{16}.

$$Z_5 =< \{l_{14}\}, \{l_{18}\}, \begin{array}{c|c} & l_{18} \\ \hline l_{14} & true \end{array}, \vee(l_{14}) > .$$

Token γ enters place l_{19} with an initial characteristic "data base of character parameters"

$$Z_6 =< \{l_{10}, l_{16}, l_{18}, l_{19}, l_{20}, l_{24}, l_{25}, l_{27}\},$$

$$\{l_{20}, l_{21}, l_{22}, l_{23}, l_{24}, l_{25}, l_{26}, l_{27}\},$$

	l_{20}	l_{21}	l_{22}	l_{23}	l_{24}	l_{25}	l_{26}	l_{27}
l_{10}	$false$	$false$	$false$	$false$	$false$	$true$	$false$	$false$
l_{16}	$false$	$false$	$false$	$false$	$true$	$false$	$false$	$false$
l_{18}	$true$	$true$	$true$	$true$	$false$	$false$	$false$	$false$
l_{19}	$false$	$false$	$false$	$false$	$false$	$false$	$true$	$false$
l_{20}	$true$	$true$	$true$	$true$	$false$	$false$	$false$	$false$
l_{24}	$false$	$false$	$false$	$false$	$true$	$false$	$false$	$false$
l_{25}	$false$	$false$	$false$	$false$	$false$	$true$	$false$	$false$
l_{27}	$false$	$false$	$false$	$false$	$false$	$false$	$false$	$true$

,

$$\wedge(l_{10}, l_{16}, \vee(l_{18}, l_{20}), l_{24}, l_{25}, l_{27}) > .$$

The tokens have the characteristics

"a particular letter parameters"

in place l_{20},

"general parameters"

in place l_{21},

"specific feature parameters"

in place l_{22},

"formalized feature parameters"

in place l_{23},

"general parameters from the data base"

in place l_{24},

"specific parameters from the data base"

in place l_{25},

"data base search"

in place l_{27}, but they do not take on any characteristic in place l_{26}.

$$Z_7 =< \{l_{21}, l_{22}, l_{23}\}, \{l_{28}\}, \begin{array}{c|c} & l_{28} \\ \hline l_{21} & true \\ l_{22} & true \\ l_{23} & true \end{array}, \wedge(l_{21}, l_{22}), l_{23}) > .$$

The tokens are given the characteristic

"best match"

in place l_{28}.

4 Conclusion

The GN-models of the real processes can be used for constructing of more general models. For example, we can describe the image processing by a GN, by another GN we can describe the processes flowing in one or more data bases containing the necessary information for the concrete process, by third GN we can represent the process of decision making, etc. The most important fact in this case is that all these processes will be described using only the GNs tool. As a further step we can extend the problem: after writer identification we can would like to inderstand the text's meaning and this process also can be described by the GNs. In all cases we can take into account that the information is imprecise and use for that reason fuzzy sets or, more general, intuitionistic fuzzy sets (see [6]) that accounts for the uncertainty the facts.

References

[1] Angelov A. *A forensic handwriting investigation and written speech parameters,* RIFSC, Sofia, 1986.

[2] Arcelli F. *Approaching character segmentation with parallel contour/skeleton analysis.* In :- Proc. 8th Scandinavian Conf. on Image analysis, 1993, Tronso, Norway, Vol. 2, 1299-1306.

[3] Atanassov K. *Generalized index matrices.* Compt. Rend. de l'Academie Bulgare des Sciences, Vol.40, 1987, No 11, 15- 18.

[4] Atanassov K. *Generalized Nets.* Singapore, New Jersey, London, World Scientific, 1991.

[5] Atanassov K. *Generalized Nets in Artificial Intelligence. Vol. 1: Generalized nets and Expert Systems.* "Prof. M. Drinov" Academic Publishing House, Sofia, 1998.

[6] Atanassov K. *Intuitionistic Fuzzy Sets,* Springer- Verlag, Heidelberg, 1999.

[7] Atanassov K. and H. Aladjov. *Generalized Nets in Artificial Intelligence. Vol. 2: Generalized nets and Machine Learning.* "Prof. M. Drinov" Academic Publishing House, Sofia, 2000.

[8] Atanassov K., E. Dincheva, D. Matev and M. Stefanova-Pavlova. *Generalized net-representation of flexible manufacturing systems.* Methods of Operations Research, Vol. 63. Proc. of the 14-th Symposium on Operations Research. Ulm, Sept. 1989, 521-528.

[9] Atanassov K., G. Gluhchev, S. Hadjitodorov, A. Shannon, and V. Vasilev. *An example of the aplication o generalized nets in Artificial Intelligence.* Compt. Rend. de Comptes Rendus de l'Academie bulgare des Sciences, Tome 56, 2003, No. 5, 13-18.

[10] Atanassov K., G. Gluhchev, S. Hadjitodorov, M. Savov and E. Szmidt. *Signature based person verification: a generalized net model.* Current Issues in Data and Knowledge Engineering (B. de Baets, R. De caluwe, G. De Tre, J. Fodor, J. Kacprzyk and S. Zadrozny, Eds.), Academicka Oficyna Wydawnicza EXIT, Warszawa, 2004, 33-41.

[11] Atanassov K., G. Gluhchev, S. Hadjitodorov, A. Shannon and V. Vassilev. *Generalized Nets and Pattern Recognition.* KvB Visual Concepts Pty Ltd, Monograph No. 6, Sydney, 2003.

[12] Gluhchev G., K. Atanassov and S. Hadjitodorov. *Handwriting analysis via generalized nets.* Proceedings of the international Scientific Conference on Energy and Information Systems and Technologies. Vol. III, Bitola, June 7-8, 2001, 758-763.

[13] Gluhchev, G., K. Atanassov, S. Hadjitodorov and E. Szmidt. *Automatic Document processing.* Issues in Intuitionistic Fuzzy Sets and Generalized Nets, Vol. 2 (K. Atanassov, J. Kacprzyk and M. Krawczak, Eds.), Wydawnictwo WSISiZ, Warszawa, 2004, 5-16.

[14] Gluhchev G., K. Atanassov, S. Hadjitodorov, E. Szmidt and A. Shannon. *Intuitionistic fuzzy generalized net model of the process of handwriting analysis.* Proc. of the Third Conf.

of the European Society for Fuzzy Logic and Technology EUSFLAT' 2003, Zittau, 10-12 Sept. 2003, 218-222.

[15] Gluhchev G., K. Atanassov, S. Hadjitodorov and V. Vasilev. *Face recognition via generalized nets.* Issues in Intuitionistic Fuzzy Sets and Generalized Nets (K. Atanassov, J. Kacprzyk and M. Krawczak, Eds.), Wydawnictwo WSISiZ, Warszawa, 2004, 57-60.

[16] Hadjitodorov S., K. Atanassov, P. Mitev, A. Shannon, G. Gluhchev and V. Vasilev. *A generalized net description for laryngeal pathology detection excluding the refusal from classification option.* Cybernetics and Information Technologies, Vol. 2, 2002, Sofia, 27-32.

[17] Hadjitodorov S. and N. Nikolova. *Generalized Net Model of the Self-organizing Map of Kohonen Classical Training Procedure.* Advances in Modelling & Analysis, B, Vol. 43, 2000, No. 1-2, 51-60.

[18] Hadjyisky L. and K. Atanassov. *A generalized net, representing the elements of one neuron network set.* AMSE Review, Vol. 14, 1990, No 4, 55-59.

[19] Hadjyisky L. and K. Atanassov. *Theorem for representation of the neuronal networks by generalized nets.* AMSE Review, Vol. 12, 1990, No 3, 47-54.

[20] Hadjyisky L. and K. Atanassov. *Generalized nets representing the elements of neuron networks.* In: Applications of generalized nets (K. Atanassov, Ed.), Singapore, World Scientific, 1993, 49-67.

[21] Hadjyisky L. and K. Atanassov. *Generalized net model of the intuitionistic fuzzy neural networks.* Advances in Modelling & Analysis, AMSE Press, Vol. 23, 1995, No 2, 59-64.

[22] Klement V. *An application system for the computer-assisted identification of handwritings.* Proc. Int. carnahan Conference on Security Technology, Zurich, 1982, 75-79.

[23] Koutanis D. and R. Rasshidi. *Petri net representation of rule based expert systems.* First Annual ESD/SMI Expert Systems Conference, 1987, 143-152.

[24] Lantsman R. *Cybernetics and Cybernetic handwriting Investigation.* Nauka, Moscow, 1969 (in Russian).

[25] Nikolov N. *Generalized nets and semantic networks.* Advances in Modelling & Analysis, AMSE Press, Vol. 27, 1995, No 1, 19-25.

[26] Pasi, G., K. Atanassov, P. Melo Pinto, R. Yager, V. Atanassova. Multi-person multi-criteria decision making: intuitionistic fuzzy approach and generalized net model. Proc. of the 10th ISPE Int. Conf. on Concurrent Engineering "Advanced Design, Production and Management Systems", 26-30 July 2003, Madeira, 1073-1078.

[27] Pasi, G., Y. Yager, K. Atanassov. Intuitionistic fuzzy graph interpretations of multi-person multi-criteria decision making: generalized net approach. Proceedings of 2004 second International IEEE Conference Intelligent Systems, Vol. 2, 434-439.

[28] Radeva V., M. Krawczak and E. Choy. *Review and bibliography on generalized nets theory and applications.* Advanced Studies in Contemporary Mathematics, Vol. 4, 2002, No. 2, 173-199.

[29] Shannon A., G. Gluhchev, K. Atanassov, P. Mitev, S. Hadjitodorov and V. Vasilev. *Generalized net representing process of speaker identification.* Artificial Intelligence, 2001, No. 4, 71-79.

[30] Shannon A., G. Gluhchev, K. Atanassov and S. Hadjitodorov. *Generalized net representing process of handwriting identification.* Cybernetics and Information Technologies, Vol. 1, 2001, Sofia, 71-80.

[31] Shannon, A., G. Gluhchev, V. Vasilev, K. Atanassov and S. Hadjitodorov. *A generalized net model of the process of face recognition with elements of intuitionistic fuzziness.* Proceedings of the Seventh International Conference on Intuitionistic Fuzzy Sets (J. Kacprzyk and K. Atanassov, Eds.), Sofia, 23-24 Aug. 2003, Notes on Intuitionistic Fuzzy Sets, Vol. 9 (2003), No. 4, 102-108.

[32] Shapiro V., G. Gluhchev and V. Sgurev. *Handwritten document image segmentation and analysis.* Pattern Recognition Letters, Vol. 14, 1993, 71-78.

[33] Stefanova-Pavlova M. *Generalized net representation of manufacturing system.* In: First Sci. Session of the Math. Found. AI Seminar, Sofia, Oct. 10, 1989, Preprint IM-MFAIS-7-89, Sofia, 1989, 60-62.

[34] Stefanova-Pavlova M. and K. Atanassov. *Generalized net models for flexible manufacturing systems.* In: Applications of generalized nets (K. Atanassov, Ed.), Singapore, World Scientific, 1993, 172-207.

Appendix: Short remarks on generalized nets

First, following [4], we shall mention that every GN-transition is described by a seven-tuple (Fig. 3):

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 3 these are $L' = \{l'_1, l'_2, \ldots, l'_m\}$ and $L'' = \{l''_1, l''_2, \ldots, l''_n\}$;

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will pass (or *transfer*) from the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [3]):

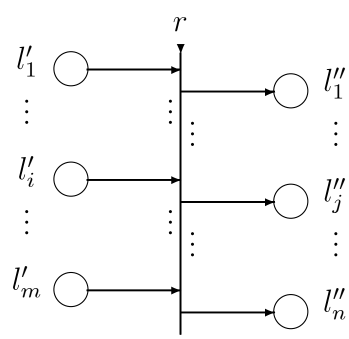

Fig. 3: GN-transition

$$r = \begin{array}{c|ccc} & l''_1 \ \ldots & l''_j & \ldots \ l''_n \\ \hline l'_1 & & & \\ \vdots & & r_{i,j} & \\ l'_i & (r_{i,j} & - & \text{predicate}\ ) \\ \vdots & (1 \leq i \leq m, & 1 \leq j \leq n) & \\ l'_m & & & \end{array} \quad ;$$

$r_{i,j}$ is the predicate which corresponds to the i-th input and j-th output places. When its truth value is "*true*", a token from i-th input place can be transferred to j-th output place; otherwise, this is not possible (for index matrices see [3]);

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \geq 0 - \text{ natural number }) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array} ;$$

(f) $\square$ is an object having a form similar to a Boolean expression. It may contain as variables the symbols which serve as labels for transition's input places, and is an expression built up of variables and the Boolean connectives $\wedge$ and $\vee$ whose semantics is defined as follows:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ - every place l_{i_1}, l_{i_2}, ..., l_{i_u} must contain at least one token,

$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ - there must be at least one token in all places l_{i_1}, l_{i_2},..., l_{i_u}, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "*true*", the transition can become active, otherwise it cannot.

The ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle,$$

$$\langle T, t^o, t^*\rangle, \langle X, \Phi, b\rangle\rangle$$

is called a *Generalized Net* (GN) if:

(a) A is a set of transitions;

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \rightarrow N$, where $N = \{0, 1, 2, \ldots\} \cup \{\infty\}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \rightarrow N$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in N, n \geq 1$ and $1 \leq k \leq n$ (obviously, L is the set of all GN-places);

(d) c is a function giving the capacities of the places, i. e., $c : L \rightarrow \ N$;

(e) f is a function which calculates the truth values of the predicates of the transition's conditions (for the GN described here let the function f have the value "$false$" or "*true*", i.e., a value from the set $\{0, 1\}$;

(f) θ_1 is a function giving the next time-moment when a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning;

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i. e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l \ ,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \rightarrow N$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale;

(l) t^o is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the GN functioning;

(n) X is the set of all initial characteristics the tokens can receive when they enter the net;

(o) Φ is a characteristic function which assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition.

(p) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \rightarrow N$.

A GN may lack some of the components, and such GNs give rise to special classes of GNs called *reduced GNs*. The omitted elements of the reduced GNs are marked by "$*$".

Different operations, relations and operators are defined over the transitions of the GNs and over the same nets.

The operators of different types, as well as the others that can be defined, have a major theoretical and practical value. On one hand, they help us in studying the properties and the behaviour of GNs. On the other hand, they facilitate the modelling of many real processes.