Twelfth Int. Conf. on IFSs, Sofia, 17-18 May 2008
NIFS 14 (2008), 1, 7-11

ON INTUITIONISTIC FUZZY NEGATIONS AND INTUITIONISTIC FUZZY EXTENDED MODAL OPERATORS. Part 1.

Chris Hinde[1] and Krassimir T. Atanassov[2]

[1] Loughborough University, Computer Science, Ashby Road,
Loughborough LE11 3TU, UK,
e-mail: *c.j.hinde@lboro.ac.uk*
[2] CLBME-Bulgarian Academy of Sciences, Acad. G. Bonchev Str., Bl. 105, Sofia-1113,
Bulgaria
e-mail: *krat@bas.bg*

Abstract

Some relations between intuitionistic fuzzy negations and intuitionistic fuzzy extended modal operation D_α are studied.

1 On some previous results

The concept of the Intuitionistic Fuzzy Set (IFS, see [1]) was introduced in 1983 as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the IFS case. One of them - operartion "negation" now there is 24 different forms (see [3]. In [1] the relations between the "classical" negation and the two standard modal operators "necessity" and "possibility" are given. Here, we shall study the same relations with the rest negations, defined over IFSs.

In some definitions we shall use functions sg and $\overline{\text{sg}}$:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

$$\overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

For two IFSs A and B the following relations are valid:

$$A \subset B \text{ iff } (\forall x \in E)(\mu_A(x) \leq \mu_B(x)\nu_A(x) \geq \nu_B(x)),$$

$$A \supset B \text{ iff } B \subset A,$$

$$A = B \text{ iff } (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)).$$

Let A be a fixed IFS. In [1] definitions of standard modal operators are given:

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\},$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.$$

The first extended modal operator is

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x)\rangle | x \in E\},$$

where $\alpha \in [0, 1]$.

In [3, 4, 5, 6, 7] the following 27 different negations are described.

$$\neg_1 A = \{\langle \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_2 A = \{\langle \overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_3 A = \{\langle \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^2\rangle | x \in E\},$$

$$\neg_4 A = \{\langle \nu_A(x), 1 - \nu_A(x)\rangle | x \in E\},$$

$$\neg_5 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(1 - \nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_6 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_7 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \mu_A(x)\rangle\rangle | x \in E\},$$

$$\neg_8 A = \{\langle 1 - \mu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_9 A = \{\langle \overline{\text{sg}}(\mu_A(x)), \mu_A(x)\rangle | x \in E\},$$

$$\neg_{10} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), 1 - \nu_A(x)\rangle\rangle | x \in E\},$$

$$\neg_{11} A = \{\langle \text{sg}(\nu_A(x)), \overline{\text{sg}}(\nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{12} A = \{\langle \nu_A(x).(\mu_A(x) + \nu_A(x)), \mu_A(x).(\mu_A(x) + \nu_A(x)^2)\rangle | x \in E\},$$

$$\neg_{13} A = \{\langle \text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{14} A = \{\langle \text{sg}(\nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{15} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{16} A = \{\langle \overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{17} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(\nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{18} A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), \mu_A(x).\text{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{19} A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{20} A = \{\langle x, \nu_A(x), 0\rangle | x \in E\},$$

$$\neg_{21} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^n\rangle | x \in E\},$$

where real number $n \in [2, \infty)$,

$$\neg_{22} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{23} A = \{\langle x, (1 - \mu_A(x)).\text{sg}(\mu_A(x)), \mu_A(x).\text{sg}(1 - \nu_A(x))\rangle | x \in E\},$$

$$\neg_{24} A = \{\langle x, (1 - \mu_A(x)).\text{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{25} A = \{\langle x, 1 - \nu_A(x), 0\rangle | x \in E\},$$

$$\neg^{\varepsilon} A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \varepsilon)\rangle | x \in E\},$$

where $\varepsilon \in [0, 1]$,

$$\neg^{\varepsilon,\eta} A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \eta)\rangle | x \in E\},$$

where $0 \leq \varepsilon \leq \eta \leq 1$.

2 Main results

Now, following and extending the idea from [9] we shall prove the the following
Theorem: For every IFS A and for every $\alpha \in [0, 1]$ the following properties are valid:
(1) $\neg_1 D_\alpha(A) = D_{1-\alpha}(\neg_1 A)$,
(2) $\neg_2 D_\alpha(A) \subset D_\alpha(\neg_2 A)$,
(3) $\neg_4 D_\alpha(A) \supset D_\alpha(\neg_4 A)$,
(4) $\neg_5 D_\alpha(A) \supset D_\alpha(\neg_5 A)$,
(5) $\neg_8 D_\alpha(A) \subset D_\alpha(\neg_8 A)$.
(6) $\neg_{11} D_\alpha(A) \supset D_\alpha(\neg_{11} A)$.
Proof: Let $0 \leq \alpha \leq 1$ and A be an IFS. Then we obtain directly that:

$$\neg_1 D_\alpha(A) = \neg_1 \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x)\rangle | x \in E\}$$

$$= \{\langle x, \nu_A(x) + (1 - \alpha).\pi_A(x), \mu_A(x) + \alpha.\pi_A(x)\rangle | x \in E\}$$

$$= D_{1-\alpha}(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\})$$

$$= D_{1-\alpha}(\neg_1 A).$$

Therefore equality (1) is valid.

The rest assertions can be proved by another manner. Let us prove, for example (8).

Let $0 \leq \alpha \leq 1$ and A be an IFS. Then:

$$\neg_8 D_\alpha(A) = \neg_8 \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x)\rangle | x \in E\}$$

$$= \{\langle x, 1 - \mu_A(x) - \alpha.\pi_A(x), \mu_A(x) + \alpha.\pi_A(x)\rangle | x \in E\}$$

and

$$D_\alpha(\neg_8 A) = D_\alpha(\{\langle x, 1 - \mu_A(x), \mu_A(x)\rangle | x \in E\})$$

$$= \{\langle x, 1 - \mu_A(x), \mu_A(x)\rangle | x \in E\}.$$

Now, we see easily that

$$1 - \mu_A(x) - (1 - \mu_A(x) - \alpha.\pi_A(x)) = \alpha.\pi_A(x)) \geq 0$$

and

$$\mu_A(x) + \alpha.\pi_A(x) - \mu_A(x) \geq 0.$$

Therefore inclusion (8) is valid.

For the nagations that are not included in Theorem 1 there are counterexamples, showing that relations $=$, $\subset$ or $\supset$ are impossible. It is interesting to note that for the ordinary modal operators $\Box$ and $\Diamond$ the relations are valid (see [9]:

(1) $\neg_1 \Box A = \Diamond \neg_1 A$,
(2) $\neg_1 \Diamond A = \Box \neg_1 A$,
(3) $\neg_2 \Box A = \Box \neg_2 A$,
(4) $\neg_2 \Diamond A \subset \Diamond \neg_2 A$,
(5) $\neg_3 \Box A \supset \Box \neg_3 A$,
(6) $\neg_3 \Diamond A \subset \Diamond \neg_3 A$,
(7) $\neg_4 \Box A \supset \Box \neg_4 A$,
(8) $\neg_4 \Diamond A = \Diamond \neg_4 A$,
(9) $\neg_5 \Box A \supset \Box \neg_5 A$,
(10) $\neg_5 \Diamond A = \Diamond \neg_5 A$,
(11) $\neg_6 \Box A \supset \Box \neg_6 A$,
(12) $\neg_6 \Diamond A = \Diamond \neg_6 A$,
(13) $\neg_7 \Box A \supset \Box \neg_7 A$,
(14) $\neg_7 \Diamond A \subset \Diamond \neg_7 A$,
(15) $\neg_8 \Box A = \Box \neg_8 A$,
(16) $\neg_8 \Diamond A \subset \Diamond \neg_8 A$,
(17) $\neg_9 \Box A \supset \Box \neg_9 A$,
(18) $\neg_9 \Diamond A \subset \Diamond \neg_9 A$,
(19) $\neg_{10} \Box A \supset \Box \neg_{10} A$,
(20) $\neg_{11} \Diamond A = \Diamond \neg_{11} A$,
(21) $\neg_{13} \Box A = \Box \neg_{12} A$,
(22) $\neg_{15} \Box A \supset \Box \neg_{15} A$,
(23) $\neg_{15} \Diamond A \subset \Diamond \neg_{15} A$,
(24) $\neg_{16} \Box A \supset \Box \neg_{16} A$,
(25) $\neg_{17} \Box A \supset \Box \neg_{17} A$,
(26) $\neg_{17} \Diamond A \subset \Diamond \neg_{17} A$,
(27) $\neg_{18} \Box A \supset \Box \neg_{18} A$,
(28) $\neg_{18} \Diamond A \subset \Diamond \neg_{18} A$,
(29) $\neg_{19} \Box A \supset \Box \neg_{19} A$,
(30) $\neg_{19} \Diamond A \subset \Diamond \neg_{19} A$,
(31) $\neg_{20} \Box A \supset \Box \neg_{20} A$,
(32) $\neg_{20} \Diamond A \subset \Diamond \neg_{20} A$,
(33) $\neg_{21} \Box A \supset \Box \neg_{21} A$,
(34) $\neg_{21} \Diamond A \subset \Diamond \neg_{21} A$,
(35) $\neg_{22} \Box A \supset \Box \neg_{22} A$,
(36) $\neg_{22} \Diamond A \subset \Diamond \neg_{22} A$,
(37) $\neg^{\varepsilon} \Box A \supset \Box \neg^{\varepsilon} A$,
(38) $\neg^{\varepsilon} \Diamond A \subset \Diamond \neg^{\varepsilon} A$,
(39) $\neg^{\varepsilon,\eta} \Box A \supset \Box \neg^{\varepsilon,\eta} A$,
(40) $\neg^{\varepsilon,\eta} \Diamond A \subset \Diamond \neg^{\varepsilon,\eta} A$.

Now, we see that about negations $\neg_1, \neg_2, \neg_4, \neg_5, \neg_8$ the behaviour of the extended modal operator D_α coincides with the behaviour of the ordinary modal operators $\Box$ and $\Diamond$, while this coincidence is not valid for the other negations.

3 Conclusion

In a next research authors will study the above properties for the case of extended intuitionistic fuzzy modal ($F-, G-$ and other) and topological operators.

References

[1] Atanassov, K. *Intuitionistic Fuzzy Sets.* Springer Physica-Verlag, Heidelberg, 1999.

[2] Atanassov, K. On intuitionistic fuzzy negations and De Morgan's laws. *Proc. of Eleventh International Conf. IPMU 2006,* Paris, July 2-7, 2006, 2355-2362.

[3] Atanassov, K., D. Dimitrov. On the negations over intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets,* Vol. 13, 2007, No. 4 (in press).

[4] Atanassova, L., On an intuitionistic fuzzy implication from Kleene-Dienes type. *Advanced Studies in Contemporary Mathematics* (in press).

[5] Atanassova, L., Modifications of an intuitionistic fuzzy implication. *Advanced Studies in Contemporary Mathematics,* Vol. 16, No. 2, 155-160.

[6] Atanassova, L., New modifications of an intuitionistic fuzzy implication. *Proc. of the First Session of Section "Informatics" of Union of Bulgarian Scientists,* Sofia, 15 Dec. 2007 (in press).

[7] Dimitrov, D., New intuitionistic fuzzy implications and their corresponding negations. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets.* Vol. 6, Warsaw School of Information Technology, 2008 (in press).

[8] R. Feys, *Modal logics.* Gauthier-Villars, Paris, 1965.

[9] Hinde, C. and K. Atanassov. On intuitionistic fuzzy negations and intuitionistic fuzzy ordinary modal operators. *Notes on Intuitionistic Fuzzy Sets,* Vol. 13, No. 4 (in press).