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Generalized real time multigraphs for communication networks: An intuitionistic fuzzy theoretical model

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Abstract: This is sequel to our previous works on RT-multigraphs [21, 22] designed with intuitionistic fuzzy theory. In this paper, we introduce the notion of 'Generalized Real Time Multigraph' (GRT-Multigraph) which is an improvement of the notion of RT multigraph. Any RT-multigraph is a special case of a GRT-multigraph. It is claimed that the model of GRT-multigraph will play a vital role in any network of communication system because of its high potential in considering real time data and information, and will open a new direction for rigorous research.
Keywords: IFS; IFN; Multigraphs; RT-multigraphs; GRT-multigraphs; neighbor node; tbl; link status; LSV; LSC; tbn; rn; communicable node, CF, EC.
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1 Introduction

There are many real life problems of networks, in particular of computer science, communication systems, transportation systems, metagraph nets, biological neuron systems, etc., which can not be modeled into graphs, but into multigraphs only, and then can be easily solved. Intuitionistic fuzzy graphs have been introduced by Atanassov [2–6, 16, 18]. Intuitionistic fuzzy multigraphs have been studied in [7, 19]. In the present days of 'big data' flow, the networks are expanding very fast in huge volumes in terms of their nodes and links/arcs. There are migrations of data from one giant network to another giant network. But, for a given alive network in many situations, its complete topology may not be always available to the communication systems at a given point of time because of the reason that few or many of its links/arcs may be temporarily disable owing to damage or external attack or blockage upon them, and of course they are under repair at that point of time. One link may be good for communication process at this moment of time but may get damaged after few hours because of several possible reasons which are usually unpredictable and hence the question of prevention or prior detection does not come into reality in general. Besides that, in most of the cases the cost parameters corresponding to its links are not crisp numbers, rather intuitionistic fuzzy numbers (or fuzzy numbers). Thus at any real time instant, the complete multigraph is not available but a submultigraph of it is available to the system for executing its communication or packets transfer.

There was no mathematical model available in the existing literature to represent such type of real time network. As a solution, in our previous work in [21,22] we introduced a mathematical model for such types of multigraphs be called by 'Real Time Multigraphs' (RT-multigraphs) in which all real time information (being updated every q quantum of time) are incorporated so that the communication/transportation system can be made very efficiently with optimal results. It was a theoretical work, a kind of intuitionistic fuzzy mathematical model. In this paper we make further consideration of real time data and information to introduce 'Generalized Real Time Multigraphs' (or GRT-Multigraphs) as a generalization of RT-multigraphs. The notion of GRT-multigraphs is the most generalized form of the crisp multigraphs, and will surely play a major role in networks, in particular in computer science, communication systems, transportation systems, etc. Clearly a GRT-multigraph is a variable representation of a network with respect to the parameter time. In this network modeling of GRT-multigraph we make an application of Atanassov's intuitionistic fuzzy numbers (IFN).

2 Preliminaries

In this section, we present basic preliminaries on the IFS theory of Atanassov [1–3] and also on the existing notion of multigraphs [8, 9, 11, 13, 20].

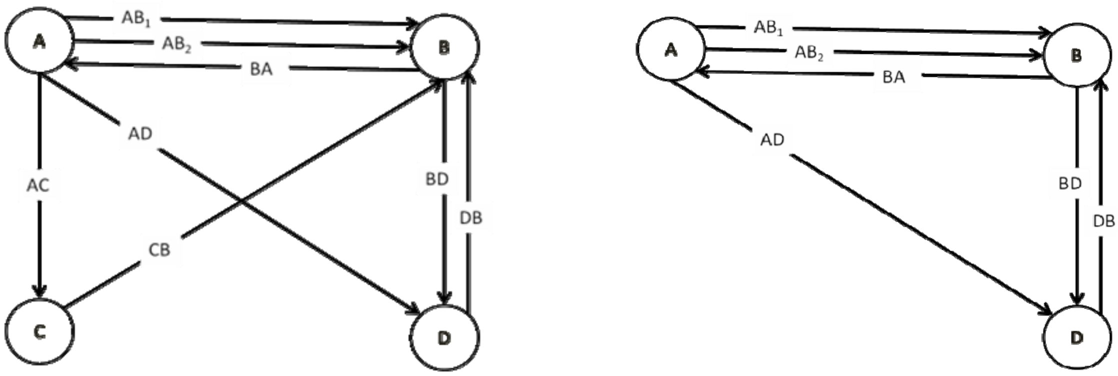
2.1 Intuitionistic fuzzy set (IFS)

The intuitionistic fuzzy set (IFS) theory of Atanassov [1–3] is now a well known powerful soft computing tool to the world. If X be a universe of discourse, an intuitionistic fuzzy set A in X is a set of ordered triplets $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A, \nu_A : X \rightarrow [0, 1]$ are functions called respectively by 'membership function' and 'non-membership function' such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall\, x \in X$. For each $x \in X$, the values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of the element x to $A \subset X$, respectively, and the amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitation part or undecided part. Of course, a fuzzy set is a particular case of the intuitionistic fuzzy set if $\pi_A(x) = 0,\ \forall\, x \in X$. For details of the classical notion of intuitionistic fuzzy set (IFS) theory, one could see the books [2, 3] authored by Atanassov.

2.2. Multigraph

A multigraph G is an ordered pair (V, E) which consists of two sets V and E, where V or $V(G)$ is the set of vertices (or, nodes), and E or $E(G)$ is the set of edges (links or arcs). Here, although multiple edges or arcs might exist between pair of vertices but in our discussion in this paper we consider that no loop exists.

Multigraphs may be of two types: undirected multigraphs and directed multigraphs. In an undirected multigraph, the edge (i, j) and the edge (j, i), if exist, are obviously identical unlike in the case of directed multigraph. For a latest algebraic study on the theory of multigraphs, the work [20] and also [8, 9, 11, 13] may be seen. Fig. 1 (a) shows below a directed multigraph $G = (V, E)$, where $V = \{A, B, C, D\}$ and $E = \{AB_1, AB_2, BA, AD, AC, CB, BD, DB\}$.



(a) **(b)**

Figure 1. (a) A multigraph G; (b) A submultigraph H of G

A multigraph $H = (W, F)$ is called a submultigraph of the multigraph $G = (V, E)$ if $W \subseteq V$ and $F \subseteq E$. Fig. 1 (b) shows a submultigraph H of the multigraph G. We now consider the modeling of a very real situation of networks and define a generalized notion of multigraphs/graphs.

3 Generalized Real Time Multigraph (GRT–Multigraph): An intuitionistic fuzzy model

In most of the real life problems of networks, be it in a communication model or a transportation model, the weights of the arcs are not always crisp but intuitionistic fuzzy numbers (IFNs) or at best fuzzy numbers. For example, Fig. 2 below shows a public road transportation model for a traveler where the cost parameters for travelling each arc have been considered as IFN which are the more generalized form of fuzzy numbers involving two independently estimated degrees: degree of acceptance and a degree of rejection.

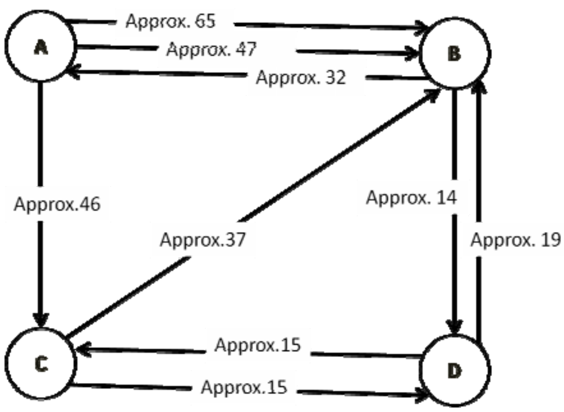


Figure 2. A multigraph G with IF weights (IFNs) of arcs.

3.1 'Neighbour' node

For a given node u the node v will be designated as a **'neighbour'** node of u if u has at least one link from u to v.

In our work here, we consider more real situations which are actually and frequently faced by the present communication systems. For example, consider an Adhoc Network or a MANET, in which there may exist multiple paths between two neighbour nodes, but because of some reasons one or more number of paths may not be in the ideal condition (may be partially damaged, or temporarily damaged). Thus, although they are available for transmission of packets by a node u to its neighbor node v, but will cause the communication delay (for a damaged condition, there will be no scope for communication). This is very useful information to the communication system if available to the sender node in advance. For this we introduce a new parameter corresponding to each link (edge) called by **'Condition Factor' (or 'link status')**.

3.2 'Condition Factor'(CF) of a Link

Consider a node u and its neighbor node v. Suppose that there are n (≥ 1) number of links from u to v outward which are uv_1, uv_2, …, uv_n. Let us designate them as 1^{st}, 2^{nd}, 3^{rd}, …, n^{th}. For each link uv_i, we define the **condition factor (CF)** or **link status** at this instant of time in the following way:

(i) The ideal (i.e. best) 'condition factor' for each link uv_r is 1, if it is available at its best condition without any damage or attack internally or externally at this moment of time. We express this information using the notation CF, like CF(uv_r) = 1.

(ii) The worst 'condition factor' for each link uv_i is 0, if it is not available, i.e. at a condition of fully damaged or blocked at this moment of time. We express this information using the notation CF, like CF(uv_r) = 0.

(iii) Otherwise, the 'condition factor' for each link uv_i is in between 0 and 1, which means that the link is available but not in its best condition. It is partially damaged or in a traffic-jam or similar many other real time circumstances (could be temporarily existing) which may cause the communication to be at slow pace.

Thus, $0 \leq \text{CF}(uv_r) \leq 1, \forall r$.

Consider the following directed RT-Multigraph [21, 22] G where the IF weights (IFNs) are shown against each link.

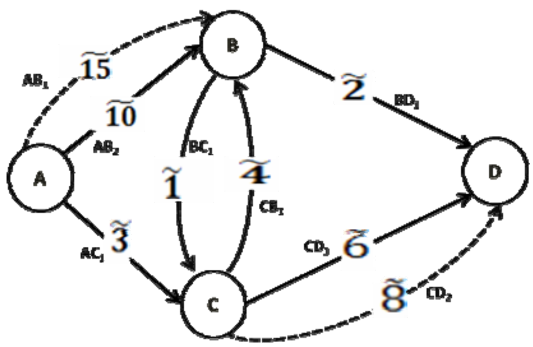


Figure 3. A RT-multigraph G having few links damaged partially/temporarily.

In our proposed mathematical model of GRT-multigraphs, we incorporate further the real time data from the network regarding the condition of each and every link (arc) to make the RT-multigraphs more dynamic, more useful, and hence more efficient to the users for making an optimal strategy for communication.

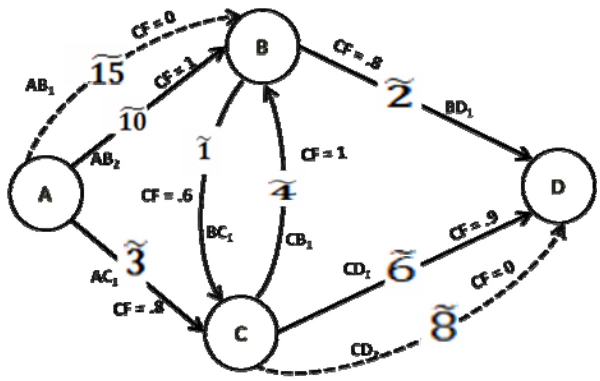


Figure 4. A GRT-multigraph G with IF weights (IFNs) of links,
but with various CF as on now.

Every node of the multigraph carries an information vector corresponding to each of its neighbour nodes. If the node u has the node v as a neighbour node then u carries the following information handy with it:

(i) Suppose that there are n (≥ 1) number of links from u to v outward which are uv_1, uv_2, …, uv_n. Let u designate them as 1st, 2nd, 3rd, …, n^{th}.

(ii) In a real life situation, because of natural phenomenon (flood, earthquake, thunder-storm, solar storm, etc. etc.) or because of some kind of external attack or technical failure or because of an predictable/unpredictable fully or partial damage of the link, it may happen in reality that during a period of time the r^{th} link uv_r of the node u to its neighbour v is not available at its best condition (r = 1, 2, 3, …, n). In our proposed model of GRT-multigraphs, this is precious information and is available with the node u here in advance.

3.3 Link Status Vector (LSV)

Corresponding to every neighbour node v of u in a GRT-multigraph, there exist a **Link Status Vector (LSV)** $I_{uv} = (i_1, i_2, i_3, \ldots, i_n)$ of u, where at any given point of time i_r happens to be some value from the closed interval [0, 1] for r = 1, 2, 3, …, n where $i_r = \mathrm{CF}(uv_r)$.

3.4 Temporarily Blocked Link (tbl)

If at a given time i_r happens to be 0, i.e. if the link uv_r is completely non-functional, then we say that the link uv_r is a **temporarily blocked link (*tbl*)** from u. The value i_r is also called the **'link status'** (or CF) of the link uv_r as mentioned earlier.

In a real situation the complete multigraph thus may not be available due to existence of non-functional or under-performance status for few links, i.e. due to existence of few *tbl*'s and few under-performer links. Few or many of the available links may not be available with CF = 1, but available with less amount of CF. Consequently not the complete topology but a sub-multigraph or a weaker multigraph of it be available for communication (Example: For communication of packets in an Adhoc Network/MANET, or for a salesman to travel many cities, or for a buss/truck carrying goods/passengers in a transportation network, etc.).

If a node u has k (≥ 0) number of neighbour nodes v_1, v_2, v_3,…, v_k, then u carries k number of LSV: I_{uv1}, I_{uv2}, I_{uv3}, …, I_{uvk}. In our mathematical model of GRT-multigraph, we propose that there is a system S for the multigraph which updates all the information vectors of all the nodes after every quantum time τ. This quantum τ is fixed (can be reset) for the system S in a multigraph, but different for different multigraphs, in general depending upon the various properties of the physical problem for which a multigraph is modelled.

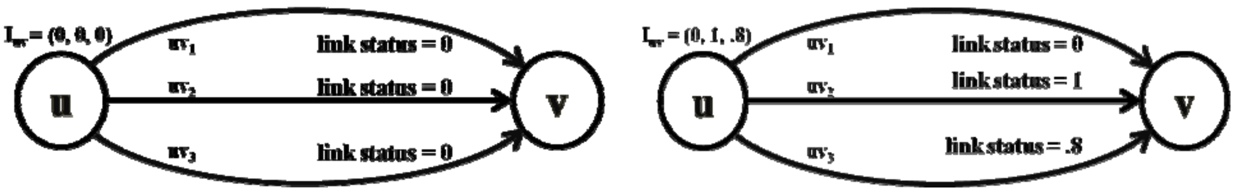
3.5 Link Status Class (LSC)

For a given node u, the collection of all LSV are called 'Link Status Class' (LSC) of u denoted by $\boldsymbol{I_u}$. If a node u has k (≥ 0) number of neighbour nodes x_1, x_2, x_3, …, x_k, then

$$\boldsymbol{I_u} = \{I_{ux1}, I_{ux2}, I_{ux3}, \ldots\ldots, I_{uxk}\}.$$

3.6 Temporarily Blocked Neighbour (tbn) & Reachable Neighbour (rn)

If v is a neighbour node of a given node u, and if I_{uv} is a null vector at a given instant of time then v is called a **temporarily blocked neighbour (*tbn*)** of u for that instant.



(a) **(b)**

Figure 5. (a) A *tbn* v of the node u; (b) A *rn* v from the node u.

However, since it is a temporary phenomenon, and if any of the links be repaired in due time, then obviously a 'blocked neighbour' may regain its 'neighbour' status at some later stage. If a neighbour v is not a *tbn*, then it is called a **reachable neighbour** (*rn*) of u. Thus v is a reachable node from u if there is at least one link having non-zero CF.

3.7 Communicable Node

For a given node u, if $I_u \neq \varphi$ and at least one member of I_u is non-null at a given time, then the node u is called a communicable node for that instant of time. If u does not have any neighbor node then $I_u = \varphi$, and in that case it is trivial that further communication is never possible. However, if $I_u \neq \varphi$ and all the members of I_u are null vectors at a point of time, then it signifies that further communication is not possible temporarily.

(iv) All the real time information mentioned / defined above will get automatically updated at every node of the multigraph at every q quantum of time (for a quantum q to be pre-fixed depending upon the properties of the network, on what kind of communication/transportation it is performing).

3.8 Effective Cost (EC) of a link

Consider a node u and its neighbor node v. Suppose that there are n (≥ 1) number of links from u to v outward, which are uv_1, uv_2, …, uv_n. Let us designate them as 1^{st}, 2^{nd}, 3^{rd},…, n^{th}. Corresponding to each link uv_r, there is a cost (weight) of the link which is an IFN n_r. If this link is not available at its best condition, then a fraction of its ideal condition is available to the system for communication. Consequently, if this link is chosen for communication of a packet from node u to node v, the effective cost of this link will not be in reality equal to n_r but a little higher side, depending upon the condition of the link uv_r at that real instant of time. Then, the **effective cost (EC)** of the link uv_r will be defined by

$$\text{EC}(uv_r) = n_r \,/\, \text{CF}(uv_r),$$

where $\text{CF}(uv_r) \neq 0$.

If $\text{CF}(uv_r) = 0$, then we say that $\text{EC}(uv_r) = \infty$. Since n_r is an IFN, $\text{EC}(uv_r)$ is also an IFN for each r.

We call such type of multigraphs by **'Generalized Real Time Multigraphs'** or **'GRT-multigraphs'** as they contain all real time information of the networks. Consequently, for a given network the GRT-multigraph is not a static multigraph but changes with time, i.e. becomes weaker sometime, regain ideal condition back, again becomes weaker, and so on. As a special case, if a network can be modelled into a graph (need not be a multigraph), then we call our proposed model as **'Generalized Real Time Graph'** or **'GRT-graph'** which is a special case of **'Generalized Real Time Multigraphs'** or **'GRT-multigraphs'.**

4 Conversion of a GRT-multigraph into an equivalent RT-multigraph

Given a GRT-multigraph, it can not be converted physically into an equivalent RT-multigraph. But for the purpose of soft-computing practices for implementing very effective communication system via a GRT-multigraph, one can mathematically convert it into a equivalent RT-multigraph and apply all the algorithms/theories of RT-multigraph [21, 22] to find the final solutions for the GRT-multigraph.

Consider a given GRT-multigraph G_{grt} at some instant of time. Now, for each link of this G_{grt} if we replace the existing cost by a new value equal to the corresponding EC value of it, we get a new multigraph G_{rt} with common V and E. This new multigraph G_{rt} can be viewed as an equivalent RT-multigraph of the GRT-multigraph G_{grt}, because in G_{rt} we shall view the condition factor to be either 0 or 1 only for each link.

Thus in G_{rt}, corresponding to every neighbour node v of a given node u, the **Link Status Vector (LSV)** of u is $I_{uv} = (i_1, i_2, i_3, \ldots, i_n)$, where at any given point of time i_r takes any of the two values only from $\{0,1\}$ for $r = 1, 2, 3, \ldots, n$ with the following significance:

- $i_r = 0$, if the link uv_r is non-functional.
- $i_r = 1$, if the link uv_r is functional.

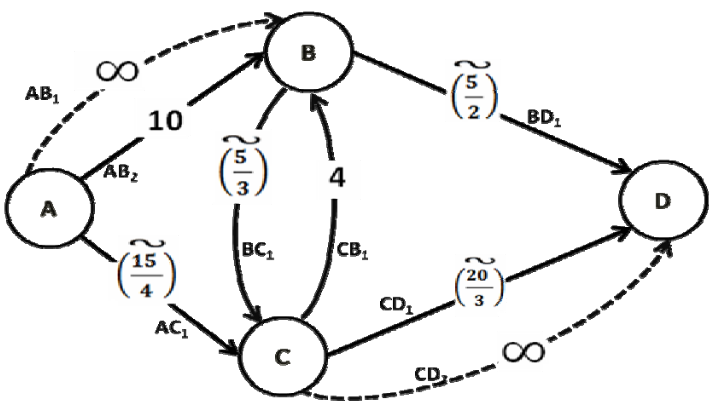


Figure 6. A GRT-multigraph converted into an equivalent RT- multigraph.

5 Conclusion and future work

In this paper, we have introduced a new notion of Graph Theory called by "GRT-multigraphs" which is a generalization of the notion of RT-multigraphs [21, 22]. In a RT-multigraph, at a given instance of time, a link is either available (*status* = 1) or not available (*status* = 0), i.e. the status of a link or CF of a link is always one score from the set {0, 1}. In a GRT-multigraph, the status or CF of a link is in the closed interval [0, 1], depending upon its condition for communication at that real instant of time. It has been shown that a GRT-multigraph can be mathematically converted into a RT-multigraph for computing purpose in order to have solutions for the problems on the GRT-multigraph.

Our next research work will be (i) to implement intuitionistic fuzzy CF in GRT-multigraph instead of crisp CF values because of the reason that condition of a link may not be always a precise quantity but an ill-defined quantity, and then (ii) to develop a method to find IF shortest path from a source vertex to a destination vertex of a GRT-multigraph.

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