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A new similarity measure of intuitionistic fuzzy sets and its application to estimate the priority weights from intuitionistic preference relations

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Abstract: The aim of this paper is to introduce a methodology for estimating the priority based weights of alternatives from the intuitionistic preference relation. A new similarity measure for intuitionistic fuzzy sets (IFSs) is also introduced and a priority method of intuitionistic preference relations is developed by employing the new similarity measure. A set of examples are provided to compare the proposed similarity measure with the existing similarity measures. Finally, the methodology is illustrated with the help of a numerical example.

Keywords: Intuitionistic fuzzy sets, Intuitionistic preference relation, Multi criteria decision making, Priority vector, Similarity measure.
AMS Classification: 03E72.

1 Introduction

Fuzzy set, proposed by Zadeh [28], assigns to each member of the universe of discourse a degree of membership between zero and one. By adding the degree of non-membership to fuzzy set, Atanassov [1] introduced intuitionistic fuzzy set (IFS) which reflects the fact that the degree of non-membership is not always equal to one minus degree of membership. There may be some degree of hesitation. Thus, there are some situations where IFS theory provides a more meaningful and applicable framework to cope with imperfect or imprecise information present in real-world applications. Subsequently Gau and Buehrer [7] introduced the concept of vague sets. However Bustince and Burillo [3] pointed out that the notion of vague set is the same as that of IFSs.

Since its introduction, the IFS theory has been studied and applied in different areas including decision making (e.g., [2, 6, 11, 12, 13, 15, 16, 20, 21, 26, 27]. Now in modeling a decision problem a situation may arise where the expert does not possess a clear idea of preferences between the alternatives. Especially, in some situations the expert may provide his/her preferences for alternatives to a certain degree, but it is possible that he/she is not so sure about it [22]. An uncertainty or hesitation may be included in their preference values over the alternatives due to subjective estimation and perception. This indeterminacy (i.e., hesitation) related to human cognitive processes, viz., thinking, reasoning etc., present in experts' preference values can be suitably captured by intuitionistic fuzzy values. Thus, in the process of multiple criteria decision making (MCDM) intuitionistic preference relation is a powerful tool to express the decision maker's intuitionistic preference information over the alternatives. Szmidt and Kacprzyk [17, 18] and Xu [22, 24] investigated decision making problems based on intuitionistic preference information. Moreover, in MCDM the problem of determining the weights (or importance) of different alternatives from the intuitionistic preference information is an interesting and important task. The priority-based weights derived from the intuitionistic preference relation may be used as the weights of the alternatives. In [23], Xu developed a method for estimating priority weights from consistent and inconsistent intuitionistic preference relations based on linear programming models. In [14], Qian and Feng established programming models for estimating interval intuitionistic priority weights from intuitionistic preference relations. Later, in [8], Gong *et al*. applied goal programming model for deriving the priority-based weights.

Till now little research has been done on the priority method of the intuitionistic preference relations. This motivates us to investigate an approach for estimating the priority-based weights of different alternatives from intuitionistic preference relations. For this purpose, a similarity measure between two IFSs has also been proposed in this paper.

This paper has been organized as follows: In section 0, a new similarity measure has been introduced and the relevant literatures have also been surveyed. A set of examples have been provided in section 0, to compare the proposed similarity measure with the existing measures. Section 0 describes an application of the proposed similarity measure to determine the alternative weights from intuitionistic preference relations. In section 0, a numerical example has been presented to illustrate the methodology and finally, the conclusions have been made in section 0.

2 New approaches to measure the similarity between IFSs

In the application of fuzzy sets as well as IFSs similarity measures play a very important role. A similarity measure is a matching function to measure the degree of similarity between two objects. Several researchers had focused on computing the similarity measure between IFSs, such as, Chen [4, 5], Hong and Kim [10] proposed a set of methods for measuring the degree of similarity between vague sets and elements. They also applied the said measures in behavior analysis problems of an organization. The measures mentioned above only reflect the influence of membership and non-membership degree to measure the similarity; they do not reflect the influence of degree of indeterminacy or hesitation. In view of this, Zhang and Fu [29] suggested some methods to measure the similarity for IFSs based on the 'background of fuzzy information handling'. In [25], Xu also developed sets of similarity measures based on the set theoretic approach.

After analyzing the aforementioned similarity measures it has been observed that, for some cases, they fail to calculate the measures of similarities correctly. Under these circumstances, the decision maker may not be able to carry out the comparison and recognition properly. This creates problem in practical applications. In order to overcome these problems of the existing similarity measures, a new similarity measure between IFSs has been proposed in the next section.

2.1 Construction of the new similarity measure

Let $A_{IFS} = \{< x_i, \mu_A(x_i), \nu_A(x_i) >: x_i \in X\}$ and $B_{IFS} = \{< x_i, \mu_B(x_i), \nu_B(x_i) >: x_i \in X\}$ be two IFSs in $X = \{x_1, x_2, ..., x_n\}$. Then the degree of similarity between the IFSs A_{IFS} and B_{IFS} may be evaluated with help of the following function:

$$S_p(A_{IFS}, B_{IFS}) = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}{1 + \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}; 1 \le p < \infty \tag{1}$$

where, $\delta_A(x_i)$ and $\alpha_A(x_i)$ are defined [29] in the following way:

$$\delta_A(x_i) = \mu_A(x_i) + \left(1 - \mu_A(x_i) - \nu_A(x_i)\right) \mu_A(x_i) \text{ and}$$
$$\alpha_A(x_i) = \nu_A(x_i) + \left(1 - \mu_A(x_i) - \nu_A(x_i)\right) \nu_A(x_i) \tag{2}$$

The larger the value of $S_p(A_{IFS}, B_{IFS})$, the more the similarity between A_{IFS} and B_{IFS}.

2.2 Properties

The proposed similarity measure given in (1) satisfies the following properties:

(P1) $0 < S_p(A_{IFS}, B_{IFS}) < 1.$

(P2) $S_p(A_{IFS}, B_{IFS}) = 1 \Leftrightarrow A_{IFS} = B_{IFS}$

(P3) $S_p(A_{IFS}, B_{IFS}) = S_p(B_{IFS}, A_{IFS})$

(P4) $S_p(A_{IFS}, C_{IFS}) \le S_p(A_{IFS}, B_{IFS})$ and $S_p(A_{IFS}, C_{IFS}) \le S_p(B_{IFS}, C_{IFS})$ if $A_{IFS} \subseteq B_{IFS} \subseteq C_{IFS}$, $C_{IFS} \in IFS(X)$.

Proof: It is obvious that (1) satisfies (P3). Hence, the proof has been omitted. The proof of (P1), (P2) and (P4) have been discussed below.

Proof of (P1): From (2) we can write $0 \le |\delta_A(x_i) - \delta_B(x_i)| \le 1$ and $0 \le |\alpha_A(x_i) - \alpha_B(x_i)| \le 1$. Now,

$$1 + \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p} > \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}$$

$$\Rightarrow 1 > \frac{\sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}{1 + \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}$$

$$\Rightarrow 1 > \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}{1 + \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}} \Rightarrow S_p(A_{IFS}, B_{IFS}) > 0$$

Again, $\frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}{1 + \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}} > 0 \Rightarrow S_p(A_{IFS}, B_{IFS}) < 1.$ □

Proof of (P2): $S_p(A_{IFS}, B_{IFS}) = 1 \Leftrightarrow \delta_A(x_i) = \delta_B(x_i)$ and $\alpha_A(x_i) = \alpha_B(x_i)$ for all $x_i \in X$. Now, for all $x_i \in X$, by utilizing (2) the following can be written

$$\delta_A(x_i) = \delta_B(x_i)$$

$$\Rightarrow \mu_A(x_i) + \left(1 - \mu_A(x_i) - \nu_A(x_i)\right)\mu_A(x_i) = \mu_B(x_i) + \left(1 - \mu_B(x_i) - \nu_B(x_i)\right)\mu_B(x_i) \tag{3}$$

$$\alpha_A(x_i) = \alpha_B(x_i)$$

$$\Rightarrow \nu_A(x_i) + \left(1 - \mu_A(x_i) - \nu_A(x_i)\right)\nu_A(x_i) = \nu_B(x_i) + \left(1 - \mu_B(x_i) - \nu_B(x_i)\right)\nu_B(x_i) \tag{4}$$

Adding (3) and (4) we will get the following:

$$\left(\mu_A(x_i) + \nu_A(x_i)\right)\left(2 - \mu_A(x_i) - \nu_A(x_i)\right) = \left(\mu_B(x_i) + \nu_B(x_i)\right)\left(2 - \mu_B(x_i) - \nu_B(x_i)\right)$$

$$\Rightarrow \mu_A(x_i) + \nu_A(x_i) = \mu_B(x_i) + \nu_B(x_i) \tag{5}$$

Subtracting (3) and (4) we will get the following

$$\left(\mu_A(x_i) - \nu_A(x_i)\right)\left(2 - \mu_A(x_i) - \nu_A(x_i)\right) = \left(\mu_B(x_i) - \nu_B(x_i)\right)\left(2 - \mu_B(x_i) - \nu_B(x_i)\right) \tag{6}$$

Therefore, from (5) and (6) the following can be obtained:

$$\mu_A(x_i) - \nu_A(x_i) = \mu_B(x_i) - \nu_B(x_i)$$

Finally, from (5) and (7) it is now clear that $\forall x_i \in X$, $\mu_A(x_i) = \mu_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$, which implies $A_{IFS} = B_{IFS}$. □

Proof of (P4): $A_{IFS} \subseteq B_{IFS} \subseteq C_{IFS}$ then $\forall x_i \in X$ the following holds: $\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$ and $\nu_A(x_i) \ge \nu_B(x_i) \ge \nu_C(x_i)$.

From (1) it may be said that, if $\forall x_i \in X$ the following two relation holds, i.e., If $|\delta_A(x_i) - \delta_C(x_i)| \ge |\delta_A(x_i) - \delta_B(x_i)|$ and $|\alpha_A(x_i) - \alpha_C(x_i)| \ge |\alpha_A(x_i) - \alpha_B(x_i)|$ holds, then we may write $S_p(A_{IFS}, C_{IFS}) \le S_p(A_{IFS}, B_{IFS})$ for $1 \le p < \infty$.

Therefore, $\forall x_i \in X$ we have to show the following: $|\delta_A(x_i) - \delta_C(x_i)| \ge |\delta_A(x_i) - \delta_B(x_i)|$ and $|\alpha_A(x_i) - \alpha_C(x_i)| \ge |\alpha_A(x_i) - \alpha_B(x_i)|$. Now since $\mu_A(x_i) \le \mu_B(x_i)$ and $\nu_A(x_i) \ge \nu_B(x_i)$ $\forall x_i \in X$, therefore, we may write $\mu_B(x_i) = \mu_A(x_i) + u$ and $\nu_A(x_i) = \nu_B(x_i) + v$ where $u > 0$, $v > 0$.

We can write

$$|\delta_C(x_i) - \delta_A(x_i)| = |\delta_C(x_i) - \delta_B(x_i) + \delta_B(x_i) - \delta_A(x_i)|. \tag{8}$$

From (8), it may said that if $\left(\delta_B(x_i) - \delta_A(x_i)\right) > 0$ and $\left(\delta_C(x_i) - \delta_B(x_i)\right) > 0$ then $|\delta_A(x_i) - \delta_C(x_i)| > |\delta_A(x_i) - \delta_B(x_i)|$.

Therefore, the aim is now to show that $\left(\delta_B(x_i) - \delta_A(x_i)\right) > 0$.

$$\begin{aligned}\delta_B(x_i) - \delta_A(x_i) &= \mu_B(x_i) + \left(1 - \mu_B(x_i) - \nu_B(x_i)\right)\mu_B(x_i) \\ &\quad - \mu_A(x_i) - \left(1 - \mu_A(x_i) - \nu_A(x_i)\right)\mu_A(x_i) \\ &= u\left(1 + \pi_A(x_i)\right) - \mu_B(x_i)(u - v)\end{aligned}$$

From above, two situations may arise:

(i) $(u - v) > 0$,

(ii) $(v - u) > 0$.

For situation (i),

$$\delta_B(x_i)-\delta_A(x_i)=u\left(1+\pi_A(x_i)\right)-\mu_B(x_i)(u-v)$$
$$\geq u\mu_B(x_i)-u\mu_B(x_i)+v\mu_B(x_i)$$
$$=v\mu_B(x_i)\geq 0$$

For situation (ii),

$$\delta_B(x_i)-\delta_A(x_i)=u\left(1+\pi_A(x_i)\right)+\mu_B(x_i)(v-u)\geq 0\,.$$

In a similar manner, it can be proved that $\left(\delta_C(x_i)-\delta_B(x_i)\right)\geq 0$. Therefore, by the non-negativity of $\left(\delta_B(x_i)-\delta_A(x_i)\right)$ and $\left(\delta_C(x_i)-\delta_B(x_i)\right)$ the following relation holds:

$$|\,\delta_C(x_i)-\delta_A(x_i)\,|=|\,\delta_C(x_i)-\delta_B(x_i)+\delta_B(x_i)-\delta_A(x_i)\,|\geq \max\{\delta_C(x_i)-\delta_B(x_i),\delta_B(x_i)-\delta_A(x_i)\}$$

Similarly, the following can be proved

$$|\,\alpha_A(x_i)-\alpha_C(x_i)\,|=|\,\alpha_A(x_i)-\alpha_B(x_i)+\alpha_B(x_i)-\alpha_C(x_i)\,|\geq \max\{\alpha_A(x_i)-\alpha_B(x_i),\alpha_B(x_i)-\alpha_C(x_i)\}$$

Finally, it is proved that $S_p(A_{IFS},C_{IFS})\leq S_p(A_{IFS},B_{IFS})$ for $1\leq p<\infty$. In a similar manner, we can prove that $S_p(A_{IFS},C_{IFS})\leq S_p(B_{IFS},C_{IFS})$ for $1\leq p<\infty$. □

2.3 Comparison with the existing methods

In this section, some examples of IFSs have been presented (see Table 1) to compare the proposed similarity measure with the four existing methods presented by Chen [4, 5], Hong and Kim [10], Zhang and Fu [29], Xu [25]. A comparison between the results of the proposed similarity measure and the results of the existing methods has been illustrated in Table 1. From Table 1 we can see some drawbacks of the existing methods and some advantages of the proposed method, which has been elaborated below.

Table 1: A comparison of the proposed similarity measure with the existing methods

Chen [4, 5]	Expression of similarity measures	$S^C(A_{IFS},B_{IFS})=1-\frac{\sum_{i=1}^{n}\lvert S_A(x_i)-S_B(x_i)\rvert}{2n}$ where $S_A(x_i)=\mu_A(x_i)-\nu_A(x_i)$, $S_B(x_i)=\mu_B(x_i)-\nu_B(x_i)$
	Example	Consider $A_{IFS}=\{<x,0.4,0>\}$ and $B_{IFS}=\{<x,0.6,0.2>\}$ $S^C(A_{IFS},B_{IFS})=1$.
	The proposed method	$p=1$, we have $S_p(A_{IFS},B_{IFS})=0.76$
Hong and Kim [10]	Expression of similarity measures	$S^H(A_{IFS},B_{IFS})=1-\frac{\sum_{i=1}^{n}\lvert\mu_A(x_i)-\mu_B(x_i)\rvert+\sum_{i=1}^{n}\lvert\nu_A(x_i)-\nu_B(x_i)\rvert}{2n}$
	Example	Consider $A_{IFS}=\{<x,1,0>\}$, $B_{IFS}=\{<x,0,0>\}$ and $C_{IFS}=\{<x,0.5,0.5>\}$ $S^H(A_{IFS},B_{IFS})=S^H(B_{IFS},C_{FS})$ $=0.5$.
	The proposed method	for $p=2$, $S_p(A_{IFS},B_{IFS})=0.5$ and $S_p(B_{IFS},C_{IFS})=0.585$.

Xu [2005]	Expression of similarity measures	$S^X(A_{IFS}, B_{IFS})$ $= \frac{\sum_{i=1}^{n}\left(\min(\mu_A(x_i), \mu_B(x_i)) + \min(\nu_A(x_i), \nu_B(x_i)) + \min(\pi_A(x_i), \pi_B(x_i))\right)}{\sum_{i=1}^{n}\left(\max(\mu_A(x_i), \mu_B(x_i)) + \max(\nu_A(x_i), \nu_B(x_i)) + \max(\pi_A(x_i), \pi_B(x_i))\right)}$
	Example	Consider $A_{IFS} = \{< x, 0.5, 0.5 >\}$, $B_{IFS} = \{< x, 0.7, 0.2 >\}$ and $C_{IFS} = \{< x, 0.2, 0.6 >\}$ $S^X(A_{IFS}, B_{IFS}) = S^X(A_{IFS}, C_{IFS})$ $= 0.538$.
	The proposed method	$S_p(A_{IFS}, B_{IFS}) = 0.641$, $S_p(A_{IFS}, C_{IFS}) = 0.675$. Calculation has been done for $p = 1$.
Zhang and Fu [29]	Expression of similarity measures	$S^Z(A_{IFS}, B_{IFS}) = 1 - \frac{1}{2n}\sum_{i=1}^{n}(\mid \delta_A(x_i) - \delta_B(x_i) \mid + \mid \alpha_A(x_i) - \alpha_B(x_i) \mid)$
	Example	Consider $A_{IFS} = \{< x, 0, 0 >\mid x \in X\}$, $B^1_{IFS} = \{< x, 0.2, 0.8 >\mid x \in X\}$, $B^2_{IFS} = \{< x, 0.4, 0.6 >\mid x \in X\}$, $B^3_{IFS} = \{< x, 0.3, 0.7 >\mid x \in X\}$ $S^Z(A_{IFS}, B^1_{IFS}) = S^Z(A_{IFS}, B^2_{IFS})$ $= S^Z(A_{IFS}, B^3_{IFS}) = 0.500$.
	The proposed method	for $p = 2$, $S_p(A_{IFS}, B^1_{IFS}) = 0.548$, $S_p(A_{IFS}, B^2_{IFS}) = 0.581$, $S_p(A_{IFS}, B^3_{IFS}) = 0.568$.

- In Example 1, two different IFSs have been provided although $S^C(A_{IFS}, B_{IFS}) = 1$. The proposed similarity measure overcomes the problem. Utilizing (1), the similarity measure is computed as $S_p(A_{IFS}, B_{IFS}) = 0.76$ (for $p = 1$).
- From Example 2 it is observed that for two different sets of IFSs, Hong and Kim's [10] method computes the same similarity values. The proposed measure (1) overcomes this problem and calculates the degree of similarity as: $S_p(A_{IFS}, B_{IFS}) = 0.5$ and $S_p(B_{IFS}, C_{IFS}) = 0.585$ (for $p = 1$).
- From Example 3, it can be seen that for two different sets of IFSs, the similarity measure proposed by Xu [25] computes the same similarity values. The proposed similarity measure (1) overcomes this problem and computes the similarity values as: $S_p(A_{IFS}, B_{IFS}) = 0.641$, $S_p(A_{IFS}, C_{IFS}) = 0.675$ (for $p = 1$).
- From Example 4, it is seen that for IFSs $A = \{< x, 0, 0 >\}$ and $B^j_{IFS} = \{< x, \mu_{B^j}(x), \nu_{B^j}(x) >\}$ (for $j = 1, 2, ...$), which satisfy $\mu_{B^j}(x) + \nu_{B^j}(x) = 1$ for all $x \in X$, we always get $S^Z(A_{IFS}, B^j_{IFS}) = 0.500$. The proposed similarity measure (1) overcomes this problem.

Therefore, from Table 1 it is clear that in all the above cases the proposed similarity measure calculates the similarity between two IFSs and overcomes the drawbacks of the existing methods.

In the next section, a priority method of intuitionistic preference relations has been developed using the proposed similarity measure of IFSs.

3 Method for estimating priority-based weights of the alternatives

3.1 The concept of intuitionistic preference relation in decision making

Frequently, in real-life situations a decision maker may not be able to accurately express his/her preferences for alternatives. Under these circumstances, it is more suitable to express the preference values of the experts with intuitionistic fuzzy values rather than exact numerical values or linguistic variables (Herrera *et al*. [9], Xu and Yager [19]). In this regard, Xu [22] introduced the notion of intuitionistic preference relation, as follows:

Definition 1: An intuitionistic preference relation R_{IPR} on the set X is represented by a matrix denoted as: $R_{IPR} = (r_{ij})_{n\times n} \subset X \times X$ with $r_{ij} = \langle (x_i, x_j), \mu(x_i, x_j), \nu(x_i, x_j) \rangle$ for all $i, j = 1, 2, .., n$. For convenience, we let $r_{ij} = (\mu_{ij}, \nu_{ij})$, for all $i, j = 1, 2, ..., n$, where r_{ij} is the intuitionistic fuzzy value, composed by the certainty degree μ_{ij} to which x_i is preferred to x_j and the certainty degree ν_{ij} to which x_i is non-preferred to x_j. Furthermore, μ_{ij} and ν_{ij} satisfy the following characteristics:

$$0 \le \mu_{ij} + \nu_{ij} \le 1,\ \mu_{ji} = \nu_{ij},\ \nu_{ji} = \mu_{ij},\ \mu_{ii} = \nu_{ii} = 0.5 \text{ for all } i, j = 1, 2, ..., n.$$

3.2 A method to estimate weights of alternatives

In order to determine the weights of alternatives in a decision making process, a methodology has been developed in this section, based on intuitionistic preference relation, which can be described as follows:

Step 1: For a MCDM problem, let 'n' numbers of alternative $A_1, A_2, ..., A_n$ are set by the expert where the weights are unknown. The expert provides his/her intuitionistic preference for every pair of alternatives (A_i, A_j) and constructs intuitionistic preference relations as follows:

$$R_{IPR} = (r_{ij})_{n\times n} \text{ where } r_{ij} = (\mu_{ij}, \nu_{ij}), 0 \le \mu_{ij} + \nu_{ij} \le 1, \text{ for all } i, j = 1, 2, ..., n.$$

From Definition 1, we may write $\mu_{ji} = \nu_{ij}, \nu_{ji} = \mu_{ij}, \mu_{ii} = \nu_{ii} = 0.5$ for all $i, j = 1, 2, ..., n$. Therefore, for 'n' number of alternatives the intuitionistic preference relations $R_{IPR} = (r_{ij})_{n\times n}$, given by the expert, may be expressed as follows:

$$
R_{IPR}=\begin{array}{c c} & \begin{array}{cccccc} A_1 & A_2 & \cdots & A_j & \cdots & A_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{array} & \begin{pmatrix} (0.5,0.5) & (\mu_{12},\nu_{12}) & \cdots & (\mu_{1j},\nu_{1j}) & \cdots & (\mu_{1n},\nu_{1n}) \\ (\mu_{21},\nu_{21}) & (0.5,0.5) & \cdots & (\mu_{2j},\nu_{2j}) & \cdots & (\mu_{2n},\nu_{2n}) \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ (\mu_{i1},\nu_{i1}) & (\mu_{i2},\nu_{i2}) & \cdots & (\mu_{ij},\nu_{ij}) & \cdots & (\mu_{in},\nu_{in}) \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ (\mu_{n1},\nu_{n1}) & (\mu_{n2},\nu_{n2}) & \cdots & (\mu_{nj},\nu_{nj}) & \cdots & (0.5,0.5) \end{pmatrix} \end{array}
$$

Step 2: Among the '*n*' alternatives let us consider the pair (A_i, A_j). If alternative A_i is *definitely preferred* over A_j, then the corresponding intuitionistic preference relation is $(1,0)$; where the certainty degree to which A_i is preferred to A_j is 1, and the certainty degree to which A_i is non-preferred to A_j is 0.

Step 3: If for the pair of alternative (A_i, A_j) the intuitionistic preference value is $r_{ij}=(\mu_{ij},\nu_{ij})$, then calculate the similarity between r_{ij} and $(1,0)$ using (1). In this way, for all the pair of alternatives $(A_i, A_j)\forall i, j=1,2,...,n,\ i\neq j$, ${}^{n}c_2$ similarity values can be calculated and sub-sequently it has been expressed in a concise manner in the form of a matrix as follows:

$$
R^{/}=\begin{array}{c c} & \begin{array}{cccccc} A_1 & A_2 & \cdots & A_j & \cdots & A_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{array} & \begin{pmatrix} S_{p11} & S_{p12} & \cdots & S_{p1j} & \cdots & S_{p1n} \\ S_{p21} & S_{p22} & \cdots & S_{p2j} & \cdots & S_{p2n} \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ S_{pi1} & S_{pi2} & \cdots & S_{pij} & \cdots & S_{pin} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \\ S_{pn1} & S_{pn2} & \cdots & S_{pnj} & \cdots & S_{pnn} \end{pmatrix} \end{array}
$$

where $S_{p\,ij}=S_p(r_{ij}, r)\,; 1\leq p<\infty;$ assuming $r=(1,0)$.

Step 4: Finally, the weight of the alternative A_i is defined by

$$
W_p(A_i)=\min_{\substack{1\leq j\leq n \\ i\neq j}}\{S_{p\,ij}\} \text{ for } 1\leq p<\infty \tag{9}
$$

Thus, the weight vectors of the alternatives $A_1, A_2,...,A_n$ are $W_p(A_1), W_p(A_2),...,W_p(A_n)$ respectively, for $1\leq p<\infty$.

4 Numerical illustration

A decision maker intends to buy an air-condition system. He has five alternatives (air-condition systems) to choose, namely $A=\{A_1, A_2, A_3, A_4, A_5\}$. Taking into consideration various factors, such as, price, functionality (i.e., different functions), user-friendliness the decision maker constructs the intuitionistic preference relations [23] as follows:

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\
R_{IPR} = (r_{ij})_{5\times 5} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \begin{pmatrix} (0.5,0.5) & (0.6,0.3) & (0.4,0.2) & (0.7,0.2) & (0.4,0.5) \\ (0.3,0.6) & (0.5,0.5) & (0.5,0.3) & (0.6,0.1) & (0.3,0.6) \\ (0.2,0.4) & (0.3,0.5) & (0.5,0.5) & (0.6,0.2) & (0.4,0.5) \\ (0.2,0.7) & (0.1,0.6) & (0.2,0.6) & (0.5,0.5) & (0.3,0.6) \\ (0.5,0.4) & (0.6,0.3) & (0.5,0.4) & (0.6,0.3) & (0.5,0.5) \end{pmatrix}
\end{array}
$$

In R_{IPR}, the element $r_{12} = (0.6, 0.3)$ is composed by the certainty degree 0.6 to which A_1 is preferred to A_2 and the certainty degree 0.3 to which A_1 is non-preferred to A_2. The other elements in R_{IPR} may be interpreted in the same way.

Utilizing (1) the similarity measure (for p = 1) between each entry of R_{IPR} and (1,0) is computed and the matrix $R^{/}$ can be constructed as follows:

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\
R^{/} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \begin{pmatrix} 0.5 & 0.599 & 0.581 & 0.690 & 0.473 \\ 0.429 & 0.5 & 0.568 & 0.740 & 0.429 \\ 0.438 & 0.554 & 0.5 & 0.658 & 0.473 \\ 0.392 & 0.377 & 0.403 & 0.5 & 0.429 \\ 0.529 & 0.599 & 0.529 & 0.599 & 0.5 \end{pmatrix}
\end{array}
$$

Finally, utilizing (9) the weights of the alternatives is obtained as follows: $W(A_1) = 0.473$, $W(A_2) = 0.429$, $W(A_3) = 0.438$, $W(A_4) = 0.377$ and $W(A_5) = 0.529$. Hence, the best air-condition system is A_5, since it has the highest priority weight.

5 Conclusion

This paper develops a new method for deriving the priority-based weights of the alternatives from intuitionistic preference relations. For this purpose, a new similarity measure of IFSs has been proposed. Furthermore, examples have also been provided to make a comparison between the proposed similarity measure and the existing methods. Finally, an air-conditioning system selection problem has been presented to illustrate the application of the proposed measure. The proposed priority method may be applicable to MADM problems in various fields, which would be the topic of our future research work.

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