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Two and three parameter represenation of intuitionistic fuzzy sets in the context of entropy and similarity

Eulalia Szmidt and Janusz Kacprzyk
Systems Research Institute, Polish Academy of Sciences
ul. Newelska 6, 01–447 Warsaw, Poland
E-mail: {szmidt, kacprzyk}@ibspan.waw.pl

Abstract

This paper is a continuation of our previous papers on entropy and similaity of the Atanassov intuitionistic fuzzy sets (A-IFSs, for short). We discuss the usefulness of taking into account all three functions (membership, non-membership and hesitation margin) describing A-IFSs while considering the entropy and similarity measures. We demonstrate on the examples that the omiting of the hesiation margins in both entropy and similarity measures considered leads sometimes to the counterintuitive results.
Keywords: Intuitionistic fuzzy sets, entropy, similarity.

1 Introduction

In our previous works (Szmidt and Kacprzyk [21], Tasseva at al. [30]), we have discussed that a chosen A-IFSs represntation (with two functions – i.e., membership and non-membership or three functions – i.e., membership, non-membership, and hesitation margins) does influence such measures as distances (Szmidt and Kacprzyk [21], [27]), entropy (Szmidt and Kacprzyk [22], [28]), and similarity (Szmidt and Kacprzyk [25], [24]), [29]). These measures play a fundamental role in inference and approximate reasoning, and in virtually all applications of fuzzy logic. The importance of those measures has motivated us to compare and examine the concrete examples showing the differences in final conclusion while differnt A-IFSs represenattions are applied.

A measure of fuzziness often used and cited in the literature is called an entropy (first mentioned by Zadeh [36]). Entropy results from from the lack of a crisp distinction between the elements belonging and not belonging to a set (i.e. the boundaries of a set under consideration are not sharply defined).

De Luca and Termini [9] introduced some requirements which capture our intuitive comprehension of a degree of fuzziness. Kaufmann (1975) (cf. [16]) proposed to measure a degree of fuzziness of a fuzzy set A by a metric distance between its membership function and the membership (characteristic) function of its nearest crisp set. Yager [35] viewed a degree of fuzziness in terms of a lack of distinction between the fuzzy set and its complement. Higashi and Klir [8] extended Yager's concept to a general class of fuzzy complements. Yager's approach was also further developed by Hu and Yu [13]. Indeed, it is the lack of distinction between sets and their complements that distinguishes fuzzy sets from crisp sets. The less the fuzzy set differs from its complement, the fuzzier it is. Kosko [15] investigated the fuzzy entropy in relation to a measure of subsethood. Fan at al. [10], [11], [12] generalized Kosko's approach.

Here we discuss measures of fuzziness for intuitionistic fuzzy sets which are a generalization of fuzzy sets. We recall a measure of entropy we introduced (Szmidt and Kacprzyk [22], [28]). We compare our approach with Zeng and Li [37] approach. We discuss the reasons of differences and the counter-intuitive results obtained in the case of Zeng and Li's entropy which boils down to entropy given by Hung [14] (cf. Szmidt and Kacprzyk [28] for further discussion).

The importance of similarity measures has motivates researchers to compare and examine the effectiveness and properties of different measures of similarity for fuzzy sets (e.g. Zwick at al. [38], Pappis and Karacapilidis [18], Chen at al. [4], Wang at al. [34], Bouchon-Meunier et al. [3], Cross and Sudkamp [5]). The analysis of similarity is also a fundamental issue while employing A-IFSs (Atanassov [1], [2]).

Like in our previous works (Szmidt and Kacprzyk [25], [24]) we discuss here results of a similarity measure which is not a standard similarity measure in the sense that it is not only a dual concept to a (general) distance measure (cf. Tversky [33]). In commonly used similarity measures, the dissimilarity behaves like a distance function. Such a standard approach, formulated for objects meant as crisp values, was later extended and used to evaluate the similarity of fuzzy sets (Cross and Sudkamp [5]). Distances were also proposed to measure the similarity between intuitionistic fuzzy sets (cf. Dengfeng and Chuntian [7], and Szmidt and Kacprzyk [24], [25]).

The measure we discuss here is a different kind of a similarity measure as it does not measure just a distance between individual intuitionistic fuzzy preferences being compared. The measure answers the question if the compared preferences are more similar or more dissimilar to each other.

We compare our approach with a similarity measure proposed by Zeng and Li [37], i.e. with the measure using two functions only (membership and non-membership) for the representation of A-IFSs, and not taking into account the complements of the elements/objects they compare with each other.

While assessing the results of comparison of our measure with the Zeng and Li [37] measure a question arises: should similarity measures between A-IFSs be just a straightforward generalization of measures between fuzzy sets? The results obtained show that, just as in the case of distances (Szmidt and Kacprzyk [27]), straightforward approaches may not work.

2 A brief introduction to A-IFSs

One of the possible generalizations of a fuzzy set in X (Zadeh [36]), given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A^{'}$, is an A-IFS, i.e. Atanassov's intuitionistic fuzzy set, (Atanassov [1], [2]) A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively.

Obviously, each fuzzy set may be represented by the following A-IFS

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{4}$$

For each A-IFS in X, we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

an *intuitionistic fuzzy index* (or a *hesitation margin*) of $x \in A$, and it expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [2]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

In our further considerations we will use the complement set A^C [2]

$$A^C = \{< x, \nu_A(x), \mu_A(x) > | x \in X\} \tag{6}$$

In our further considerations we will use the normalized Hamming distance between fuzzy sets A, B in $X = \{x_{1,}, \ldots, x_n\}$ Szmidt and Baldwin [19], [20], Szmidt and Kacprzyk [21], [27]:

$$l_{IFS}(A, B) \quad = \quad \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \tag{7}$$

For (7) we have: $0 \leq l_{IFS}(A, B) \leq 1$. Clearly the normalized Hamming distance (7) satisfies the conditions of the metric. In Szmidt and Kacprzyk [21], Szmidt and Baldwin [19], [20], and especially in Szmidt and Kacprzyk [27] it is shown why when calculating distances between IFSs we should take into account all three functions describing A-IFSs.

Applications of A-IFSs to group decision making, negotiations, etc. are presented in (Szmidt and Kacprzyk [23, 25, 26]).

3 Examples of entropy and similarity measures without hesitation margins

In this Section we cite the examples of the entropy and similarity measures which do not take into account hesitation margins.

3.1 Zeng and Li's entropy measure

The entropy measures the whole missing information which may be necessary to have no doubts when classifying an element, i.e. to say that an element fully belongs or fully does not belong to a set considered. We cite here Zeng and Li's entropy measure [37] for an A-IFSs A (notation used in [37] is changed so that it is consistent with that in this paper):

$$E_{ZL}(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} (|\mu_A(x_i) + \mu_A(x_i) + \pi_A(x_i) - 1| \tag{8}$$

Having in mind that for A-IFSs we have $\mu_{.}(x_i) + \nu_{.}(x_i) + \pi_{.}(x_i) = 1$, Zeng and Li's entropy measure (8) becomes

$$E_{ZL}(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} (|\mu_A(x_i) - \nu_A(x_i)| \tag{9}$$

In other words, Zeng and Li's similarity measure (9) does not take into account the values of $\pi_A(x_i)$. Only the values of the memberships and non-memberships are taken into account.

In Szmidt and Kacprzyk [28] we discussed in more detail the above measure (9). Although all the mathematical "constructions" of this measure are correct, the question arises if we may use any mathematically correct approach to represent the measures which by definition are to render some properties that have a concrete semantic meaning, and are in most cases to be useful. It seems that the mathematical correctness is in this context for sure a necessary but not a sufficient condition.

3.2 Zeng and Li's similarity measure

We cite here Zeng and Li's similarity measure [37] between two A-IFSs A and B (again, the terms and symbols used in [37] are changed so that they are consistent with those in this paper):

$$s_{ZL}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |(\mu_A(x_i) + \pi_A(x_i)) - (\mu_B(x_i) + \pi_B(x_i))|) \tag{10}$$

Having in mind that for A-IFSs we have

$$\mu_{.}(x_i) + \pi_{.}(x_i) = 1 - \nu_{.}(x_i)$$

Zeng and Li's similarity measure (10) becomes

$$s_{ZL}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |1 - \nu_A(x_i) - 1 + \nu_B(x_i))|) =$$

$$= \quad 1 - \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|) \tag{11}$$

In other words, Zeng and Li's similarity measure (11) does not take into account the values of $\pi_A(x_i) - \pi_B(x_i))$. Only the values of the memberships and non-memberships are taken into account. The measure (11) also does not take into account the complements of the sets considered. Certainly, the above measure is correct for crisp sets, and for fuzzy sets for which the values of π are always equal to zero. But later we will show that if $\pi \neq 0$, i.e. for A-IFSs, the measure (11) may produce quite counter-intuitive results.

4 Examples of entropy and similarity measures with hesitation margins

Now we will recall briefly another approaches (cf. Szmidt and Kacprzyk [28]) to measuring entropy and similarity of intuitionistic fuzzy sets. The measures we will consider are not only mathematically correct but at the same time render the sense of entropy, and similarity not only as a pure mathematical construction but as the measures to be useful in practice.

4.1 Szmidt and Kacprzyk's entropy for A-IFSs

In Szmidt and Kacprzyk [28] we gave a motivation and revised some conditions for entropy measures for A-IFSs. Here we only recall one of the possible entropy measures fulfilling the new conditions (cf. Szmidt and Kacprzyk [28]) and rendering the very meaning of entropy.

Entropy for an A-IFS A with n elements may be given as (Szmidt and Kacprzyk [22]):

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{d(F_i, F_{i,near})}{d(F_i, F_{i,far})} \tag{12}$$

where $d(F_i, F_{i,near})$ is a *distance* from F_i to its the nearer point $F_{i,near}$ among $M(1,0,0)$ and $N(0,1,0)$, and $d(F_i, F_{i,far})$ is the *distance* from F_i to its the farer point $F_{i,far}$ among $M(1,0,0)$ and $N(0,1,0)$.

A ratio-based measure of entropy (12) satisfies the axioms formulated in Szmidt and Kacprzyk [28]. For the detailed explanations we refer an interested reader to Szmidt and Kacprzyk [21], [22], [24], [28].

4.2 Szmidt and Kacprzyk's similarity for A-IFSs

In [29] we have proposed a similarity measure having the advantages of the similarity measure proposed in Szmidt and Kacprzyk [25], [24]) and whose the numerical values are consistent with the common scientific tradition (i.e. they are from interval [0, 1] which was not fulfilled for the measures discussed in [25], [24]).

Similarity measure (Szmidt and Kacprzyk [29]) between two A-IFSs A and B with n elements, can be expressed as

$$Sim(l_{IFS}(A,B), l_{IFS}(A,B^C)) = 1 + \frac{2}{n}\sum_{i=1}^{n}\frac{l_{IFS}((A(x_i), B(x_i))}{l_{IFS}((A(x_i), B(x_i)) + l_{IFS}((A(x_i), B(x_i)^C)} \tag{13}$$

Measure (13) is discussed in detail in (Szmidt and Kacprzyk [29]). Here we would like to stress mainly that the measure (13) takes into account hesitation margins. In the next Section we will show on examples that the hesitation margins play an important role while measuring similarity between A-IFSs.

5 Comparison of the results

5.1 Comparison of the entropy measures

Now we will verify if the results produced by (9) and (12) are consistent with our intuition. We examine entropy of single elements x_i of an A-IFS, each described via (μ_i, ν_i, π_i), namely:

$$x_1 : (0.7, 0.3, 0) \quad (14)$$
$$x_2 : (0.6, 0.2, 0.2) \quad (15)$$
$$x_3 : (0.5, 0.1, 0.4) \quad (16)$$
$$x_4 : (0.4, 0, 0.6) \quad (17)$$

We assume that x_i represents the $i-th$ house we consider to buy. On the one extreme, for house x_1 (the first house) 70% of the attributes have desirable values, and 30% of attributes have undesirable values. On the other extreme, for house x_4 we only know that it has 40% of the desirable attributes and we do not know about 60% of the attributes we are interested in. The entropy calculated due to (9) gives the following results:

$$E_{ZL}(x_1) = 1 - |0.7 - 03| = 0.6 \quad (18)$$
$$E_{ZL}(x_2) = 1 - |0.6 - 0.2| = 0.6 \quad (19)$$
$$E_{ZL}(x_3) = 1 - |0.5 - 0.1| = 0.6 \quad (20)$$
$$E_{ZL}(x_4) = 1 - |0.4 - 0| = 0.6 \quad (21)$$

Results (18)–(21) suggest that the entropy of all $x_1, \dots, x_4$ is the same though this is counter-intuitive! It seems that the entropy of the situation expressed by x_1, i.e., 70% positive attributes, 30% negative attributes is less than the entropy of x_4, i.e., 40% of positive attributes, and 60% unknown. Case (x_1) is "clear" in the sense that we know for sure that 30% negative attributes prevents house x_1 to be our "dream house" while in case of (x_4) we only know for sure that it has 40% of desirable attributes, and 60% is unknown. So we may conclude that it is quite possible that (x_4) may: fulfill in 100% our demands (if all 60% of the unknown attributes happen to be desirable), or may fulfill in 40% our demands and does not fulfill 60% of our demands (if 60% of unknown attributes turn out to be undesirable), or in general – 40%+α can fulfill and 0%+β does not fulfill our demands where $\alpha + \beta = 60\%$ and α, $\beta \geq 0$. So we intuitively feel that it is easier to classify house x_1 as fulfilling our demands (30% is missing) than to classify house x_4 to the set of houses fulfilling (worth buying) or not fulfilling (not worth buying) our demands.

The entropy calculated from (12) gives the following results:

$$E(x_1) = \frac{|1 - 0.7| + |0 - 0.3| + |0 - 0|}{|0 - 0.7| + |1 - 0.3| + |0 - 0|} = 0.43 \quad (22)$$
$$E(x_2) = \frac{|1 - 0.6| + |0 - 0.2| + |0 - 0.2|}{|0 - 0.6| + |1 - 0.2| + |0 - 0.2|} = 0.5 \quad (23)$$
$$E(x_3) = \frac{|1 - 0.5| + |0 - 0.1| + |0 - 0.4|}{|0 - 0.5| + |1 - 0.1| + |0 - 0.4|} = 0.56 \quad (24)$$
$$E(x_4) = \frac{|1 - 0.4| + |0 - 0| + |0 - 0.6|}{|0 - 0.4| + |1 - 0| + |0 - 0.6|} = 0.6 \quad (25)$$

Results (22)–(25) seem to better reflect our intuition - the purchase decision is the easiest in the first case (entropy is the smallest) and the most difficult in the fourth case (the biggest entropy). This may be depicted as in Fig. 1. It is worth stressing that entropy (12) is a special case of the similarity measure (we refere an interested reader to Szmidt and Kacprzyk [24] for more details).

It seems that when calculating entropy of A-IFSs one should take into account all three functions (membership, non-membership and hesitation margin) describing an A-IFSs. Only then full information

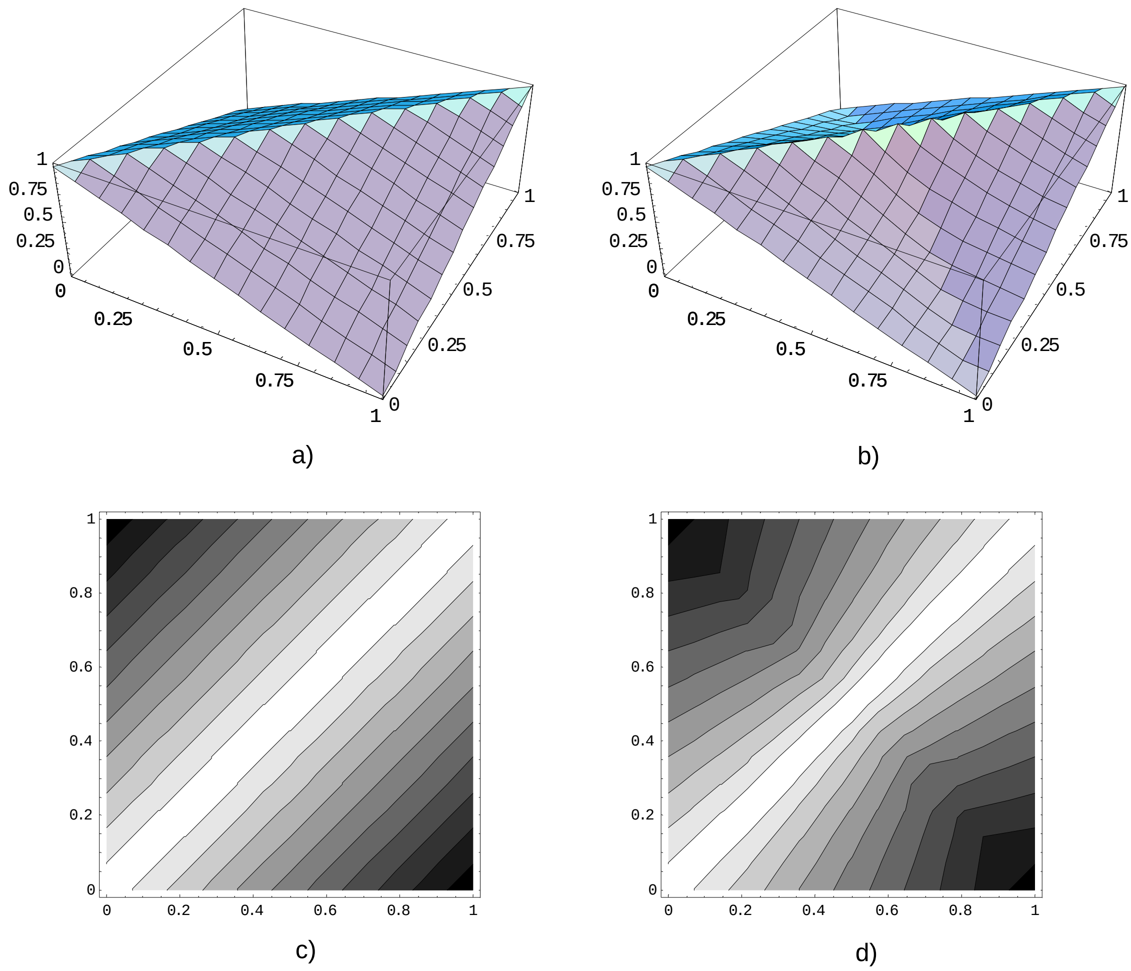

Figure 1: Entropy calculated from (9): a) and c)– countour plot, entropy calculated from (12): b) and d) – countour plot;

preventing from univocal classification of an element as belonging or not belonging to a set is taken into account (due to the very sense of entropy). This point of view has been also justified in, e.g., image processing via A-IFSs (cf. Vlachos and Sergiadis [32]).

5.2 Comparison of the similarity measures

In Section 2 we pointed out some possible applications of A-IFSs, namely to model different voting situations. So let an element x of an A-IFS characterized via (μ, ν, π) expresses the situation of voting when μ expresses those who vote for, ν those who vote against, and π those who abstain. We could use a measure of similarity to compare different voting situation. To make the results as clear as possible we examine here the similarity of one-element A-IFSs.

Let us examine the similarity of
$E = (x, 0.7, 0, 0.3)$ i.e., 70% vote for and 30% abstain,
and the following A-IFSs:
$L = (x, 0.7, 0.3, 0)$ – 70% vote for, 30% against,
$I = (x, 0.65, 0.25, 0.1)$ – 65% vote for, 25% vote against, and 10% abstain,

$J = (x, 0.6, 0.2, 0.2)$ – 60% vote for, 20% vote against, and 20% abstain,
$K = (x, 1, 0, 0)$ – 100% vote for,

It seems that the above situations are different. But from Zeng and Li's similarity measure (10) we obtain

$$s_{ZL}(E, L) = 1 - 0.5(|0.7 - 0.7| + |0 - 0.3|) = 0.85 \tag{26}$$
$$s_{ZL}(E, I) = 1 - 0.5(|0.7 - .65| + |0 - .25|) = 0.85 \tag{27}$$
$$s_{ZL}(E, J) = 1 - 0.5(|0.7 - 0.6| + |0 - 0.2|) = 0.85 \tag{28}$$
$$s_{ZL}(E, K) = 1 - 0.5(|0.7 - 1| + |0 - 0|) = 0.85 \tag{29}$$

The results (26)–(29) obtained from Zeng and Li's similarity measure (10) are this time counter-intuitive. It is difficult to accept that the situation (E) in which 70% vote for and 30% abstain is to the same extent similar to such different situations as: (K) 100% vote for – (29), and (L) 70% vote for and 30% against(!) – (26).

On the other hand from the measure (13) which we proposed here, we obtain

$$Sim(E, L) = 1 - \frac{0.3}{0.3 + 0.7} = 0.7 \tag{30}$$
$$Sim(E, I) = 1 - \frac{0.25}{0.25 + 0.65} = 0.72 \tag{31}$$
$$Sim(E, J) = 1 - \frac{0.2}{0.2 + 0.6} = 0.75 \tag{32}$$
$$Sim(E, K) = 1 - \frac{0.3}{0.3 + 1} = 0.77 \tag{33}$$

The results (30)–(33) are intuitively appealing. First, contarary to the results obtained from Zeng and Li's similarity measure (10), we have obtained different values of the similarity between different situations of voting. We observe that the values of the similarity increase in the function of decreasing values of those who vote against. Now similarity between (E) 70% vote for and 30% abstain, and (K) 100% vote for is bigger (equal to 0.77 – (33)) than (0.7, i.e.,) similarity between (E) 70% vote for and 30% abstain , and 70% vote for, 30% against (L) – (30). This result seems intuitively correct.

Figure 2 illustrates the descibed phenomenon and explains it qualitatively. This figure shows similarity between element $(0.7, 0.2, 01)$ and other possible elements (μ, ν, π) belonging to an A-IFS. In Figure 2 a) we can see the shape of Zeng and Li's similarity measure (10), and in Figure 2 b) – the shape of Szmidt and Kacprzyk's similarity measure (13). In Figures 2 c) and d) we can see their respective contour plots. As it could be expected, the measure (13) taking into account all three parameters describing an A-IFs is able to render more details ("sees" the differences better) among different elements than the measure (10) (examples above confirm it). The same fact (advantages of a three-parameter A-IFS representation over a two-parameter A-IFS representation) has been already pointed out in our previous works (e.g. concerning distances – cf. Szmidt and Kacprzyk [27]).

To sum up, it seems that similarity measure between A-IFs (13) proposed in this paper gives more intuitively appealing results than Zeng and Li's (10) measure.

6 Concluding remarks

We considered the problem of measuring entropy and similarity for A-IFSs. It turns out that just the same as it was while considering the possible representations of A-IFSs (Szmidt and Kacprzyk [21], Tasseva at al. [30]), distances between A-IFSs (Szmidt and Kacprzyk [21], [27])), while considering entropy and similarity it is expedient to use all three functions (membership, non-membership and hesitation margin). Omitting e.g., hesitation margin may lead to counter-intuitive results.

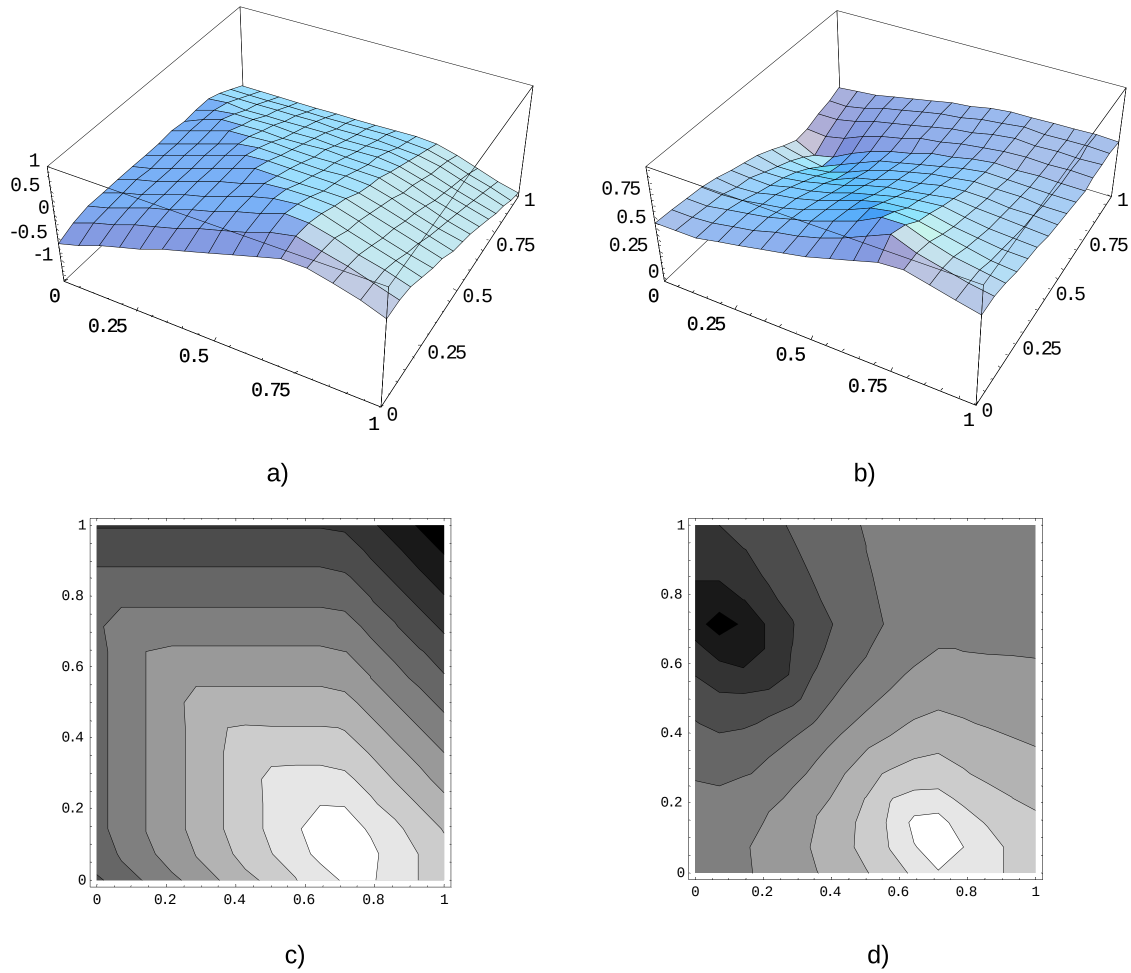

Figure 2: Similarity between element (0.7, 0.2, 0.1) and other possible elements calculated from Zeng and Li's (10) – a) and c), and Szmidt and Kacprzyk's (13) – b) and d) similarity measures.

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