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A note on the unconscientious experts' evaluations in the intuitionistic fuzzy environment

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Abstract: In the intuitionistic fuzzy environment, unconscientious opinions may cause problems in the data processing. In this paper, new ways of correction of the unconscientious experts evaluations are proposed.

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1 Introduction

Intuitionistic fuzzy (IF) sets are first introduced by Krassimir T. Atanassov in 1983. The latest developments of the theory are presented in the monograph [1]. In one of the subsections are discussed the issues regarding the use of experts' opinions to determination of the membership degree and the non-membership degree, with which the evaluated variant belong/not belong to the IF set of variants satisfying certain criterion.

The problem arises if an expert is *more than 100% sure* that the variant belongs either to the set or to the complement of this set.

More precisely, we can describe this fact in terms of membership, and non-membership functions as follows.

Let E_i , $I = 1, \ldots, n$, be an $i^{\text{-th}}$ expert from the group of n experts. Following Atanassov ([1], p.12) we call the expert E_i *unconscientious*, if among his estimations $\{\langle \mu_{i,j}, \nu_{i,j} \rangle \mid j \in J_i\}$, where $J = \bigcup_{i=1}^{n} J_i$ is an index set (related to the evaluated variants), there exists an estimation for which $\mu_{i,j} \le 1$ and $\nu_{i,j} \le 1$, but $\mu_{i,j} + \nu_{i,j} > 1$.

Let us call the IF value $\langle \mu_{i,j}, \nu_{i,j} \rangle$ for which $\mu_{i,j} \le 1$ and $\nu_{i,j} \le 1$, but $\mu_{i,j} + \nu_{i,j} > 1$ an *unconscientious evaluation* (UE) of $j^{\text{-th}}$ variant (feature, event) by the $i^{\text{-th}}$ expert.

From now on, the UE $\langle \mu_{i,j}, \nu_{i,j} \rangle$ we denote, for shortly, as UE $\langle \mu, \nu \rangle$.

To apply the intuitionistic fuzzy sets theory to the processing of evaluations, the UE $\langle \mu, \nu \rangle$ must be adjusted (convert) to the correct IF value $\langle \overline{\mu}, \overline{\nu} \rangle$ where $\overline{\mu}, \overline{\nu}, \overline{\pi} \in [0, 1]$ and $\overline{\mu} + \overline{\nu} \in [0, 1]$, with hesitation margin $\overline{\pi} = 1 - \overline{\mu} - \overline{\nu}$.

Atanassov notes that the fact of existence of this kind of problems by the evaluation of events distinguishes the decision aid in the intuitionistic fuzzy environment from the decision aid in the (classical) fuzzy environment, where such unconscientious evaluations do not exist (or are easy to correction).

No general condition has been given that should be fullfiled in order to the conversion can being considered as proper.

I think that the conversion's mapping should fullfill at least the property given below.

Property (P1):

a) if $\mu \geq \nu$, then $\overline{\mu} \geq \overline{\nu}$;

b) if $\mu \leq \nu$, then $\overline{\mu} \leq \overline{\nu}$.

In the terms of intuitionistic fuzzy logic, when we consider the UE $\langle \mu, \nu \rangle$ as the logical truth-value of the hadjudicate about the possession of a given attribute by the object, the property (P1) can be written as:

Property (P1′):

a) if $\langle \mu, \nu \rangle$ is an IFT, then $\langle \overline{\mu}, \overline{\nu} \rangle$ is an IFT;

b) if $\langle \mu, \nu \rangle$ is an IFcT, then $\langle \overline{\mu}, \overline{\nu} \rangle$ is an IFcT.

Let us recall, that we call the IF value $\langle a, b \rangle$ an Intuitionistic Fuzzy Tautology (IFT) *iff* $a \geq b$, and, similarly, an Intuitionistic Fuzzy co-Tautology (IFcT) *iff* $a \leq b$.

I think that in the case of unconscientious experts' evaluations another problem should be considered.

Problem 1: If $\langle \mu, \nu \rangle$ is an UE, then, for the corrected value $\langle \overline{\mu}, \overline{\nu} \rangle$, should be:

a) $\overline{\pi} = 0$;

b) $\overline{\pi} > 0$;

c) $\overline{\pi}$ does not have to meet any conditions.

I am not able to solve Problem 1.

On the one hand, it can be concluded that an *expert* is a serious man, and he does not specify that is more than 100% sure. The *expert* is, at most, 100% sure of his opinion, and the surplus of more than 100% is irrelevant. It seems to be rational because this type of *expert's* mistake can happen just by accident.

On the other hand, it is reasonable that the *unconscientious* expert is, in fact, *unsure* and his estimation should be considered as *uncertain* with the hesitation degree greater than 0. In this case the degree $\overline{\pi}$ should be an increasing (non-decreasing?) function of the sum $\mu + \nu$. It seems to be rational too, because the greater sum $\mu + \nu$ means the greater *un-precision* of the evaluation of the variant by the expert.

Problem 1 can be described also in terms of *accuracy* of the IF value $\langle \overline{\mu}, \overline{\nu} \rangle$. The accuracy is defined as: *accuracy*($\langle \overline{\mu}, \overline{\nu} \rangle$) = $\overline{\mu} + \overline{\nu}$. In the problem 1 we would consider the question: Should *accuracy*($\langle \overline{\mu}, \overline{\nu} \rangle$) be equivalent to 1, or should it be less than 1, or whether does not have to meet any conditions.

In the cited monograph [1], Atanassov proposed five ways for the adjustment of the values in the *unconscientious experts' case*.

Let $\langle \mu, \nu \rangle$ be an UE.

Way 1. We calculate the values $\overline{\mu}, \overline{\nu}$ as follows:

$$\overline{\mu} = \frac{\mu}{\mu+\nu}, \quad \overline{\nu} = \frac{\nu}{\mu+\nu}.$$

It is clear that $\overline{\mu} + \overline{\nu} = 1$, what means that the hesitation margin is, by this conversion, reduced to zero. The property (P1) is fulfilled.

The modification of this way is given in the form

$$\overline{\mu} = \frac{\mu}{2}, \quad \overline{\nu} = \frac{\nu}{2}.$$

In this case, the hesitation margin $\overline{\pi}$ belongs to the interval $[0, \frac{1}{2})$.

Way 2. We calculate the values $\overline{\mu}, \overline{\nu}$ as follows:

$$\overline{\mu} = \mu - \frac{\min(\mu, \nu)}{2}, \quad \overline{\nu} = \nu - \frac{\min(\mu, \nu)}{2}.$$

It is easy to show that $\overline{\mu} + \overline{\nu} = \max(\mu, \nu) \leq 1$, and $\overline{\pi} \in [0, \frac{1}{2})$. Moreover, $\overline{\pi} = 0$ *iff* $\max(\mu, \nu) = 1$. The property (P1) is fulfilled.

Ways 3, 4 and **5** (see [1], pp.14–16) are designated on the basis of cardinalities of sets and in this paper are omitted.

2 New ways of the correction of unconscientious evaluations

The next way can be given as below.

Way 6. We calculate the corrected degrees as

$$\overline{\mu} = 1 - \nu,$$
$$\overline{\nu} = 1 - \mu.$$

For the UE $\langle \mu, \nu \rangle$ we have $\mu, \nu \in [0, 1]$ and $\mu + \nu > 1$.

So $\overline{\mu}, \overline{\nu} \in [0, 1]$ and $\overline{\mu} + \overline{\nu} = (1 - \mu) + (1 - \nu) = 2 - (\mu + \nu) \in [0, 1)$. Furthermore, $\overline{\pi} = 1 - \overline{\mu} - \overline{\nu} = \mu + \nu - 1 \in (0, 1]$, what means that there exist always a positive hesitation margin of corrected degree. It is also: the greater $\mu + \nu$, the greater is the hesitation margin $\overline{\pi}$.

The property (P1) is fulfilled.

The other formula can be obtained by an averaging operator @, defined (see [1], p.17) for two IF values $\langle a, b\rangle$ and $\langle c, d\rangle$ in the form

$$\langle a, b\rangle \,@\, \langle c, d\rangle = \langle \frac{a+c}{2}, \frac{b+d}{2} \rangle.$$

Then we obtain the Way 7. The value $\langle \bar{\mu}, \bar{\nu} \rangle$ is, in this case, the @-average of the UE value $\langle \mu, \nu\rangle$ and the corrected value $\langle 1 - \nu, 1 - \mu\rangle$ obtained in the Way 6. We should note that the operator @, defined for two IF values, must be extended for the UE $\langle \mu, \nu \rangle$ and the IF value $\langle 1 - \nu, 1 - \mu \rangle$. But

$$\frac{1+\mu-\nu}{2} \in (0, 1), \frac{1-\mu+\nu}{2} \in (0, 1)$$

and

$$\frac{1+\mu-\nu}{2} + \frac{1-\mu+\nu}{2} = 1,$$

therefore,

$$\langle \frac{1+\mu-\nu}{2}, \frac{1-\mu+\nu}{2} \rangle$$

is an IF value.

Way 7. We calculate the corrected degrees as

$$\bar{\mu} = \frac{1+\mu-\nu}{2}, \ \bar{\nu} = \frac{1-\mu+\nu}{2}.$$

The hesitation margin $\bar{\pi}$ in this case equals to 0. The property (P1) is fulfilled. Way 7 is obtained by Atanassov as some modification of Way 2.

If we replace the operator @ by some its generalization $@_\lambda$, defined in the form

$$\langle a, b\rangle \,@_\lambda\, \langle c, d\rangle = \langle \frac{a+c+\lambda-1}{2\lambda}, \frac{b+d+\lambda-1}{2\lambda} \rangle,$$

where $\lambda \geq 1$ is a real parameter, we obtain a class of transformations of the unconscientious evaluations.

First, we should note, that the operator $@_\lambda$: IFVs × IFVs → IFVs, where IFVs is a set of intuitionistic fuzzy values, is well defined.

It is

$$\frac{a+c+\lambda-1}{2\lambda}, \frac{b+d+\lambda-1}{2\lambda} \in [\frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}],$$

and

$$\frac{\lambda-1}{2\lambda} \in [0, \tfrac{1}{2}), \quad \frac{\lambda+1}{2\lambda} \in (\tfrac{1}{2}, 1].$$

Also the sum

$$\frac{a+c+\lambda-1}{2\lambda} + \frac{b+d+\lambda-1}{2\lambda} \in [0, 1],$$

because

$$0 \le 1 - \frac{1}{\lambda} \le \frac{2\lambda - 2}{2\lambda} \le \frac{a + c + b + d + 2\lambda - 2}{2\lambda} \le \frac{2 + 2\lambda - 2}{2\lambda} = 1.$$

Of course $@ = @_\lambda$ for $\lambda = 1$.

Using the $@_\lambda$ operator for pairs $\langle \mu, \nu \rangle$ and $\langle 1 - \nu, 1 - \mu \rangle$, we obtain:

$$\langle \overline{\mu}_\lambda, \overline{\nu}_\lambda \rangle = \langle \mu, \nu \rangle \, @_\lambda \, \langle 1 - \nu, 1 - \mu \rangle = \langle \frac{\lambda + \mu - \nu}{2\lambda}, \frac{\lambda + \nu - \mu}{2\lambda} \rangle.$$

As before, we note that the operator $@_\lambda$, defined for IF values, must be extended for the UE $\langle \mu, \nu \rangle$ and the IF value $\langle 1 - \nu, 1 - \mu \rangle$. But

$$\frac{\lambda + \mu - \nu}{2\lambda} \in (0, 1), \ \frac{\lambda + \nu - \mu}{2\lambda} \in (0, 1),$$

and

$$\frac{\lambda + \mu - \nu}{2\lambda} + \frac{\lambda + \nu - \mu}{2\lambda} = 1,$$

therefore,

$$\langle \frac{\lambda + \mu - \nu}{2\lambda}, \frac{\lambda + \nu - \mu}{2\lambda} \rangle$$

is an IF value.

Way 8. As the result of above reasoning we can calculate the corrected degrees as

$$\overline{\mu}_\lambda = \frac{\lambda + \mu - \nu}{2\lambda}, \ \overline{\nu}_\lambda = \frac{\lambda + \nu - \mu}{2\lambda}.$$

In this case is always $\overline{\pi}_\lambda = 1 - \overline{\mu}_\lambda - \overline{\nu}_\lambda = 0$.

The property (P1) holds because

$$\mu \ge \nu \ \textit{iff} \ \mu - \nu \ge \nu - \mu \ \textit{iff} \ \frac{\lambda + \mu - \nu}{2\lambda} \ge \frac{\lambda + \nu - \mu}{2\lambda} \ \textit{iff} \ \overline{\mu}_\lambda \ge \overline{\nu}_\lambda.$$

The special case of Way 8 is given by Atanassov as Way 1. Namely, if we replace the parameter $\lambda \ge 1$ by the sum $\mu + \nu > 1$, we obtain

$$\overline{\mu} = \frac{\mu}{\mu + \nu}, \quad \overline{\nu} = \frac{\nu}{\mu + \nu}.$$

The other correction is given as Way 9.

Way 9. We calculate the corrected degrees as:

$$\overline{\mu} = \frac{1 + score(\langle \mu, \nu \rangle)}{2 \cdot accuracy(\langle \mu, \nu \rangle)} = \frac{1 + \mu - \nu}{2(\mu + \nu)},$$

$$\overline{\nu} = \frac{1 - score(\langle \mu, \nu \rangle)}{2 \cdot accuracy(\langle \mu, \nu \rangle)} = \frac{1 - \mu + \nu}{2(\mu + \nu)}.$$

In the *unconscientious* experts' case, we have $\mu + \nu > 1$, so

$$0 < \frac{1 + \mu - \nu}{2(\mu + \nu)} < 1$$

and, similarly,

$$0 < \frac{1-\mu+\nu}{2(\mu+\nu)} < 1.$$

Additionally, we have

$$\bar{\pi} = 1 - \bar{\mu} - \bar{\nu} = 1 - \frac{1}{\mu+\nu} \in (0, \tfrac{1}{2}],$$

and *accuracy*($\langle \bar{\mu}, \bar{\nu} \rangle$)$\in [\tfrac{1}{2}, 1)$. The hesitation margin $\bar{\pi}$ is an increasing function of the sum $\mu + \nu$.

The property (P1) holds because:

$\langle \mu, \nu \rangle$ is an IFT *iff* $\mu \geq \nu$ *iff* $\frac{1+\mu-\nu}{2(\mu+\nu)} \geq \frac{1-\mu+\nu}{2(\mu+\nu)}$ *iff* $\langle \bar{\mu}, \bar{\nu} \rangle$ is an IFT.

The boundary values obtained by Way 1, Way 2, and Ways 6 ÷ 9 correction are given in the Table 1.

For comparison of the methods we calculate corrected values for the specific unconscientious evaluations – as in Table 2.

Table 1. The corrected values $\langle \bar{\mu}, \bar{\nu} \rangle$ in the boundary cases.

Method of conversion	**Boundary unconscientious evaluations**		
	$\mu + \nu = 1^+$	$\mu = 1$ **and** $\nu \in (0, 1]$	$\nu = 1$ **and** $\mu \in (0, 1]$
Way 1	$\langle \mu^-, \nu^- \rangle$	$\langle \frac{1}{1+\nu}, \frac{\nu}{1+\nu} \rangle$	$\langle \frac{\mu}{1+\mu}, \frac{1}{1+\mu} \rangle$
Way 2	$\langle \frac{\mu}{2}, 1^+ - \frac{3\mu}{2} \rangle$, for $\mu \leq \nu$ $\langle 1^+ - \frac{3\nu}{2}, \frac{\nu}{2} \rangle$, for $\mu \geq \nu$	$\langle \frac{2-\nu}{2}, \frac{\nu}{2} \rangle$	$\langle \frac{\mu}{2}, \frac{2-\mu}{2} \rangle$
Way 6	$\langle \mu^-, \nu^- \rangle$	$\langle 1-\nu, 0 \rangle$	$\langle 0, 1-\mu \rangle$
Way 7	$\langle \mu^-, \nu^- \rangle$	$\langle 1-\frac{\nu}{2}, \frac{\nu}{2} \rangle$	$\langle \frac{\mu}{2}, 1-\frac{\mu}{2} \rangle$
Way 8	$\langle \frac{\lambda+2\mu-1^+}{2\lambda}, \frac{\lambda+2\nu-1^+}{2\lambda} \rangle$	$\langle \frac{\lambda+1-\nu}{2\lambda}, \frac{\lambda-1+\nu}{2\lambda} \rangle$	$\langle \frac{\lambda-1+\mu}{2\lambda}, \frac{\lambda+1-\mu}{2\lambda} \rangle$
Way 9	$\langle \mu^-, \nu^- \rangle$	$\langle \frac{1-\frac{\nu}{2}}{1+\nu}, \frac{\frac{\nu}{2}}{1+\nu} \rangle$	$\langle \frac{\frac{\mu}{2}}{1+\mu}, \frac{1-\frac{\mu}{2}}{1+\mu} \rangle$

Table 2. The special cases of the corrected values $\langle \overline{\mu}, \overline{\nu} \rangle$.

Method of conversion	**The specific unconscientious evaluations**		
	$\langle \mathbf{1, 0^+} \rangle$	$\langle \mathbf{0^+, 1} \rangle$	$\langle \mathbf{1, 1} \rangle$
Way 1	$\langle 1^-, 0^+ \rangle$	$\langle 0^+, 1^- \rangle$	$\langle \frac{1}{2}, \frac{1}{2} \rangle$
Way 2	$\langle 1^-, 0^+ \rangle$	$\langle 0^+, 1^- \rangle$	$\langle \frac{1}{2}, \frac{1}{2} \rangle$
Way 6	$\langle 1^-, 0 \rangle$	$\langle 0, 1^- \rangle$	$\langle 0, 0 \rangle$
Way 7	$\langle 1^-, 0^+ \rangle$	$\langle 0^+, 1^- \rangle$	$\langle \frac{1}{2}, \frac{1}{2} \rangle$
Way 8	$\langle \frac{\lambda + 1^-}{2\lambda}, \frac{\lambda - 1^-}{2\lambda} \rangle$	$\langle \frac{\lambda - 1^-}{2\lambda}, \frac{\lambda + 1^-}{2\lambda} \rangle$	$\langle \frac{1}{2}, \frac{1}{2} \rangle$
Way 9	$\langle 1^-, 0^+ \rangle$	$\langle 0^+, 1^- \rangle$	$\langle \frac{1}{4}, \frac{1}{4} \rangle$

3 Conclusion

In the intuitionistic fuzzy environment unconscientious opinions may cause problems in the data processing. In this paper new ways of correction of the unconscientious experts evaluations are proposed. The basic property, which should be fulfilled in order to the conversion can being considered as proper, is given.

References

[1] Atanassov, K. T., *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.