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Intuitionistic Fuzzy Voronoi Diagrams – Definition and Properties II.

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Abstract: It is defined new theorems over Intuitionistic Fuzzy Voronoi diagrams (IFVD). The new theorems of IFVD are proved. The base of IFVD theorems is extended.

Keywords: Voronoi Diagrams, Intuitionistic Fuzzy Voronoi Diagrams.

In article[1] we proved properties and theorems over IFVD. In this article we will continue with proofs of some new theorems over IFVD.

Theorem 2. Let P is set of different points in the plane and over the set is built Intuitionistic Fuzzy Voronoi Diagram. The average number of edges on the borderline of Voronoi region is smaller than 6.

Proof: Let's apply Euler's formula [2] about reduced IFVD. Every vertex has at least 3 edges connected with it [Fig.1].

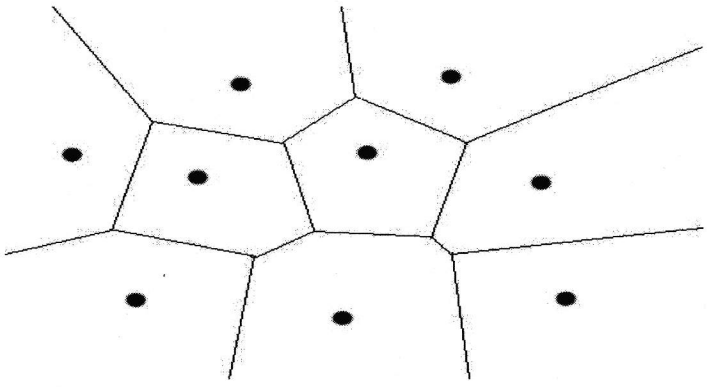

Fig.1. IFVD diagram

Since every edge is counted 2 times then we have :

$$n_e \geq \frac{3n_v}{2}. \tag{1}$$

Replacing this inequality (1) into Euler's formula and taking into consideration that $n_f = n+1$ then we will receive:

$$n_v \leq 2n-2 \text{ and } n_e \leq 3n-3, \tag{2}$$

n_v is number of nodes, n_e is number of arcs and n_f is number of faces.

Summing edges of all *n+1* cells, every edge will be counted two times and the received result will be $2n_e \leq 6n-6$. In that case the average number of edges in closed region will be reduced from

$$\frac{(6n-6)}{(n+1)} < 6. \tag{3}$$

Theorem 3. Let P is set of n different points in a plane. If all points are collinear then IFVD will be consisted of *n-1* parallel lines. In other case IFVD will contain segments or/and half-lines.

Proof: According to property 4 [1] if P is set from n different collinear points in a plane then the IFVD will consist of *n-1* parallel lines i.e.

$$n_v = 0,\ n_e = n-1. \tag{4}$$

According to definition IFVD [1] has like edges part of lines or/and half-lines parallel to symmetrical axises of segments connecting every two points of set P, i.e. edges of IFVD are parallel of edges of Voronoi Diagrams, on which IFVD is derivated. Hence in this case IFVD will consist of *n-1* parallel lines.

Now we will direct our attention to common case when all points are not collinear. Let us presume that IFVD has an edge e which is a line. Let e is borderline between Intuitionistic Fuzzy Voronoi cells $IFV(p_i)$ and $IFV(p_j)$. Let $p_k \in P$ and is not collinear with p_i and p_j. Symmetrical axis of segment connecting points is not parallel to e. Therefore the edge of IFVD which is parallel or coincide with the diagram also is not parallel to e and consequently cross it. Hence permission that in Voronoi diagram has an edge which is a line is mistaken. Therefore we can conclude that edges of IFVD are segment or/and half-lines.

The last thing we have to prove is that IFVD is connected. If we have a case in which this is feasible then will exist Intuitionistic Fuzzy Voronoi cell $IFV(p_i)$ which

separates plane on a half. Since IFVD cell's is convex then $IFV(p_i)$ will be separated and will consist of two parallel lines. But we proved that edge of IFVD can not be a line and now we have a contradiction. Hence permission is wrong and therefore IFVD is connected.

References

1. Todorova, L., A. Antonov, St. Hadjitodorov. Intuitionistic fuzzy Voronoi Diagrams-definition and properties. Proceedings of the Eighth International Conference on Intuitionistic Fuzzy Sets- part 2, Varna, 20-21 June, pp. 56-60, 2004.
2. Gibbons, A.. *Algorithmic Graph Theory*. Cambridge Universite Press, Cambridge, 1985.