Notes on Intuitionistic Fuzzy Sets
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On statistical concepts of intuitionistic fuzzy soft set theory via utility

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Abstract: In this paper, we establish the foundation of intuitionistic fuzzy soft statistics with the help of utility theory of mathematical economics. We use ideas of (α, β)-cut with respect to utility theory to prove results related to intuitionistic fuzzy soft mean, intutionistic fuzzy soft covariance, intuitionistic fuzzy soft attribute correlation coefficients, etc. Suitable examples are provided in each case. Concepts of utility-wise representation of intuitionistic fuzzy soft have been discussed. Here, we also discuss the generating process of a new intuitonistic fuzzy soft set from the old one with respect to utility theory and prove some important theorems.
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1 Introduction

Mathematical theories are based on various abstract ideas. Here, one has freedom to develop certain abstract environments by neglecting many facts; for example in physics, the frictional effect of air on a free falling body is often neglected to make the calculations easier, but this fact is fully impossible in real life. Similarly, medical science, economics, engineering, social sciences, etc., are full of uncertainties. Zadeh [19] initiated the study of uncertainties with the introduction of fuzzy sets in 1965. Later, Atanassov introduced intuitionistic fuzzy set theory [5].

Molodtsov [14] introduced the concept of soft set theory in the year 1999 and investigated various applications in game theory, smoothness of functions, operation researches, Perron integration, probability theory, theory of measurement, etc.

Later Maji et al. [10] defined fundamental operations of soft sets. Pei and Miao [16], Chen [7] pointed out errors in some of the results of Maji et al. [11] and introduced some new notions and properties. At present, investigations of different properties and applications of soft set theory have attracted many researchers from various backgrounds. Since then many applications of soft set theory can be found in other branches of science and social science. Fuzzy soft set was introduced by Maji et al. [10] as a hybrid structure of soft set with fuzzy set. Later, Intuitionistic fuzzy soft sets were introduced by Maji et al. [12]. One may refer to Mitchell [13], Szmidt and Kacprzyk [17], Huang [8] for researches related to correlation coefficients on intuitionistic fuzzy sets. One may refer to [9, 18] for some works on fuzzy soft sets.

Applications of uncertainty-based statistical ideas related to sociological issues viz. human trafficking and illegal immigration can found in Acharjee and Mordeson [3], Mordeson et al. [15], Acharjee et al. [4]. Moreover, hybrid structures related to soft set can be found in Acharjee [1], Acharjee and Tripathy [2], and many others.

In this paper, we establish the foundation of intuitionistic fuzzy soft statistics. Here, we try to connect two different domains of nature, i.e., uncertainties, which are present in large scale data, and preference (i.e., utility based on human choice behavior) by developing statistical ideas based on intuitionistic fuzzy soft set theory. Utility theory is applicable in various domains of computational social sciences and information systems. One may find uses of utility theory in computational social choice theory, mathematical psychology, robotics, decision making, etc. It is to be understood that our statistical ideas connect attributes, linguistic variables, [0,1], etc. and, thus, it may have potentiality of applications in various areas of science and social science.

2 Preliminaries

The following definitions are due to Çağman et al. [6].

Definition 2.1. [6] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x) : x \in E, f_A(x) \in P(U)\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, f_A is called an *approximate function* of the soft set F_A. The value of $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. We will denote the set of all soft sets over U as $S(U)$.

Definition 2.2. [6] Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called a *soft empty set*, denoted by $F_\emptyset$. $f_A(x) = \emptyset$ means there is no element in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 2.3. [6] Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called an *A-universal soft set*, denoted by $F_{\widetilde{A}}$.

If $A = E$, then the A-universal soft set is denoted by $F_{\widetilde{E}}$.

Definition 2.4. [6] Let $F_A, F_B \in S(U)$. Then, F_A is a *soft subset* of F_B, denoted by $F_A \widetilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.5. [6] Let $F_A, F_B \in S(U)$. Then, F_A and F_B are soft equal, denoted by F_A=F_B, if and only if $f_A(x)$=$f_B(x)$ for all $x \in E$.

Definition 2.6. [6] Let $F_A, F_B \in S(U)$. Then, the *soft union* $F_A \widetilde{\cup} F_B$, *the soft intersection* $F_A \widetilde{\cap} F_B$ and the *soft difference* $F_A \widetilde{\setminus} F_B$ of F_A and F_B are defined by the approximation functions $f_{A\widetilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\widetilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ and $f_{A\widetilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$, respectively, and the soft complement $F_A^{\widetilde{c}}$ of F_A is defined by the approximate function, $f_A^{\widetilde{c}}(x) = f_A^c(x)$, where $f_A^c(x)$ is the complement of the set $f_A(x)$; that is $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

It is easy to see that $(F_A^{\widetilde{c}})^{\widetilde{c}} = F_A$ and $F_{\emptyset}^{\widetilde{c}} = F_{\widetilde{E}}$.

Example 2.1. [6] Let us consider a universe $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$. Let $A = \{e_1, e_2, e_3\}$. We define a soft set $(F, A) = \{(e_1, \{a, b\}), (e_2, \{a, c\}), (e_3, \{a, b, c\})\}$.

Then, the representation of (F, A) in tabular form is shown in Table 1:

	$F(e_1)$	$F(e_2)$	$F(e_3)$
a	1	1	1
b	1	0	1
c	0	1	1

Table 1. The representation of (F, A)

Definition 2.7. [19] Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be the universe of discourse; then a fuzzy set A in X is defined as $A = \{\langle x, \mu_A(x)\rangle | x \in X\}$ where $\mu_A : X \to [0, 1]$ is the membership degree.

Definition 2.8. [5] An intuitionistic fuzzy set A in X can be written as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ are the membership degree and non-membership degree, respectively, satisfying the requirement $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Then, $1 - \mu_A(x) - \nu_A(x)$ is called the hesitancy degree of the element $x \in X$ to the set A; denoted by $\pi_A(x)$. $\pi_A(x)$ is called the intuitionistic index of x to A. Greater $\pi_A(x)$ indicates more vagueness on x. Obviously, when $\pi_A(x) = 0$ $\forall x \in X$, the IFS generates into an ordinary fuzzy set. In the sequel, all IFSs of X is denoted by IFSs(X).

Definition 2.9. [5] For $A \in$ IFSs(X) and $B \in$ IFSs(X), some relations between them are defined as:

(i) $A \subseteq B$ iff $\forall\, x \in X, \mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x)$

(ii) $A = B$ iff $\forall$, $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$,

(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$, where A^c is the complement of A.

3 Main results

In this section, we introduce utility based statistical concepts in intuitionistic fuzzy soft sets. Throughout this paper, we shall write IFSS and IFS in short to represent "intuitionistic fuzzy soft set" and "intuitionistic fuzzy set", respectively. We shall denote $I = \{1, 2, 3, \ldots, n\}$

3.1 Some new definitions

Definition 3.1. If (F, A) be an IFSS over a universe U, where $F(e_i)$ is an IFS for the attribute $e_i \in A, i \in I$, then the intuitionistic fuzzy soft mean of (F, A) is denoted by $\widetilde{F_A} = \{(A, F(A))\}$; where $F(A) = \{\frac{x_k}{(\min\{a_k^i\}, \max\{b_k^i\})} \, |k \in \Delta, i \in I\}$.

Here, a_k^i and b_k^i are membership value and non-membership value of x_k, respectively, for the attribute e_i, where $e_i \in A$.

If $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$; then $(\alpha, \beta)\overrightarrow{F_A} = ((\alpha_1, \beta_1), (\alpha_2, \beta_2), \ldots, (\alpha_n, \beta_n), \ldots)$; where $\alpha_k = 1$ if $\min\{a_k^i\} \geq \alpha$; otherwise 0 and $\beta_k = 0$ if $\max\{b_k^i\} \leq \beta$; otherwise 1 for $i \in I$.

Example 3.1. Let us consider an IFSS (F, A) over a universe U, where $(F, A) = \{(e_1, \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 0.6, 0.4\rangle, \langle x_3, 0.3, 0.3\rangle\}), (e_2, \{\langle x_1, 0.3, 0.6\rangle, \langle x_2, 0.6, 0.4\rangle, \langle x_3, 0.4, 0.1\rangle\}), (e_3, \{\langle x_1, 0.3, 0.5\rangle, \langle x_2, 0.6, 0.4\rangle, \langle x_3, 0.2, 0.1\rangle\})\}$.

Then, $(0.3, 0.2)\overrightarrow{F_A}$= ((1, 1), (1, 1), (0, 1)); $(0.4, 0.7)\overrightarrow{F_A}$= ((0, 0), (1, 0), (0, 0)).

Definition 3.2. If (F, A) be an IFSS and $e_i \in A, i \in I$, then $(\alpha, \beta)\overrightarrow{F(e_i)} = (\gamma_1, \gamma_2, \ldots, \gamma_j, \ldots)$, where $\gamma_j = (\alpha_j, \beta_j)$. In this case, $\alpha_j = 1$ if $F_j^1(e_i)(x_j) \geq \alpha$ and 0; otherwise. Again, $\beta_j = 0$ if $F_j^2(e_i)(x_j) \leq \beta$ and 1; otherwise.

Here, $F_j^1(e_i)(x_j)$ indicates the membership value of x_j in j^{th} place of $F(e_i)$ $\forall j \in \Delta$. Similarly, $F_j^2(e_i)(x_j)$ indicates the non-membership value of x_j in j^{th} place of $F(e_i)$ $\forall j \in \Delta$.

Example 3.2. Consider Example 3.1, here $(0.3, 0.2)\overrightarrow{F(e_1)}$ = ((1,0), (1,1), (1,1)), $(0.4, 0.5)\overrightarrow{F(e_2)}$ = ((0,1), (1,0), (1,0))

Definition 3.3. Let (F, A) be an IFSS, then scale of (F, A) is denoted by h and it is defined as $h = \max\{F_j^1(e_i)(x_j), F_j^2(e_i)(x_j) | e_i \in A, x_j \in U, i \in I, j \in \Delta\}$.

Example 3.3. Consider Example 3.1, here $h0.9$.

Definition 3.4. Let U be a universe and E be the set of attributes, where $A \subseteq E$ and $|A| = n$. If (F, A) be an IFSS over U, then (α, β)-cut IFSS standard deviation of (F, A) is denoted by $\sigma((\alpha, \beta)\overrightarrow{(F, A)})$ and it is defined as

$$\left(\sqrt{\frac{1}{n} \sum_{i \in I, k \in \Delta} || \, (\alpha, \beta)\overrightarrow{F_k^1(e_i)} - (\alpha, \beta)\overrightarrow{F_{k,A}^1} \, ||^2}, \sqrt{\frac{1}{n} \sum_{i \in I, k \in \Delta} || \, (\alpha, \beta)\overrightarrow{F_k^2(e_i)} - (\alpha, \beta)\overrightarrow{F_{k,A}^2} \, ||^2} \right),$$

where $|| \, (\alpha, \beta)\overrightarrow{F_k^1(e_i)} - (\alpha, \beta)\overrightarrow{F_{k,A}^1} \, ||^2 = \langle (\alpha, \beta)\overrightarrow{F_k^1(e_i)} - (\alpha, \beta)\overrightarrow{F_{k,A}^1}, (\alpha, \beta)\overrightarrow{F_k^1(e_i)} - (\alpha, \beta)\overrightarrow{F_{k,A}^1} \rangle$ and so on for other part.

Here, $(\alpha, \beta)\overrightarrow{F_k^1(e_i)}$ and $(\alpha, \beta)\overrightarrow{F_{k,A}^1}$ indicate the first coordinate of $(\alpha, \beta)\overrightarrow{F_k(e_i)}$ and $(\alpha, \beta)\overrightarrow{F_{k,A}}$ respectively, where k indicates the k-th ordered pair representation of the membership and the non-membership values for $(\alpha,\beta)\overrightarrow{F_k(e_i)}$ and $(\alpha,\beta)\overrightarrow{F_A}$, respectively, with respect to (α,β)-cut. Similarly, we can describe other case.

Example 3.4. Let us consider Example 3.1., where

$$(F,A) = (e_1, \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 0.5, 0.4\rangle, \langle x_3, 0.3, 0.3\rangle\}), (e_2, \{\langle x_1, 0.4, 0.6\rangle, \langle x_2, 0.5, 0.5\rangle, \langle x_3, 0.4, 0.3\rangle\}), (e_3, \{\langle x_1, 0.3, 0.7\rangle, \langle x_2, 0.6, 0.4\rangle, \langle x_3, 0.2, 0.1\rangle\})\}.$$

Let $\alpha = 0.3, \beta = 0.2$; then $(0.3,0.2)\overrightarrow{F(e_1)} = ((1,0),(1,1),(1,1))$; $(0.3,0.2)\overrightarrow{F(e_2)} = ((1,1),(1,1),(1,1))$ and $(0.3,0.2)\overrightarrow{F(e_3)} = ((1,1),(1,1),(0,0))$. Now, $(0.3,0.2)\overrightarrow{F_A} = ((1,1),(1,1),(0,1))$.
Thus,

$$\sqrt{\tfrac{1}{n}\sum_{i\in I,k\in\Delta} ||\,(0.3,0.2)\overrightarrow{F_k^1(e_i)} - (0.3,0.2)\overrightarrow{F_{k,A}^1}\,||^2} = \sqrt{\frac{1}{3}(0+0+0+0+0+0+1+1+0)} = \sqrt{\frac{2}{3}}$$

and

$$\sqrt{\tfrac{1}{n}\sum_{i\in I,k\in\Delta} ||\,(0.3,0.2)\overrightarrow{F_k^2(e_i)} - (0.3,0.2)\overrightarrow{F_{k,A}^2}\,||^2} = \sqrt{\frac{1}{3}(1+0+0+0+0+0+0+0+1)} = \sqrt{\frac{2}{3}}.$$

Thus,

$$\sigma((0.3,0.2)\overrightarrow{(F,A)}) = \left(\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right).$$

The above result indicates that standard deviation of $(0.3,0.2)\overrightarrow{(F,A)}$ is $\sqrt{\frac{2}{3}}$ from both membership and non-membership values.

3.2 Concept of utility-wise representation of intuitionistic fuzzy soft set

Consider an IFSS (F,A) over a universe U and $\mathbf{R}$ is the set of real numbers. We define an α-cut level utility function $\mu_\alpha : U \to \mathbf{R}$ as $x \succeq y \iff \mu_\alpha(x) \geq \mu_\alpha(y)$ for $x, y \in U$ and so on with fundamental notions of utility theory. Similarly, we define a β-cut level utility function $\nu_\beta : U \to \mathbf{R}$ as $x \succeq y \iff \nu_\beta(y) \geq \nu_\beta(x)$ for $x, y \in U$. Together we call them as (α,β)-cut level utility function.

If $F^1(e)(x) \geq \alpha$ and $F^2(e)(x) \leq \beta$ for $x \in U, e \in A$, then the utility wise representation of x in $F(e)$ is shown in Table 2.

	$F(e)$	$F(e)^c$
$x^{(\alpha,\beta)}$	$(\mu_\alpha(x), \nu_\beta(x))$	$(1-\mu_\alpha(x), 1-\nu_\beta(x))$

Table 2. The utility wise representation of x in $F(e)$

If $F^1(e)(x) < \alpha$, then we assume $\mu(x) = 0$ and if $F^2(e)(x) > \beta$, then we assume $\nu(x) = 1$ for the particular case beyond any pre-assumption of $\mu(x)$ and $\nu(x)$. Here, $x^{\alpha,\beta)}$ denotes $x \in U$ with (α,β)-cut level utility.

Example 3.5 Consider an IFSS (F, A), where $(F, A) = (e_1, \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 0.5, 0.4\rangle, \langle x_3, 0.3, 0.3\rangle\}), (e_2, \{\langle x_1, 0.4, 0.6\rangle, \langle x_2, 0.5, 0.5\rangle, \langle x_3, 0.4, 0.3\rangle\}), (e_3, \{\langle x_1, 0.3, 0.7\rangle, \langle x_2, 0.6, 0.4\rangle, \langle x_3, 0.2, 0.1\rangle\})\}$.

We define $(0.3, 0.2)$-cut level utility function as 0.3-cut level utility function $\mu_{0.3} : U \to \mathbf{R}$ as $\mu_{0.3}(x_1) = 5$, $\mu_{0.3}(x_2) = -2$, $\mu_{0.3}(x_3) = 3$, and 0.2-cut level utility function $\nu_{0.2} : U \to \mathbf{R}$ as $\nu_{0.2}(x_1) = 6$, $\nu_{0.2}(x_2) = 1$, $\nu_{0.2}(x_3) = 2$.

Then, the utility wise representation of (F, A) at $(0.3, 0.2)$-cut level is shown in Table 3.

	$F(e_1)$	$F(e_2)$	$F(e_3)$
$x_1^{(0.3,0.2)}$	$(5, 6)$	$(5, 1)$	$(5, 1)$
$x_2^{(0.3,0.2)}$	$(-2, 1)$	$(-2, 1)$	$(-2, 1)$
$x_3^{(0.3,0.2)}$	$(3, 1)$	$(3, 1)$	$(0, 2)$

Table 3. The utility wise representation of (F, A) at $(0.3, 0.2)$-cut level

In this case, we call $(5, -2, 3)$ as membership utility origin of (F, A) and $(6, 1, 2)$ as non-membership utility origin of (F, A) at 0.3-cut utility level of membership and 0.2-cut utility level of non-membership, respectively.

3.3 Generating process of a new intuitionistic fuzzy soft set from the old one with respect to utility based (α, β)-cut

(1) Consider an IFSS (F, A), whose (α, β)-cut level representation is denoted by $(\alpha, \beta)\overrightarrow{(F, A)}$ over a universe U with elements x_i, where $i \in \Delta$,$e_j \in A, j \in I$.
Then, we can generate an IFSS $(G_F, A)_{(\alpha,\beta)}$ with (α, β)-cut level of representation $(\alpha, \beta)\overrightarrow{(G_F, A)}$ if $(\gamma_1, \gamma_2, \dots, \gamma_i, \dots)$+$(\alpha, \beta)\overrightarrow{(F, A)}$ exists, where $(\gamma_1, \gamma_2, \dots, \gamma_i, \dots) \neq (\tilde{0}, \tilde{0}, \tilde{0}, \dots, \tilde{0}, \dots)$, $\gamma_i \in \mathbf{R} \times \mathbf{R}$, $i \in \Delta$. Here, $\tilde{0} = (0, 0)$ and $\gamma_i = (\alpha_i, \beta_i)$.

Now, we discuss the generating process. We define (α, β)-cut level utility function where α-cut level utility function is $\mu_\alpha : U \to \mathbf{R}$ such that if $\alpha_i + F_i^1(e_j) \geq \mu_\alpha(x_i), i \in \Delta, j \in I$, then $x_i \in G_i(e_j)$ with the membership value of $\min\{$membership value of x_i in $F_i(e_j), \alpha\}$. Otherwise, x_i has membership value 0 in $G_i(e_j)$.

Similarly, we can define β-cut level utility function $\nu_\beta : U \to \mathbf{R}$ such that if $\beta_i + F_i^2(e_j) \leq \nu_\beta(x_i), i \in \Delta, j \in I$, then $x_i \in G_i(e_j)$ with the non-membership value of $\min\{$non-membership value of x_i in $F_i(e_j), \beta\}$. Otherwise, x_i has non-membership value 0 in $G_i(e_j)$. Now,

$$(\gamma_1, \gamma_2, \dots, \gamma_j, \dots) + (\alpha, \beta)\overrightarrow{(F, A)} = ((\alpha_1, \beta_1), (\alpha_2, \beta_2), \dots, (\alpha_j, \beta_j), \dots) + \{((\alpha, \beta)\overrightarrow{F_1(e_1)}, (\alpha, \beta)\overrightarrow{F_2(e_1)}, \dots, (\alpha, \beta)\overrightarrow{F_j(e_1)}, \dots), ((\alpha, \beta)\overrightarrow{F_1(e_2)}, (\alpha, \beta)\overrightarrow{F_2(e_2)}, \dots, (\alpha, \beta)\overrightarrow{F_j(e_2)}, \dots), \dots, ((\alpha, \beta)\overrightarrow{F_1(e_n)}, (\alpha, \beta)\overrightarrow{F_2(e_n)}, \dots, (\alpha, \beta)\overrightarrow{F_j(e_n)}, \dots)\}$$
$$= \{((\alpha_1, \beta_1) + (\alpha, \beta)\overrightarrow{F_1(e_1)}, (\alpha_2, \beta_2) + (\alpha, \beta)\overrightarrow{F_2(e_1)}, \dots, (\alpha_j, \beta_j) + (\alpha, \beta)\overrightarrow{F_j(e_1)}, \dots), ((\alpha_1, \beta_1) + (\alpha, \beta)\overrightarrow{F_1(e_2)}, (\alpha_2, \beta_2) + (\alpha, \beta)\overrightarrow{F_2(e_2)}, ..., (\alpha_j, \beta_j) + (\alpha, \beta)\overrightarrow{F_j(e_2)}), \dots), \dots,$$

$((\alpha_1,\beta_1)+(\alpha,\beta)\overrightarrow{F_1(e_n)},(\alpha_2,\beta_2)+(\alpha,\beta)\overrightarrow{F_2(e_n)},\ldots,(\alpha_j,\beta_j)+(\alpha,\beta)\overrightarrow{F_j(e_n)},\ldots))\}.$

$=\{((\alpha_1+(\alpha,\beta)\overrightarrow{F_1^1(e_1)},\beta_1+(\alpha,\beta)\overrightarrow{F_1^2(e_1)}),(\alpha_2+(\alpha,\beta)\overrightarrow{F_2^1(e_1)},\beta_2+(\alpha,\beta)\overrightarrow{F_2^2(e_1)}),\ldots,$
$(\alpha_j+(\alpha,\beta)\overrightarrow{F_j^1(e_1)},\beta_j+(\alpha,\beta)\overrightarrow{F_j^2(e_1)}),\ldots),((\alpha_1+(\alpha,\beta)\overrightarrow{F_1^1(e_2)},\beta_1+(\alpha,\beta)\overrightarrow{F_1^2(e_2)}),(\alpha_2+$
$(\alpha,\beta)\overrightarrow{F_2^1(e_2)},\beta_2+(\alpha,\beta)\overrightarrow{F_2^2(e_2)}),\ldots,(\alpha_j+(\alpha,\beta)\overrightarrow{F_j^1(e_2)},\beta_j+(\alpha,\beta)\overrightarrow{F_j^2(e_2)}),\ldots),\ldots,$
$((\alpha_1+(\alpha,\beta)\overrightarrow{F_1^1(e_n)},\beta_1+(\alpha,\beta)\overrightarrow{F_1^2(e_n)}),(\alpha_2+(\alpha,\beta)\overrightarrow{F_2^1(e_n)},\beta_2+(\alpha,\beta)\overrightarrow{F_2^2(e_n)}),\ldots,$
$(\alpha_j+(\alpha,\beta)\overrightarrow{F_j^1(e_n)},\beta_j+(\alpha,\beta)\overrightarrow{F_j^2(e_n)}),\ldots))\}.$

The new IFSS $(G_F,A)_{(\alpha,\beta)}$ with representation $(\alpha,\beta)\overrightarrow{(G_F,A)}=(\gamma_1,\gamma_2,\ldots,\gamma_j,\ldots)$+$(\alpha,\beta)\overrightarrow{(F,A)}$ is called (α,β)-cut generated fuzzy soft set of (F,A). The intuitionistic fuzzy soft mean of $(G_F,A)_{(\alpha,\beta)}$ is denoted by $\widetilde{G_A}$ with (α,β)-cut level representation $(\alpha,\beta)\overrightarrow{G_A}$.

(2) Let the scale of (F,A) be h. Then, similarly as discussed above, we can construct another (α,β)-cut generated IFSS $(G_F,A)^h_{(\alpha,\beta)}$ of (F,A) with (α,β)-cut level representation $(\alpha,\beta)\overrightarrow{(G_F,A)}_h=\frac{(\alpha,\beta)\overrightarrow{(G_F,A)}}{h}$ and intuitionistic fuzzy soft mean $\widetilde{G_A}^h$.

Now,

$$(\alpha,\beta)\overrightarrow{(G_F,A)}_h=$$
$$\{((\frac{\alpha_1+(\alpha,\beta)\overrightarrow{F_1^1(e_1)}}{h},\frac{\beta_1+(\alpha,\beta)\overrightarrow{F_1^2(e_1)}}{h}),(\frac{\alpha_2+(\alpha,\beta)\overrightarrow{F_2^1(e_1)}}{h},\frac{\beta_2+(\alpha,\beta)\overrightarrow{F_2^2(e_1)}}{h}),\ldots,$$
$$(\frac{\alpha_j+(\alpha,\beta)\overrightarrow{F_j^1(e_1)}}{h},\frac{\beta_j+(\alpha,\beta)\overrightarrow{F_j^2(e_1)}}{h}),\ldots),((\frac{\alpha_1+(\alpha,\beta)\overrightarrow{F_1^1(e_2)}}{h},\frac{\beta_1+(\alpha,\beta)\overrightarrow{F_1^2(e_2)}}{h}),$$
$$(\frac{\alpha_2+(\alpha,\beta)\overrightarrow{F_2^1(e_2)}}{h},\frac{\beta_2+(\alpha,\beta)\overrightarrow{F_2^2(e_2)}}{h}),\ldots,(\frac{\alpha_j+(\alpha,\beta)\overrightarrow{F_j^1(e_2)}}{h},\frac{\beta_j+(\alpha,\beta)\overrightarrow{F_j^2(e_2)}}{h}),\ldots)$$
$$,\ldots,((\frac{\alpha_1+(\alpha,\beta)\overrightarrow{F_1^1(e_n)}}{h},\frac{\beta_1+(\alpha,\beta)\overrightarrow{F_1^2(e_n)}}{h}),(\frac{\alpha_2+(\alpha,\beta)\overrightarrow{F_2^1(e_n)}}{h},\frac{\beta_2+(\alpha,\beta)\overrightarrow{F_2^2(e_n)}}{h}),\ldots,$$
$$(\frac{\alpha_j+(\alpha,\beta)\overrightarrow{F_j^1(e_n)}}{h},\frac{\beta_j+(\alpha,\beta)\overrightarrow{F_j^2(e_n)}}{h}),\ldots)\}$$

Example 3.6. Let us consider an IFSS (F,A) where $(F,A)=\{(e_1,\{\langle x_1,0.5,0.4\rangle,\langle x_2,0.3,0.1\rangle\}),(e_2,\{\langle x_1,0.4,0.6\rangle,\langle x_2,0.3,0.5\rangle\})$. We define $(0.3,0.1)$-cut level utility as follows: $\mu_{0.3}(x_1)=2,\mu_{0.3}(x_2)=-2,\nu_{0.1}(x_1)=1,\nu_{0.1}(x_2)=2$.

Then, $(0.3,0.1)\overrightarrow{(F,A)}=\{((1,1),(1,0)),((1,1),(1,1))\}$. If we consider $(\gamma_1,\gamma_2)=((1,4),(2,-2))$, then $(\gamma_1,\gamma_2)+(0.3,0.1)\overrightarrow{(F,A)}=\{((2,5),(3,-2)),((2,5),(3,-1))\}$.
So, $(G_F,A)_{(0.3,0.1)}=\{(e_1,\{\langle x_1,0.3,0\rangle,\langle x_2,0.3,0.1\rangle\}),(e_2,\{\langle x_1,0.3,0\rangle,\langle x_2,0.3,0.1\rangle\})$; which is the new IFSS at $(0.3,0.1)$-level of utility.

Theorem 3.1. Let (F,A) be an IFSS over U and $(G_F,A)_{(\alpha,\beta)}$ be an (α,β)-cut level generated IFSS of (F,A), then $\sigma((\alpha,\beta)\overrightarrow{(F,A)})=\sigma((\alpha,\beta)\overrightarrow{(G_F,A)})$

Proof. Let $(\gamma_1,\gamma_2,\ldots,\gamma_i,\ldots)\neq(\tilde{0},\tilde{0},\tilde{0},\ldots,\tilde{0},\ldots)$ where $\gamma_i\in\mathbf{R}\times\mathbf{R}$, $i\in\Delta$. Here, $\tilde{0}=(0,0)$ and $\gamma_i=(\alpha_i,\beta_i)$.

We define α-cut level utility function as $\mu_\alpha : U \to \mathbf{R}$ as $\mu_\alpha(x_i) = \theta_i$, such that $x_i \in G_i(e_j)$ with membership value $\min$ {membership value of x_i in $F_i(e_j), \alpha$} if $\alpha_i + F_i^1(e_j) \geq \theta_i$, $i \in \Delta, j \in I$.

Similarly, we define β-cut level utility function as $\nu_\beta : U \to \mathbf{R}$ as $\nu_\beta(x_i) = \psi_i$, such that $x_i \in G_i(e_j)$ with non-membership value $\min$ {non-membership value of x_i in $F_i(e_j), \beta$} if $\beta_i + F_i^2(e_j) \leq \psi_i, i \in \Delta, j \in I$.

Now,

$$(\alpha,\beta)\overrightarrow{G_A} = ((\min\{\alpha_1 + (\alpha,\beta)\overrightarrow{F_1^1(e_1)}, \alpha_1 + (\alpha,\beta)\overrightarrow{F_1^1(e_2)}, \ldots, \alpha_1 + (\alpha,\beta)\overrightarrow{F_1^1(e_n)}\}, \min\{\beta_1 + (\alpha,\beta)\overrightarrow{F_1^2(e_1)}, \beta_1 + (\alpha,\beta)\overrightarrow{F_1^2(e_2)}, \ldots, \beta_1 + (\alpha,\beta)\overrightarrow{F_1^2(e_n)}\}), (\min\{\alpha_2 + (\alpha,\beta)\overrightarrow{F_2^1(e_1)}, \alpha_2 + (\alpha,\beta)\overrightarrow{F_2^1(e_2)}, \ldots, \alpha_2 + (\alpha,\beta)\overrightarrow{F_2^1(e_n)}\}, \min\{\beta_2 + (\alpha,\beta)\overrightarrow{F_2^2(e_1)}, \beta_2 + (\alpha,\beta)\overrightarrow{F_2^2(e_2)}, \ldots, \beta_2 + (\alpha,\beta)\overrightarrow{F_2^2(e_n)}\}), \ldots, (\min\{\alpha_j + (\alpha,\beta)\overrightarrow{F_j^1(e_1)}, \alpha_j + (\alpha,\beta)\overrightarrow{F_j^1(e_2)}, \ldots, \alpha_j + (\alpha,\beta)\overrightarrow{F_j^1(e_n)}\}, \min\{\beta_j + (\alpha,\beta)\overrightarrow{F_j^2(e_1)}, \beta_j + (\alpha,\beta)\overrightarrow{F_j^2(e_2)}, \ldots, \beta_j + (\alpha,\beta)\overrightarrow{F_j^2(e_n)}\}), \ldots)$$

$$= ((\alpha_1 + (\alpha,\beta)\overrightarrow{F_1^1(e_{j_1})}, \beta_1 + (\alpha,\beta)\overrightarrow{F_1^2(e_{k_1})}), (\alpha_2 + (\alpha,\beta)\overrightarrow{F_2^1(e_{j_2})}, \beta_2 + (\alpha,\beta)\overrightarrow{F_2^2(e_{k_2})}), \ldots, (\alpha_j + (\alpha,\beta)\overrightarrow{F_j^1(e_{j_j})}, \beta_j + (\alpha,\beta)\overrightarrow{F_j^2(e_{k_j})}, \ldots) \text{ (say)}$$

Thus,

$$\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta} || (\alpha,\beta)\overrightarrow{G_k^1(e_i)} - (\alpha,\beta)\overrightarrow{G_{k,A}^1} ||^2}$$
$$= \sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta} || \alpha_k + (\alpha,\beta)\overrightarrow{F_k^1(e_i)} - \{\alpha_k + (\alpha,\beta)\overrightarrow{F_k^1(e_{j_k})}\} ||^2}$$
$$= \sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta} || (\alpha,\beta)\overrightarrow{F_k^1(e_i)} - (\alpha,\beta)\overrightarrow{F_k^1(e_{j_k})} ||^2}.$$

Similarly, $\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta} || (\alpha,\beta)\overrightarrow{G_k^2(e_i)} - (\alpha,\beta)\overrightarrow{G_{k,A}^2} ||^2}$

$$= \sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta} || \beta_k + (\alpha,\beta)\overrightarrow{F_k^2(e_i)} - \{\beta_k + (\alpha,\beta)\overrightarrow{F_k^2(e_{j_k})}\} ||^2}$$
$$= \sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta} || (\alpha,\beta)\overrightarrow{F_k^2(e_i)} - (\alpha,\beta)\overrightarrow{F_k^2(e_{j_k})} ||^2}.$$

Hence, proved. □

Example 3.7. Let us consider Example 3.6, then $(0.3, 0.1)\overrightarrow{(G_F, A)} = \{((2,5),(3,-2)),((2,5),(3,-1))\}$. Then, $(G_F, A)_{(0.3,0.1)} = \{(e_1, \{\langle x_1, 0.3, 0\rangle, \langle x_2, 0.3, 0.1\rangle\}), (e_2, \{\langle x_1, 0.3, 0\rangle, \langle x_2, 0.3, 0.1\rangle\})\}$, then clearly, $\sigma((0.3, 0.1)\overrightarrow{(F, A)}) = \sigma((0.3, 0.1)\overrightarrow{(G_F, A)}) = (0, \frac{1}{\sqrt{2}})$.

Theorem 3.2. Let (F, A) be an IFSS over U and $(G_F, A)^h_{(\alpha,\beta)}$ be an (α, β)-cut level generated IFSS of (F, A), then $\sigma((\alpha,\beta)\overrightarrow{(G_F, A)^h}) = \frac{1}{h} \times \sigma((\alpha,\beta)\overrightarrow{(G_F, A)})$.

Definition 3.5. Let us consider an IFSS over a universe U with (α, β)-cut level of representation $(\alpha, \beta)\overrightarrow{(F, A)}$; then

$\alpha\overrightarrow{F_A} = (\min\{(\alpha, \beta)\overrightarrow{F_1^1(e_1)}, (\alpha, \beta)\overrightarrow{F_1^1(e_2)}, \ldots, (\alpha, \beta)\overrightarrow{F_1^1(e_n)}\}, \min\{(\alpha, \beta)\overrightarrow{F_2^1(e_1)},$
$(\alpha, \beta)\overrightarrow{F_2^1(e_2)}, \ldots, (\alpha, \beta)\overrightarrow{F_2^1(e_n)}\}, \ldots, \min\{(\alpha, \beta)\overrightarrow{F_j^1(e_1)}, (\alpha, \beta)\overrightarrow{F_j^1(e_2)}, \ldots, (\alpha, \beta)\overrightarrow{F_j^1(e_n)}\}, \ldots)$

and

$\beta\overrightarrow{F_A} = (\min\{(\alpha, \beta)\overrightarrow{F_1^2(e_1)}, (\alpha, \beta)\overrightarrow{F_1^2(e_2)}, \ldots, (\alpha, \beta)\overrightarrow{F_1^2(e_n)}\}, \min\{(\alpha, \beta)\overrightarrow{F_2^2(e_1)}, (\alpha, \beta)\overrightarrow{F_2^2(e_2)}$
$, \ldots, (\alpha, \beta)\overrightarrow{F_2^2(e_n)}\}, \ldots, \min\{(\alpha, \beta)\overrightarrow{F_j^2(e_1)}, (\alpha, \beta)\overrightarrow{F_j^2(e_2)}, \ldots, (\alpha, \beta)\overrightarrow{F_j^2(e_n)}\}, \ldots).$

Thus,

$\alpha\overrightarrow{G_A} =$
$(\min\{\alpha_1 + (\alpha, \beta)\overrightarrow{F_1^1(e_1)}, \alpha_1 + (\alpha, \beta)\overrightarrow{F_1^1(e_2)}, \ldots, \alpha_1 + (\alpha, \beta)\overrightarrow{F_1^1(e_n)}\}, \min\{\alpha_2 + (\alpha, \beta)\overrightarrow{F_2^1(e_1)},$
$\alpha_2 + (\alpha, \beta)\overrightarrow{F_2^1(e_2)}, \ldots, \alpha_2 + (\alpha, \beta)\overrightarrow{F_2^1(e_n)}\}, \ldots, \min\{\alpha_j + (\alpha, \beta)\overrightarrow{F_j^1(e_1)}, \alpha_j + (\alpha, \beta)\overrightarrow{F_j^1(e_2)},$
$\ldots, \alpha_j + (\alpha, \beta)\overrightarrow{F_j^1(e_n)}\}, \ldots)$ and $\beta\overrightarrow{G_A} = (\min\{\beta_1 + (\alpha, \beta)\overrightarrow{F_1^2(e_1)}, \beta_1 + (\alpha, \beta)\overrightarrow{F_1^2(e_2)}, \ldots, \beta_1$
$+ (\alpha, \beta)\overrightarrow{F_1^2(e_n)}\}, \min\{\beta_2 + (\alpha, \beta)\overrightarrow{F_2^2(e_1)}, \beta_2 + (\alpha, \beta)\overrightarrow{F_2^2(e_2)}, \ldots, \beta_2 + (\alpha, \beta)\overrightarrow{F_2^2(e_n)}\}, \ldots,$
$\min\{\beta_j + (\alpha, \beta)\overrightarrow{F_j^2(e_1)}, \beta_j + (\alpha, \beta)\overrightarrow{F_j^2(e_2)}, \ldots, \beta_j + (\alpha, \beta)\overrightarrow{F_j^2(e_n)}\}, \ldots),$

where $\gamma_i = (\alpha_i, \beta_i), i \in \Delta$.

Example 3.8. Let $(F, A) = \{(e_1, \{\langle x_1, 0.3, 0.3\rangle, \langle x_2, 0.4, 0.5\rangle, \langle x_3, 0.3, 0.1\rangle\}), (e_2, \{\langle x_1, 0.3, 0.7\rangle, \langle x_2, 0.4, 0.6\rangle, \langle x_3, 0.4, 0.1\rangle\})\}$ and $(\alpha, \beta) = (0.3, 0.2)$. Then, $0.3\overrightarrow{F_A} = (1, 1, 1)$ and $0.2\overrightarrow{F_A} = (1, 1, 0)$.

3.4 Intuitionistic fuzzy soft coefficient of variation

Definition 3.6. If (F, A) be an IFSS over U, then (α, β)-cut level of intuitionistic fuzzy soft coefficient of variation is denoted by (α, β)IFSCV$\overrightarrow{(F, A)}$ and it is defined as

$$(\alpha, \beta)\text{IFSCV}\overrightarrow{(F, A)} = \{\frac{\| \sigma((\alpha, \beta)\overrightarrow{(F, A)} \|}{\max\{\| \alpha\overrightarrow{F_A} \|^2, \{\| \beta\overrightarrow{F_A} \|^2\}}\} \times 100$$

Theorem 3.6. Let (F, A) be an IFSS over U, then (α, β) IFSCV $\overrightarrow{(G_F, A)} = \{\frac{\sigma(\|(\alpha,\beta)\overrightarrow{(F,A)}\|)}{\max\{\theta_1,\theta_2\}}\} \times 100$, where $\theta_i = \| \psi_i \|^2 + 2\langle\psi_i, (\alpha, \beta)\overrightarrow{F_A^i}\rangle + \| (\alpha, \beta)\overrightarrow{F_A^i} \|^2$, $\psi_1 = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_i, \ldots.)$, $\psi_2 = (\beta_1, \beta_2, \beta_3, \ldots, \beta_i, \ldots.)$ and $i = 1, 2$.

Proof. In proof of Theorem 3.1, we found that
$(\alpha, \beta)\overrightarrow{G_A} = ((\alpha_1 + (\alpha, \beta)\overrightarrow{F_1^1(e_{j_1})}, \beta_1 + (\alpha, \beta)\overrightarrow{F_1^2(e_{k_1})}), (\alpha_2 + (\alpha, \beta)\overrightarrow{F_2^1(e_{j_2})}, \beta_2 + (\alpha, \beta)\overrightarrow{F_2^2(e_{k_2})}),$
$\ldots, (\alpha_j + (\alpha, \beta)\overrightarrow{F_j^1(e_{j_j})}, \beta_j + (\alpha, \beta)\overrightarrow{F_j^2(e_{k_j})}), \ldots)$ (say) , where $j_j, k_j \in I$.

Then, $\alpha\overrightarrow{G_A} = (\alpha_1 + (\alpha,\beta)\overrightarrow{F_1^1(e_{j_1})}, \alpha_2 + (\alpha,\beta)\overrightarrow{F_2^1(e_{j_2})}, \ldots, \alpha_j + (\alpha,\beta)\overrightarrow{F_j^1(e_{j_j})}, \ldots)$ and $\beta\overrightarrow{G_A} = (\beta_1 + (\alpha,\beta)\overrightarrow{F_1^2(e_{k_1})}, \beta_2 + (\alpha,\beta)\overrightarrow{F_2^2(e_{k_2})}, \ldots, \beta_j + (\alpha,\beta)\overrightarrow{F_j^2(e_{k_j})}, \ldots)$

So, easily $\alpha\overrightarrow{G_A} = \psi_1 + (\alpha,\beta)\overrightarrow{F_A^1}$ and $\beta\overrightarrow{G_A} = \psi_2 + (\alpha,\beta)\overrightarrow{F_A^2}$.

Thus, $\theta_1 = || \alpha\overrightarrow{G_A} ||^2 = || \psi_1 + (\alpha,\beta)\overrightarrow{F_A^1} ||^2 = || \psi_1 ||^2 + 2\langle\psi_1, (\alpha,\beta)\overrightarrow{F_A^1}\rangle + || (\alpha,\beta)\overrightarrow{F_A^1} ||^2$, since $\langle\psi_1, (\alpha,\beta)\overrightarrow{F_A^1}\rangle = \langle(\alpha,\beta)\overrightarrow{F_A^1}, \psi_1\rangle$.

Similarly, $\theta_2 = || \beta\overrightarrow{G_A} ||^2 = || \psi_2 + (\alpha,\beta)\overrightarrow{F_A^2} ||^2 = || \psi_2 ||^2 + 2\langle\psi_2, (\alpha,\beta)\overrightarrow{F_A^2}\rangle + || (\alpha,\beta)\overrightarrow{F_A^2} ||^2$, since $\langle\psi_2, (\alpha,\beta)\overrightarrow{F_A^2}\rangle = \langle(\alpha,\beta)\overrightarrow{F_A^2}, \psi_2\rangle$.

Again, from theorem 3.1, we have $\sigma((\alpha,\beta)\overrightarrow{(F,A)}) = \sigma((\alpha,\beta)\overrightarrow{(G_F,A)})$. Thus, proved. □

Theorem 3.7. Let (F,A) be an IFSS over U, then

$$(\alpha,\beta)\text{IFSCV}\overrightarrow{(G_F,A)_h} = h \times (\alpha,\beta)\text{IFSCV}\overrightarrow{(G_F,A)}.$$

Example 3.9. Let us consider Example 3.7., where $\sigma((0.3,0.1)\overrightarrow{(G_F,A)}) = (0, \frac{1}{\sqrt{2}})$. So,

$$(0.3,0.1)\text{IFSCV}\overrightarrow{(G_F,A)} = \frac{50}{13\sqrt{2}}.$$

In our example $h = 0.9$, thus $\frac{(0.3,0.1)\overrightarrow{(G_F,A)}}{h} = \{((\frac{2}{0.9}, \frac{5}{0.9}), (\frac{3}{0.9}, \frac{-2}{0.9})), ((\frac{2}{0.9}, \frac{5}{0.9}), (\frac{3}{0.9}, \frac{-1}{0.9}))\}$ and $(0.3,0.1)\overrightarrow{G_A} = ((\frac{2}{0.9}, \frac{5}{0.9}), (\frac{3}{0.9}, \frac{-1}{0.9}))$. Hence, $\sigma((0.3,0.1)\overrightarrow{(G_F,A)_{0.9}}) = (0, \frac{1}{0.9\times\sqrt{2}})$.

Now, we have $(0.3,0.1)IFSCV\overrightarrow{(G_F,A)_{0.9}} = 0.9 \times \frac{50}{13\sqrt{2}}$.

Remark 3.1 Scaling of an (α,β)-cut generated IFSS may not generate distinct (α,β)-cut generated IFSS.

The above remark can be understood from the following example.

Example 3.10. Let us consider Example 3.7, where $(G_F,A)_{(0.3,0.1)} = \{(e_1, \{\langle x_1, 0.3, 0\rangle, \langle x_2, 0.3, 0.1\rangle\}), (e_2, \{\langle x_1, 0.3, 0\rangle, \langle x_2, 0.3, 0.1\rangle\})\}$. Here, $h = 0.9$.

It can be checked that $(G_F,A)^{0.9}_{(0.3,0.1)} = \{(e_1, \{\langle x_1, 0.3, 0\rangle, \langle x_2, 0.3, 0.1\rangle\}), (e_2, \{\langle x_1, 0.3, 0\rangle, \langle x_2, 0.3, 0.1\rangle\})\}$.

3.5 Intuitionistic fuzzy soft covariance with (α,β)-cut

Definition 3.7. (i) Let us consider a universe U with the set of attributes E. Let (F,A) and (G,B) be IFSSs, where $A, B \subseteq E, |A| = n > |B| = m$. We extend B to $C = B \cup \{f_{m+1}, f_{m+2}, \ldots, f_n\}$ such that $G_i^1(f_k)(x_i) = 0$ and $G_i^2(f_k)(x_i) = 0\ \forall k \in \{m+1, m+2, \ldots, n\}$. Then, the (α,β)*-cut level intuitionistic fuzzy soft covariance of* (F,A) *and* (G,B) is denoted by $(\alpha,\beta)\text{IFSCov}(\overrightarrow{(F,A)}, \overrightarrow{(G,B)}_{|A|>|B|})$ and it is defined as $(\alpha,\beta)\text{IFSCov}(\overrightarrow{(F,A)}, \overrightarrow{(G,B)}_{|A|>|B|}) = (\frac{1}{n}\{|| \Delta_1 ||^2 + || \Delta_2 ||^2 + \cdots + || \Delta_n ||^2\}, \frac{1}{n}\{|| \Delta'_1 ||^2 + || \Delta'_2 ||^2 + \cdots + || \Delta'_n ||^2\})$, where $\Delta_j = (\min\{\alpha F_1^1(e_j), \alpha G_1^1(f_j)\}, \min\{\alpha F_2^1(e_j), \alpha G_2^1(f_j)\}, \ldots, \min\{\alpha F_i^1(e_j), \alpha G_i^1(f_j)\}, \ldots)$, $\Delta'_j = (\min\{\beta F_1^2(e_j), \beta G_1^2(f_j)\}, \min\{\beta F_2^2(e_j), \beta G_2^2(f_j)\}, \ldots, \min\{\beta F_i^2(e_j), \beta G_i^2(f_j)\}, \ldots)$ and $e_j \in A, f_j \in C, i \in \Delta, j \in I$.

The attributes $f_{m+1}, f_{m+2}, \ldots, f_n$ with $G_i^1(f_k)(x_i) = 0$ and $G_i^2(f_k)(x_i) = 0\ \forall k \in \{m+1, m+2, \ldots, n\}, i \in \Delta$ are called *intuitionistic fuzzy soft statistical dummy attributes* for B relative to A.

(ii) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two IFSSs, where $A, B \subseteq E, |A| = n = |B|$. Then, (α, β)*-cut level intuitionistic fuzzy soft covariance of* (F, A) *and* (G, B) is denoted by (α, β)IFSCov$(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|=|B|})$ and it is defined as (α, β)IFSCov$(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|=|B|})$ = $(\frac{1}{n}\ \{\|\ \Delta_1\ \|^2 + \|\ \Delta_2\ \|^2 + \cdots + \|\ \Delta_n\ \|^2\}, \frac{1}{n}\ \{\|\ \Delta'_1\ \|^2 + \|\ \Delta'_2\ \|^2 + \cdots + \|\ \Delta'_n\ \|^2\})$, where Δ_j = $(\min\{\alpha F_1^1(e_j), \alpha G_1^1(f_j)\}, \min\{\alpha F_2^1(e_j), \alpha G_2^1(f_j)\}, \ldots, \min\{\alpha F_i^1(e_j), \alpha G_i^1(f_j)\}, \ldots)$, Δ'_j = $(\min\{\beta F_1^2(e_j), \beta G_1^2(f_j)\}, \min\{\beta F_2^2(e_j), \beta G_2^2(f_j)\}, \ldots, \min\{\beta F_i^2(e_j), \beta G_i^2(f_j)\}, \ldots)$ and $e_j \in A, f_j \in B, i \in \Delta, j \in I$.

The above definition can be redefined if $A = B$. In this case $e_j = f_j\ \forall j \in I$.

Definition 3.8. (i) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two IFSSs, where $A, B \subseteq E, \mid A \mid = n > \mid B \mid = m$. We extend B to $C = B \cup \{f_{m+1}, f_{m+2}, \ldots, f_n\}$ such that $G_i^1(f_k)(x_i) = 0$ and $G_i^2(f_k)(x_i) = 0\ \forall k \in \{m+1, m+2, \ldots, n\}$. Then, (F, A) and (G, B) are said to be $\epsilon_{(\alpha,\beta)}$*-approximation independent intuitionistic fuzzy soft sets* if $\Delta_j = 0$, $\Delta'_j = 0$, where $0 = (0, 0, \ldots, 0, \ldots)$, $e_j \in A$, and $f_j \in C, i \in \Delta, \forall j \in I$.

(ii) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two IFSSs, where $A, B \subseteq E, \mid A \mid = n = \mid B \mid$. Then, (F, A) and (G, B) are said to be $\epsilon_{(\alpha,\beta)}$*-approximation independent intuitionistic fuzzy soft sets* if $\Delta_j = 0$, $\Delta'_j = 0$ where $0 = (0, 0, \ldots, 0, \ldots)$, $e_j \in A$, and $f_j \in C, i \in \Delta, \forall j \in I$.

Theorem 3.6. (i) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two IFSSs, where $A, B \subseteq E, \mid A \mid = n > \mid B \mid = m$. Then, (α, β)IFSCov$(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|>|B|})$ = (0, 0)$\Leftrightarrow$ (F, A) and (G, B) are $\epsilon_{(\alpha,\beta)}$-approximation independent intuitionistic fuzzy soft sets.

(ii) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two two IFSSs, where $A, B \subseteq E, \mid A \mid = n = \mid B \mid$. Then, (α, β)IFSCov$(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|=|B|})$ = (0,0) $\Leftrightarrow$ (F, A) and (G, B) are $\epsilon_{(\alpha,\beta)}$-approximation independent intuitionistic fuzzy soft sets.

Proof. (i) (α, β)IFSCov$(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|>|B|}) = (0, 0)$
$\Leftrightarrow (\frac{1}{n}\ \{\|\ \Delta_1\ \|^2 + \|\ \Delta_2\ \|^2 + \cdots + \|\ \Delta_n\ \|^2\}, \frac{1}{n}\ \{\|\ \Delta'_1\ \|^2 + \|\ \Delta'_2\ \|^2 + \cdots + \|\ \Delta'_n\ \|^2\}) = (0, 0)$
$\Leftrightarrow \|\ \Delta_j\ \|^2 = 0$ and $\|\ \Delta'_j\ \|^2 = 0\ \forall j \in I$
$\Leftrightarrow \|\ \Delta_j\ \| = 0$ and $\|\ \Delta'_j\ \| = 0\ \forall j \in I$
$\Leftrightarrow \Delta_j = (0, 0, \ldots, 0, \ldots)$ and $\Delta'_j = (0, 0, \ldots, 0, \ldots) \forall j \in I$
$\Leftrightarrow \Delta_j = 0$ and $\Delta'_j = 0 \forall j \in I$
$\Leftrightarrow (F, A)$ and (G, B) are $\epsilon_{(\alpha,\beta)}$-approximation independent intuitionistic fuzzy soft sets. □

4 Intuitionistic fuzzy soft attribute correlation coefficient and utility based (α, β)-cut

Definition 4.1. Let (F, A) be an IFSS with at least two attributes e_1, $e_2 \in A$, then the intuitionistic fuzzy soft attribute correlation coefficient of $(\alpha,\beta)\overrightarrow{F(e_1)}$ and $(\alpha,\beta)\overrightarrow{F(e_2)}$ is denoted by IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)}, (\alpha,\beta)\overrightarrow{F(e_2)})$ and it is defined as IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)}, (\alpha,\beta)\overrightarrow{F(e_2)}) = (\frac{\sum\limits_{i\in\Delta}\Delta_i}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||}, \frac{\sum\limits_{i\in\Delta}\Delta_{i'}}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{F(e_2)}||})$; where $\Delta_i = \min\{\alpha F_i(e_1), \alpha F_i(e_2)\}$, $\Delta_i' = \min\{\beta F_i(e_1), \beta F_i(e_2)\}$, $||\alpha\overrightarrow{F(e_j)}|| = \sqrt{\langle\alpha\overrightarrow{F(e_j)}, \alpha\overrightarrow{F(e_j)}\rangle} = \sqrt{\sum\limits_{i\in I}(\alpha F_i(e_j))^2}$, $||\alpha\overrightarrow{F(e_1)}|| \neq 0$, $||\alpha\overrightarrow{F(e_2)}|| \neq 0$ and $||\beta\overrightarrow{F(e_j)}|| = \sqrt{\langle\beta\overrightarrow{F(e_j)}, \beta\overrightarrow{F(e_j)}\rangle} = \sqrt{\sum\limits_{i\in I}(\beta F_i(e_j))^2}$, $||\beta\overrightarrow{F(e_1)}|| \neq 0$, $||\beta\overrightarrow{F(e_2)}|| \neq 0$. If any of $||\alpha\overrightarrow{F(e_1)}||$, $||\alpha\overrightarrow{F(e_2)}||$, $||\beta\overrightarrow{F(e_1)}||$, $||\beta\overrightarrow{F(e_2)}||$ is 0, then IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)}, (\alpha,\beta)\overrightarrow{F(e_2)})$ is not possible. In this case, we shall use notation "(∞,∞)", no matter what is Δ_i or Δ_i'.

Example 4.1. Consider an IFSS (F, A), where $(F, A) = \{(e_1, \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.2\rangle\})$, $(e_2, \{\langle x_1, 0.3, 0.4\rangle, \langle x_2, 0.5, 0.2\rangle\})\}$, then IFSACC$((0.4, 0.3)\overrightarrow{F(e_1)}, (0.4, 0.3)\overrightarrow{F(e_2)}) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

From this onward, if $a, b, c, d \in \mathbf{R}$, and $(a, b) \leq (c, d)$, then we mean that $a \leq c$ and $b \leq d$.

Theorem 4.1. If (F, A) be any IFSS with at least two attributes e_1, $e_2 \in A$ over a universe U, then $(0, 0)) \leq$ IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)}, (\alpha,\beta)\overrightarrow{F(e_2)}) \leq (1, 1)$.

Proof. We know,

$$\sum_{i\in\Delta}\Delta_i = \sum_{i\in\Delta}\min\{\alpha F_i(e_1), \alpha F_i(e_2)\} \leq \sum_{i\in\Delta}(\alpha F_i(e_1).\alpha F_i(e_2))$$
$$\leq \sqrt{\sum_{i\in\Delta}(\alpha F_i(e_1))^2}.\sqrt{\sum_{i\in\Delta}(\alpha F_i(e_2))^2} = ||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||.$$

So, $\frac{\sum\limits_{i\in\Delta}\Delta_i}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||} \leq 1$. Similarly, we can show that $\frac{\sum\limits_{i\in\Delta}\Delta_i'}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{F(e_2)}||} \leq 1$.

Thus, IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)}, (\alpha,\beta)\overrightarrow{F(e_2)}) \leq (1, 1)$.

Again, $\sum\limits_{i\in\Delta}\Delta_i \geq 0$, $||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}|| > 0$, thus $\frac{\sum\limits_{j\in I}\Delta_i}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||} \geq 0$.
Similarly, $\frac{\sum\limits_{i\in\Delta}\Delta_i'}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{F(e_2)}||} \geq 0$. Hence, proved. □

Theorem 4.2. Let (F, A) be any IFSS with at least two attributes e_1,$e_2 \in A$, then IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)}, (\alpha,\beta)\overrightarrow{F(e_2)})$ = IFSACC$((\alpha,\beta)\overrightarrow{F(e_2)}, (\alpha,\beta)\overrightarrow{F(e_1)})$.

Proof. Proof can be obtained from Definition 4.1. □

Theorem 4.3. If $F(e_1), G(e_1)$ and $H(e_1)$ are three IFSs of three IFSSs (F, A), (G, A) and (H, A), over U such that $F(e_1) \subseteq G(e_1) \subseteq H(e_1)$ and $e_1 \in A$. Then;

(i) IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)},(\alpha,\beta)\overrightarrow{H(e_1)}) \leq$ IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)},(\alpha,\beta)\overrightarrow{G(e_1)})$

(ii) IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)},(\alpha,\beta)\overrightarrow{H(e_1)}) \leq$ IFSACC$((\alpha,\beta)\overrightarrow{G(e_1)},(\alpha,\beta)\overrightarrow{H(e_1)})$

(iii) IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)},(\alpha,\beta)\overrightarrow{G(e_1)}) \leq$ IFSACC$((\alpha,\beta)\overrightarrow{H(e_1)},(\alpha,\beta)\overrightarrow{G(e_1)})$,

if $||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{H(e_1)}|| - ||\alpha\overrightarrow{G(e_1)}||^2 \leq 0$ and $||\beta\overrightarrow{G(e_1)}||^2 - ||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{H(e_1)}|| \leq 0$

Proof. (i) Let IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)},(\alpha,\beta)\overrightarrow{H(e_1)})$

$$= \left(\frac{\sum_{i\in\Delta}{\Delta_i}^1}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{H(e_1)}||}, \frac{\sum_{i\in\Delta}{\Delta'_i}^1}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{H(e_1)}||}|\right);$$

where ${\Delta_i}^1=\min\{\alpha F_i(e_1),\alpha H_i(e_1)\} = \alpha F_i(e_1)$, ${\Delta'_i}^1=\min\{\beta F_i(e_1),\beta H_i(e_1)\} = \beta H_i(e_1)$, $||\alpha\overrightarrow{F(e_1)}|| \neq 0$, $||\alpha\overrightarrow{H(e_1)}|| \neq 0$, $||\beta\overrightarrow{F(e_1)}|| \neq 0$, $||\beta\overrightarrow{H(e_1)}|| \neq 0$.

Again, we consider IFSACC$((\alpha,\beta)\overrightarrow{F(e_1)},(\alpha,\beta)\overrightarrow{G(e_1)})$

$$= \left(\frac{\sum_{i\in\Delta}{\Delta_i}^2}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{G(e_1)}||}, \frac{\sum_{i\in\Delta}{\Delta'_i}^2}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{G(e_1)}||}|\right);$$

where ${\Delta_i}^2=\min\{\alpha F_i(e_1),\alpha G_i(e_1)\} = \alpha F_i(e_1)$, ${\Delta'_i}^2= min\{\beta F_i(e_1),\beta G_i(e_1)\} = \beta G_i(e_1)$, $||\alpha\overrightarrow{F(e_1)}|| \neq 0$, $||\alpha\overrightarrow{G(e_1)}|| \neq 0$, $||\beta\overrightarrow{F(e_1)}|| \neq 0$, $||\beta\overrightarrow{G(e_1)}|| \neq 0$.

Now, $\frac{\sum_{i\in\Delta}{\Delta_i}^1}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{H(e_1)}||}$ - $\frac{\sum_{i\in\Delta}{\Delta_i}^2}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{G(e_1)}||}$

= $\frac{\sum_{i\in\Delta}{\Delta_i}^1}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{H(e_1)}||}$ - $\frac{\sum_{i\in\Delta}{\Delta_i}^1}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{G(e_1)}||}$

= $\frac{\sum_{i\in\Delta}{\Delta_i}^1}{||\alpha\overrightarrow{F(e_1)}||}\{\frac{1}{||\alpha\overrightarrow{H(e_1)}||} - \frac{1}{||\alpha\overrightarrow{G(e_1)}||}\} \leq 0$, since $G(e_1) \subseteq H(e_1)$, then $||\alpha\overrightarrow{G(e_1)}|| \leq ||\alpha\overrightarrow{H(e_1)}||$.

Thus, $\frac{\sum_{i\in\Delta}{\Delta_i}^1}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{H(e_1)}||} \leq \frac{\sum_{i\in\Delta}{\Delta_i}^2}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{G(e_1)}||}$

Now, it is to be noted that ${\Delta'_i}^1 = 0$ or $1\ \forall i \in \Delta$. Thus, squaring of ${\Delta'_i}^1$ does not alter the sum and so on for other cases.

Again, $\frac{\sum_{i\in\Delta}{\Delta'_i}^1}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{H(e_1)}||}$ - $\frac{\sum_{i\in\Delta}{\Delta'_i}^2}{||\beta\overrightarrow{F(e_1)}||.||\beta\overrightarrow{G(e_1)}||}$

= $\frac{1}{||\beta\overrightarrow{F(e_1)}||}.\{\frac{\sum_{i\in\Delta}{\Delta'_i}^1}{||\beta\overrightarrow{H(e_1)}||}$ - $\frac{\sum_{i\in\Delta}{\Delta'_i}^2}{||\beta\overrightarrow{G(e_1)}||}\}$

$$= \frac{1}{||\beta \overrightarrow{F(e_1)}||} . \{ \frac{\sum_{i \in \Delta} (\Delta_i'^1)^2}{||\beta \overrightarrow{H(e_1)}||} - \frac{\sum_{i \in \Delta} (\Delta_i'^2)^2}{||\beta \overrightarrow{G(e_1)}||} \}$$

$$= \frac{1}{||\beta \overrightarrow{F(e_1)}||} . \{ \frac{||\beta \overrightarrow{H(e_1)}||^2}{||\beta \overrightarrow{H(e_1)}||} - \frac{||\beta \overrightarrow{G(e_1)}||^2}{||\beta \overrightarrow{G(e_1)}||} \}$$

$$= \frac{1}{||\beta \overrightarrow{F(e_1)}||} . \{ ||\beta \overrightarrow{H(e_1)}|| - ||\beta \overrightarrow{G(e_1)}|| \} \leq 0$$

So, $\frac{\sum_{i \in \Delta} \Delta_i'^1}{||\beta \overrightarrow{F(e_1)}||.||\beta \overrightarrow{H(e_1)}||} \leq \frac{\sum_{i \in \Delta} \Delta_i'^2}{||\beta \overrightarrow{F(e_1)}||.||\beta \overrightarrow{G(e_1)}||}$. Hence, proved.

Similarly, we can prove remaining parts. □

Now, we state the following theorem without discussing the proof.

Theorem 4.4. IFSACC$((\alpha, \beta)\overrightarrow{F(e_1)}, (\alpha, \beta)\overrightarrow{F(e_2)}) = (1, 1) \Leftrightarrow \alpha \overrightarrow{F(e_1)} = \alpha \overrightarrow{F(e_2)}$ and $\beta \overrightarrow{F(e_1)} = \beta \overrightarrow{F(e_2)}$

5 Conclusion

This paper introduces foundational concepts of intuitionistic fuzzy soft statistics along with correlation coefficient on intuitionistic fuzzy soft set. Here, we try to connect two crucial areas of contemporary science viz. uncertainty mathematics and statistics with one of the most applicable areas of economics viz. utility theory. Uses of the binary digital representation in this new statistical idea open the possibilities of applications in computer science, quantum computing, mathematical psychology, mathematical sociology, human trafficking, illegal immigration, human-computer interactions etc., where attributes play crucial roles. However, this new statistical area must be developed systematically to have broader applications for the betterment of human race.

References

[1] Acharjee, S. (2017). New results for Kharal's similarity measures of soft sets, *New Math. Nat. Comput*, 13 (01), 55–60.

[2] Acharjee, S., & Tripathy, B. C. (2017). Some results on soft bitopology, *Bol. Soc. Paran. Math*, 35 (1), 269–279.

[3] Acharjee, S., & Mordeson, J. N. (2017). Soft statistics with respect to utility and application in human trafficking, *New Math. Nat. Comput.*, 13 (03), 289–310.

[4] Acharjee, S., Sarma, D. J., Hanneman, R. A., Mordeson, J. N., & Malik, D. S. (2018). Fuzzy soft attribute correlation coefficient and application to data of human trafficking, *Proyecciones Jour. of Math.*, 37 (4), 637–681.

[5] Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer Physica-Verlag, Heidelberg.

[6] Çağman, N., Karataş, S., & Enginoglu, S. (2011). Soft topology, *Comput. Math. Appl.*, 62, 351–358.

[7] Chen, D. (2005). The parametrization reduction of soft sets and its application, *Comput. Math. Appl.*, 49, 757–763.

[8] Huang, H. L. (2017). An improved correlation coefficient of intuitionistic fuzzy sets, *Jour. Intel. Syst.*, 13 pages, DOI: `https://doi.org/10.1515/jisys-2017-0094`.

[9] Li, H., & Shen, Y. (2012). Similarity measures of fuzzy soft sets based on different distances, *Fifth Int. Symp. Comput. Intel. and Design.*, 28-29 Oct 2012, Hangzhou, China, 527– 529.

[10] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets,*Jour. Fuzzy Math.*, 9 (3), 589–602.

[11] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory, *Comput. Math. Appl.*, 45, 555–562.

[12] Maji, P. K., Roy, A. R., & Biswas, R. (2004). On Intuitionistic Fuzzy Soft Sets, *Jour. Fuzzy Math.*, 12 (3), 669–683.

[13] Mitchell, H. B. (2004). A correlation coefficient for intuitionistic fuzzy sets, *Int. J. Intel. Syst.*, 19 (5), 483–490.

[14] Molodtsov, D. (1999). Soft set theory – first results, *Comput. Math. Appl.*, 37, 19–31.

[15] Mordeson, J. N., Mallenby, M., Mathew, S., & Acharjee, S. (2017). Human trafficking: Policy intervention, *New Math. Nat. Comput.*, 13 (03), 341–358.

[16] Pei, D., & Miao, D. (2005). From soft sets to information systems, *Proceedings of the IEEE international conference on granular computing*, 25-27 July 2005, Beijing, China, 2, 617–621.

[17] Szmidt, E., & Kacprzyk, J. (2010). Correlation of Intuitionistic Fuzzy Sets, *International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, June 28-July 2, 2010, Dortmund, Germany, LNAI 6178, 169–177.

[18] Xiao, Z., & Zou, Y. (2014). A comparative study of soft sets with fuzzy sets and rough sets, *Jour. Intel. Fuzzy Syst.*, 27, 425–434.

[19] Zadeh, L. A. (1965). Fuzzy sets, *Inform. Contrl.*, 8, 338–353.