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A SET-THEORETICAL OPERATION OVER INTUITIONISTIC FUZZY SETS

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Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Different relations and operations are introduced over the IFSs. Some of them are the following

$$\begin{array}{lcl} A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \le \mu_B(x) \& \nu_A(x) \ge \nu_B(x)), \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)), \\ \overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\ A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\ A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \end{array}$$

$$\begin{aligned}
A+B &= \{\langle x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\},\\
A.B &= \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x)\rangle | x \in E\},\\
A@B &= \{\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2}\rangle | x \in E\}.
\end{aligned}$$

In this short remark we introduce a new operator, defined over IFSs. It is an analogous of operations "substraction" and "division" and has the form for every two given IFSs A and B:

$$A|B = \{\langle \min(\mu_A(x), \nu_B(x)), \max(\mu_B(x), \nu_A(x))\rangle | x \in E\}.$$

First, we must check that in a result of the operation we obtain an IFS. Really, for two given IFSs A and B and for each $x \in E$, if $\mu_B(x) \leq \nu_A(x)$, then

$$\min(\mu_A(x), \nu_B(x)) + \max(\mu_B(x), \nu_A(x)) = \min(\mu_A(x), \nu_B(x)) + \nu_A(x) \leq \mu_A(x) + \nu_A(x) \leq 1;$$

if $\mu_B(x) > \nu_A(x)$, then

$$\min(\mu_A(x), \nu_B(x)) + \max(\mu_B(x), \nu_A(x)) = \min(\mu_A(x), \nu_B(x)) + \mu_B(x) \leq \nu_B(x) + \nu_A(x) \leq 1.$$

By similar way we can prove the following assertions.

Theorem 1: For every two IFSs A and B:
(a) $A|E^* = O^*$,
(b) $A|O^* = A$,
(c) $E^*|A = \overline{A}$,
(d) $O^*|A = O^*$,
(e) $(A|B) \cap C = (A \cap C)|B = A \cap (C|B)$,
(f) $(A|B) \cup C = (A \cup C) \cap \overline{B|C} = (A \cup C)|(B|C)$,
(g) $(A \cap B)|C = (A|C) \cap (B|C)$,
(h) $(A \cup B)|C = (A|C) \cup (B|C)$.

Obviously

$$O^*|U^* = O^*,\ O^*|E^* = O^*,\ U^*|O^* = U^*,$$
$$U^*|E^* = O^*,\ E^*|O^* = E^*,\ E^*|U^* = O^*.$$

Two open problems at the moment are the following:
1. Are there relations between operations " $-$ " and " : " from one side and opeastion "|" from other?
2. Are there relations between operations " $+$ " and "." from one side and opeastion "|" from other?

Theorem 2: For every three IFSs A, B and C:
(a) $(A|B)|C = (A|C)|B$,
(b) $(A|C) \cap B = A \cap (B|C)$,
(c) $\overline{A|B} = \overline{A \cap \overline{B}} = \overline{A} \cup B$.

References:
[1] K. Atanassov, *Intuitionistic Fuzzy Sets,* Springer, Heidelberg, 1991.