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EXTENDED LEVEL OPERATORS OVER INTUITIONISTIC FUZZY SETS

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1 Introduction

Following the idea for a fuzzy set of level α (e.g. [3]), in [2] the definition of a set of (α, β)-level, generated by an Intuitionistic Fuzzy Set (IFS, see, e.g. [1]) A, can be introduced, where $\alpha, \beta \in [0, 1]$ are fixed numbers for which $\alpha + \beta \leq 1$. Formally, this set has the form:

$$N_{\alpha,\beta}(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E \ \&\ \mu_A(x) \geq \alpha \ \&\ \nu_A(x) \leq \beta\}.$$

In [2, 1] two particular cases of the above set are defined ($\alpha \in [0, 1]$ is a fixed number):

- $$N_\alpha(A) = \{\langle x, \mu_A(A), \nu_A(x)\rangle | x \in E \ \&\ \mu_A(x) \geq \alpha\}$$
  called *"a set of level of membership α"* generated by A;
- $$N^\alpha(A) = \{\langle x, \mu_A(A), \nu_A(x)\rangle | x \in E \ \&\ \nu_A(x) \leq \alpha\}$$
  called *"a set of level of non-membership α"* generated by A.

From the above definition it directly follows for every IFS A and for every $\alpha, \beta \in [0, 1]$, such that $\alpha + \beta \leq 1$, the validity of

$$N_{\alpha,\beta}(A) \subseteq \left\{ \begin{array}{c} N^\beta(A) \\ N_\alpha(A) \end{array} \right\} \subset A$$

where "$\subseteq$" is a relation in the set-theoretical sense and

$$N_{\alpha,\beta}(A) = N_\alpha(A) \cap N^\beta(A).$$

2 Main results

Now, we shall introduce three different extensions of the above set. We directly see that this operator decrement the number of the elements of the given IFS A, saving only these elements of E that have degrees, satisfying the condition $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$ (see Fig. 1).

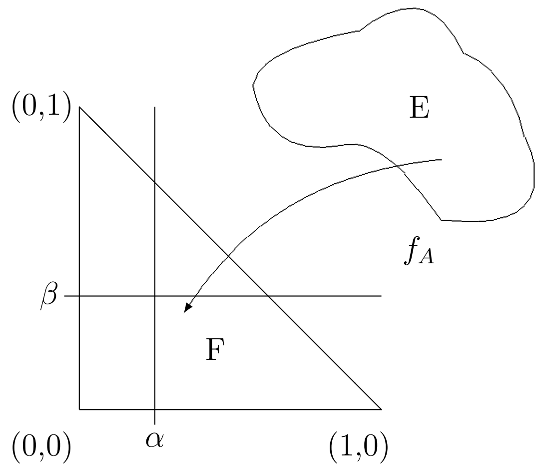


Fig. 1.

First, we change this condition with a more general one. Let f be a function. We define operator

$$N^f_{\alpha,\beta}(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E \ \& f(\mu_A(x), \nu_A(x)) \geq \langle \alpha, \beta\rangle\}.$$

Obviously, then f is the identity, $N^f_{\alpha,\beta}(A)$ coincide with $N_{\alpha,\beta}(A)$.

Second, we extend the first modification, changing function f with a predicate p, i.e.,

$$N^p_{\alpha,\beta}(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E \ \& p((\mu_A(x), \nu_A(x)), (\alpha, \beta))\rangle\}.$$

When predicate p has the form

$$p((a, b), (c, d)) = \text{“}f(a, b) \geq (c, d)\text{”},$$

then $N^p_{\alpha,\beta}(A)$ coincides with $N^f_{\alpha,\beta}(A)$.

In the three cases above numbers α, β are given beforehand and fixed. Now, we shall introduce three other extensions - of each one of the above cases. We shall change the two given constants α and β with elements of a given IFS B. The three new level-operators will have, respectively, the forms:

$$N_B(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E \ \& \ \mu_A(x) \geq \nu_B(x) \ \& \ \nu_A(x) \leq \nu_B(x)\},$$

$$N^f_B(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E \ \& f(\mu_A(x), \nu_A(x)) \geq \langle \mu_B(x), \nu_B(x)\rangle\},$$

$$N^p_B(A) = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E \ \& p((\mu_A(x), \nu_A(x)), (\mu_B(x), \nu_B(x)))\rangle\}.$$

We shall formulate some assertions, related to the new level-operators

Theorem 1: For every three IFSs A, B and C,

(a) if $B \subset C$, then

$$N_B(A) \supset N_C(A),$$

$$N_B^f(A) \supset N_C^f(A),$$

(b) if $A \subset B$, then

$$N_C(A) \subset N_C(B),$$

$$N_C^f(A) \subset N_C^f(B).$$

Corollary 1: The same inclusions are valid for $N_{\alpha,\beta}^f(A)$ and $N_{\gamma,\delta}^f(A)$; for $N_{\alpha,\beta}(A)$ and $N_{\gamma,\delta}(A)$, where

$$\alpha \leq \gamma \text{ and } \beta \geq \delta;$$

for $N_{\alpha,\beta}^f(A)$ and $N_{\alpha,\beta}^f(B)$; and for $N_{\alpha,\beta}(A)$ and $N_{\alpha,\beta}(B)$, where the two IFSs A and B satisfy inclusion $A \subset B$.

Theorem 2: For every two IFSs A and B,

$$A = N_B^p(A) \cup N_B^{\neg p}(A),$$

where $\neg p$ is the negation of predicate p.

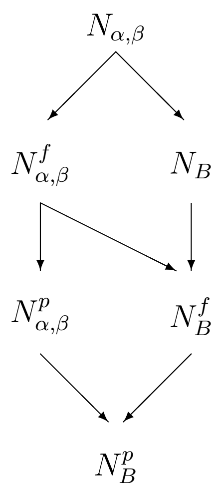
Corollary 2: The same equality is valid for $N_{\alpha,\beta}^p(A)$ and $N_{\alpha,\beta}^{\neg p}(A)$, where α and β are fixed numbers from $[0, 1]$.

Theorem 3: For every two IFSs A and B and for every n predicates $p_1, p_2, ..., p_n$:

$$N_B^{p_1 \vee p_2 \vee ... \vee p_n}(A) = \bigcup_{1 \leq i \leq n} N_B^{p_i}(A).$$

$$N_B^{p_1 \& p_2 \& ... \& p_n}(A) = \bigcap_{1 \leq i \leq n} N_B^{p_i}(A).$$

let us denote the fact that operator X generalizes operator Y by $X \rightarrow Y$. Then, the relations between the above discussed operators are:



References

[1] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Heidelberg, 1999.

[2] Atanassov K., Level operators on intuitionistic fuzzy sets, BUSEFAL Vol. 54, 1993, 4-8.

[3] Kaufmann A., Introduction a la theorie des sour-ensembles flous, Paris, Masson, 1977.