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ON THE INTUITIONISTIC FUZZY LOGIC OPERATIONS

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Some operations of the Intuitionistic Fuzzy Logic (IFL) are introduced in [1,2] and their basic properties are studied. While operations "$\neg$" (negation), "&" (conjunction) and "$\vee$" (disjunction) are defined in one form, the operation "$\supset$" has ten ones. Here, we shall introduce other variants of the binary (i.e., without the unary operation "$\neg$") operations and shall show some of their properties.

Following [1], we shall note that to each proposition (in the classical sense) we can assign its truth value: truth - denoted by 1, or falsity - 0. In the case of fuzzy logics this truth value is a real number in the interval [0, 1] and can be called "truth degree" of a particular proposition. In [1] we added one more value - "falsity degree" - which will be in the interval [0, 1] as well. Thus one assigns to the proposition p two real numbers $\mu(p)$ and $\gamma(p)$ with the following constraint to hold:

$$\mu(p) + \gamma(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p), \gamma(p) \rangle.$$

Hence the function V: S --> [0, 1] x [0, 1] gives the truth and falsity degrees of all propositions in S.

We assume that the evaluation function V assigns to the logical truth T: $V(T) = \langle 1, 0 \rangle$, and to F: $V(F) = \langle 0, 1 \rangle$.

The evaluation of the negation $\neg p$ of the proposition p is defined through:

$$V(\neg p) = \langle \gamma(p), \mu(p) \rangle.$$

When the values V(p) and V(q) of the propositions p and q are known, the evaluation function V can be extended also for the operations "&", "$\vee$" through the definition :

$$V(p \,\&\, q) = \langle \min(\mu(p), \mu(q)), \max(\nu(p), \nu(q))\rangle,$$

$$V(p \vee q) = \langle \max(\mu(p), \mu(q)), \min(\nu(p), \nu(q))\rangle.$$

Here we shall introduce the following other variant of these two operations:

$$V(p \,\&\, q) = \langle \mu(p) + \mu(q) - \mu(p).\mu(q), \nu(p).\nu(q)\rangle,$$

$$V(p \vee q) = \langle \mu(p).\mu(q), \nu(p) + \nu(q) - \nu(p).\nu(q)\rangle.$$

Below we shall show their basic properties.

Depending on the way of defining the operation "$\supset$", the following ten different variants of IFPC are described in [1,2]:

1) $V(p \supset q) = \langle 1 - (1 - \mu(q)).sg(\mu(p) - \mu(q)), \nu(q).sg(\mu(p) - \mu(q)).sg(\nu(q) - \nu(p))\rangle,$

where:

$$sg(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases};$$

2) $V(p \supset q) = \langle \max(\nu(p), \mu(q)), \min(\mu(p), \nu(q))\rangle;$

3) $V(p \supset q) = \langle \max(1 - \mu(p), \nu(q)), \min(\mu(p), 1 - \nu(q))\rangle$

4) $V(p \supset q) = \langle \nu(p) + \mu(p).\mu(q), \mu(p).\nu(q)\rangle$

5) $V(p \supset q) = \langle \min(1, \nu(p)+\mu(q)), \mathrm{mpx}(0, 1-\nu(p)-\mu(q))\rangle$

6) $V(p \supset q) = \begin{cases} \langle \nu(p), \mu(p)\rangle & , \text{ if } \mu(q) \le \nu(q) \\ \langle \mu(q), \nu(q)\rangle & , \text{ if } \mu(p) \ge \nu(p) \\ \langle \mathrm{mpx}(\nu(p), \mu(q)), \min(\mu(p), \nu(q)\rangle, & \text{otherwise} \end{cases}$

7) $V(p \supset q) = \begin{cases} \langle 1, 0\rangle & , \text{ if } \mu(p) \le \mu(q) \,\&\, \nu(p) \ge \nu(q) \\ \langle \mu(q), \nu(p)\rangle & , \text{ if } \mu(p) > \mu(q) \,\&\, \nu(p) \ge \nu(q) \\ \langle \mu(p), \nu(q)\rangle & , \text{ if } \mu(p) \le \mu(q) \,\&\, \nu(p) < \nu(q) \\ \langle 0, 1\rangle & , \text{ if } \mu(p) > \mu(q) \,\&\, \nu(p) < \nu(q) \end{cases}$

8) $V(p \supset q) = \langle \dfrac{\nu(p).\mu(q)}{\nu(p).\mu(q)+(1-\nu(p)).(1-\mu(q))}, \dfrac{\mu(p).\nu(q)}{\mu(p).\nu(q)+(1-\mu(p)).(1-\nu(q))} \rangle;$

9) $V(p \supset q) = \langle \dfrac{\nu(p).\mu(q)}{(1-\mu(p)).(1-\nu(q))+(1-\nu(p)).(1-\mu(q))}, \dfrac{\mu(p).\nu(q)}{(1-\nu(p)).(1-\mu(q))+(1-\mu(p)).(1-\nu(q))} \rangle;$

10) $V(p \supset q) = \langle \dfrac{\nu(p)+\mu(q)}{2-(\mu(p)+\nu(q))}, \dfrac{\mu(p)+\nu(q)}{2-(\nu(p)+\mu(q))} \rangle;$

To this list of definitions we can add also:

11) $V(p \supset q) = \langle \frac{2 - \mu(p) - \nu(q)}{2}, \frac{\mu(p) + \nu(q)}{2} \rangle$;

12) $V(p \supset q) = \langle \frac{\nu(p) + \mu(q)}{2}, \frac{\mu(p) + \nu(q)}{2} \rangle$;

13) $V(p \supset q) = \langle \nu(p) + \mu(q) - \nu(p).\mu(q), \mu(p).\nu(q) \rangle$.

Let

$$\neg V(p) = V(\neg p),$$
$$V(p) \wedge V(q) = V(p \& q),$$
$$V(p) \vee V(q) = V(p \veebar q),$$
$$V(p) \to V(q) = V(p \supset q).$$

Following [1], a given propositional form A (c.f. [3]: each proposition is a propositional form; if A is a propositional form then $\neg A$ is a propositional form; if A and B are propositional forms, then A & B, $A \veebar B$, $A \supset B$ are propositional forms) will be called a tautology if $V(A) = \langle 1, 0 \rangle$, for all valuation functions V and an intuitionistic fuzzy tautology (IFT), if $\mu(A) \geq \nu(A)$.

For the first forms of "&", "$\veebar$" and "$\supset$" it is proved in [1] that if A, B and C are arbitrary propositional forms, then:

(a) $A \supset A$,

(b) $A \supset (B \supset A)$,

(c) $A \& B \supset A$,

(d) $A \& B \supset B$,

(e) $A \supset (A \veebar B)$,

(f) $B \supset (A \veebar B)$,

(g) $A \supset (B \supset (A \& B))$,

(h) $(A \supset C) \supset ((B \supset C) \supset ((A \veebar B) \supset C))$,

(i) $\neg\neg A \supset A$,

(j) $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$

are tautologies and for the first forms of "&" and "$\veebar$" and of the second form of "$\supset$" (a)-(j) and

(k) $(\neg A \supset \neg B) \supset ((\neg A \supset B) \supset A)$

and IFTs. Therefore, in the last case we can assert that IFL with the last forms of the operations is a model of the axioms of the propositional calculus, while for the first forms of these operations it follows that IFL is not such model (because of definition of "$\supset$").

Analogically it can be shown that not one of the implications with numbers 3, 4, ..., 12 can be a basis of a model of the propositional calculus axioms. For example, 7-th, 8-th, ..., 12-th implicvations do not satisfy axiom (b). Therefore, the unique implication which satisfies all above axioms is the second implication (see [1]). For it and for the second forms of operations "&" and "∨" is valid that (c) - (f), but (g) and (h) are not valid. For example, if V(A) = V(B) = <0.4, 0.5>, then

$$V(A \supset (B \supset (A \,\&\, B))) = \langle 0.4,\ 0.5\rangle,$$

i.e., in this model the axiom $A \supset (B \supset (A \,\&\, B))$ is not an IFT (and therefore it is not a tautology, too).

On the other hand, in [1,2] it is shown that for the implications which numbers 1, 6, 7, 8, 9 and 10 the Modus Ponens (MP) is valid, while for the implications with numbers 2, 3, 4 and 5 - not. In similar way we can show that the last fact is valid for implications with numbers 11 and 13, while the implication 12 satisfies the MP.

It should be noted that the definitions of the operation "⊃" with numbers 6, 7, ..., 10 have the following drawbacks: they are what is usually called "external" operations (unlike conjunction, negation, etc.) and their evaluation requires exact comparison of real numbers - they are not continuous; on the other hand they seem to be not very functional - in the sense that in logical calculations they are very cumbersome.

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