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Intuitionistic M-Fuzzy Sub-Bigroup and its Bi-Level M-Sub-Bigroups

N.Palaniappan[1], R.Muthuraj[2]& P.Sundararajan[3]

[1] Professor & Chairperson, School of Mathematics, Alagappa University, Karaikudi – 630003, Tamilnadu , India; e-mail: palaniappan1950@yahoo.co.in

[2] Department of Mathematics, PSNA College of Engineering & Technology , Dindigul– 624622, Tamilnadu, India; e-mail: rmr1973@yahoo.co.in

[3] Department of Mathematics, K.S.R.College of Technology, Tiruchengod, India; e-mail: ponsundar@gmail.com

ABSTRACT: In this paper, we introduce the concept of intuitionistic M-fuzzy sub-bigroup of an M-bigroup and bi-level M-subset of an intuitionistic M-fuzzy sub-bigroup and discussed some of its properties.

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KEYWORDS: M-group, intuitionistic M-fuzzy subgroup of an M-group, intuitionistic M-fuzzy sub-bigroup of an M-bigroup, bi-level M-subset.

INTRODUCTION:

The concept of fuzzy set was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld gave the idea of fuzzy subgroups and Ranjith Biswas gave the idea of intuitionistic fuzzy subgroups.

The notion of bigroup was first introduced by P.L.Maggu in 1994. W.B. Vasantha Kandasamy and D.Meiyappan introduced concept of fuzzy sub-bigroup of a bigroup. Author N. Jacobson introduced the concept of M-group, M-subgroup.

1. Preliminaries

This section contains some definitions and results to be used in the sequel.

1.1 Definition[10]

A set $(G, +, \bullet)$ with two binary operations $+$ and $\bullet$ is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that

i. $G = G_1 \cup G_2$,

ii. $(G_1 , +)$ is a group,

iii. $(G_2 , \bullet)$ is a group.

A non-empty subset H of a bigroup $(G , + , \bullet)$ is called a sub-bigroup, if H itself is a bigroup under the operations $+$ and $\bullet$ defined on G.

1.2 Definition[6]

A group with operators is an algebraic system consisting of a group G , a set M and a function defined in the product set $M \times G$ and having values in G such that, if ma denotes the element in G determined by the element a of G and the element m of M, then m(ab) = (ma)(mb) holds for all a, b G and m M. We shall use the phrases "G is an M-group" to a group with operators.

A subgroup H of an M-group G is said to be an M-subgroup if mx H for all m M and x H.

1.3 Definition

A set (G , + , •) with two binary operation + and • is called an M-bigroup if there exist two proper subsets G_1 and G_2 of G such that

i. $G = G_1 \cup G_2$

ii. $(G_1, +)$ is an M-group.

iii. $(G_2, \bullet)$ is an M-group.

A non-empty subset H of an M-bigroup (G , + , •) is called an M-sub-bigroup, if H itself is an M-bigroup under + and • operations defined on G.

1.4 Definition[3]

Let G be a group. An intuitionistic fuzzy subset $A = \langle x , \mu_A(x) , \nu_A(x) \rangle$ of G is said to be an intuitionistic fuzzy subgroup (IFSG) of G if the following conditions are satisfied:

(i) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$, $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$ for all x and $y \in G$.

(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$, $\nu_A(x^{-1}) \leq \nu_A(x)$ for all $x \in G$.

1.5 Definition[3]

Let A be an intuitionistic fuzzy subset of S. For $\alpha , \beta \in [0,1]$, the level subset of A is the set,

$$A_{<\alpha , \beta>} = \{ x \in S : \mu_A (x) \geq \alpha \text{ and } \nu_A(x) \leq \beta \}.$$

1.6 Definition[3]

Let G be a group and A be an intuitionistic fuzzy subgroup of G .

Let $Im (\mu_A) = \{ \alpha_i : \mu_A(x) = \alpha_i$ for every $x \in G \}$ and $Im (\nu_A) = \{ \beta_i : \mu_A(x) = \beta_i$ for every $x \in G \}$. Then $\{ A_{<\alpha_i , \beta_i>} \}$ are the only level subgroups of A.

1.7 Definition

Let G be an M-group. Then an intuitionistic fuzzy sub-bigroup A of G is said to be an intuitionistic M-fuzzy subgroup of G if for all x and $y \in G$,

i. $\mu_A(mx) \geq \mu_A(x)$,

ii. $\nu_A(mx) \leq \nu_A(x)$.

1.8 Definition

Let G be an M-group and A be an intuitionistic M-fuzzy subgroup of G .

Let $Im (\mu_A) = \{ \alpha_i : \mu_A(x) = \alpha_i$ for every $x \in G \}$ and $Im (\nu_A) = \{ \beta_i : \mu_A(x) = \beta_i$ for every $x \in G \}$. Then $\{ A_{<\alpha_i , \beta_i>} \}$ are the only level M-subgroups of A.

1.9 Definition

Let $G = (G_1 \cup G_2 , + , \bullet)$ be a bigroup. Then an intuitionistic fuzzy subset A is said to be an intuitionistic fuzzy sub-bigroup of the bigroup G if there exists two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that

i. $A = A_1 \cup A_2$.

ii. $(A_1, +)$ is an intuitionistic fuzzy subgroup of $(G_1, +)$.

iii. $(A_2, \bullet)$ is a an intuitionistic fuzzy subgroup $(G_2, \bullet)$.

1.10 Definition

Let $G = (G_1 \cup G_2 , + , \bullet)$ be an M-bigroup. Then an intuitionistic fuzzy sub-bigroup A of G is said to be intuitionistic M-fuzzy sub-bigroup if

i. $\mu_A (m + x) \geq \mu_A(x)$ for all $x \in G_1$ and $m \in M$.

ii. $\nu_A (m + x) \leq \nu_A(x)$ for all $x \in G_1$ and $m \in M$.

iii. $\mu_A(m \bullet x) \geq \mu_A(x)$ for all $x \in G_2$ and $m \in M$.

iv. $\nu_A(m \bullet x) \leq \nu_A(x)$ for all $x \in G_2$ and $m \in M$.

2. The notion of bi-level M-subset of an intuitionistic M-fuzzy sub-bigroup: Definition

Let $G = (G_1 \cup G_2, +, \bullet)$ be an M-bigroup and $A = (A_1 \cup A_2, +, \bullet)$ be an intuitionistic M-fuzzy sub-bigroup of G. The bi-level M-subset of the intuitionistic M-fuzzy sub-bigroup A of G is defined as:

$A_{<\alpha,\beta>} = A_{1<\alpha,\beta>} \cup A_{2<\alpha,\beta>}$, for every $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, where e_1 denotes the identity element of the group $(G_1, +)$ and e_2 denotes the identity element of the group $(G_2, \bullet)$.

3 Remark

The conditions $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$ are essential for the bi-level M-subset to be an M-sub-bigroup, for if $\alpha \notin [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \notin [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, the bi-level M-subset need not in general be an M-sub-bigroup of an M-bigroup G.

3.1 Theorem

Every bi-level M-subset of an intuitionistic M-fuzzy sub-bigroup A of an M-bigroup G is an M-sub-bigroup of G.

Proof

Let $A = (A_1 \cup A_2, +, \bullet)$ be an intuitionistic M-fuzzy sub-bigroup of $G = (G_1 \cup G_2, +, \bullet)$. Consider the bi-level M-subset $A_{<\alpha,\beta>}$ of an intuitionistic M-fuzzy sub-bigroup A for every $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, where e_1 denotes the identity element of the group $(G_1, +)$ and e_2 denotes the identity element of the group $(G_2, \bullet)$. Then $A_{<\alpha,\beta>} = A_{1<\alpha,\beta>} \cup A_{2<\alpha,\beta>}$, where $A_{1<\alpha,\beta>}$ and $A_{2<\alpha,\beta>}$ are M-subgroups of G_1 and G_2 respectively. Hence by the definition of M-sub-bigroup $A_{<\alpha,\beta>}$ is an M-sub-bigroup of $(G, +, \bullet)$.

3.2 Theorem

Any M-sub-bigroup H of an M-bigroup G can be realized as a bi-level M-sub-bigroup of some intuitionistic M-fuzzy sub-bigroup of G.

Proof

Let $G = (G_1 \cup G_2, +, \bullet)$ be an M-bigroup.

Let $H = (H_1 \cup H_2, +, \bullet)$ be an M-sub-bigroup of G.

Define $\mu_{A1} : H_1 \rightarrow [0,1]$ and $\nu_{A1} : H_1 \rightarrow [0,1]$ by

$$\mu_{A1}(x) = \begin{cases} \alpha & \text{for } x \in H_1 \\ 0 & \text{for } x \notin H_1 \end{cases} \qquad \nu_{A1}(x) = \begin{cases} 0 & \text{for } x \in H_1 \\ \beta & \text{for } x \notin H_1 \end{cases}$$

, and define

$\mu_{A2} : H_2 \rightarrow [0,1]$ and $\nu_{A2} : H_2 \rightarrow [0,1]$ by

$$\mu_{A2}(x) = \begin{cases} \alpha & \text{for } x \in H_2 \\ 0 & \text{for } x \notin H_2 \end{cases} \qquad \nu_{A2}(x) = \begin{cases} 0 & \text{for } x \in H_2 \\ \beta & \text{for } x \notin H_2 \end{cases},$$

where $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$.
Let $x, y \in G$.
Suppose $x, y \in H$, then

i. $x, y \in H_1 \Rightarrow x + y \in H_1$
$\mu_{A1}(x + y) = \alpha$, $\mu_{A1}(x) = \alpha$, $\mu_{A1}(y) = \alpha$ and
$\nu_{A1}(x + y) = 0$, $\nu_{A1}(x) = 0$, $\nu_{A1}(y) = 0$ then
$\mu_{A1}(x + y) \geq \min\{\mu_{A1}(x), \mu_{A1}(y)\}$
$\nu_{A1}(x + y) \leq \max\{\nu_{A1}(x), \nu_{A1}(y)\}$.

ii. $x, y \in H_2 \Rightarrow xy \in H_2$
$\mu_{A2}(xy) = \alpha$, $\mu_{A2}(x) = \alpha$, $\mu_{A2}(y) = \alpha$ and
$\nu_{A2}(xy) = 0$, $\nu_{A2}(x) = 0$, $\nu_{A2}(y) = 0$ then
$\mu_{A2}(xy) \geq \min\{\mu_{A2}(x), \mu_{A2}(y)\}$
$\nu_{A2}(xy) \leq \max\{\nu_{A2}(x), \nu_{A2}(y)\}$.

iii. $x \in H_1$ and $y \notin H_1 \Rightarrow x + y \notin H_1$
$\mu_{A1}(x + y) = 0$, $\mu_{A1}(x) = \alpha$, $\mu_{A1}(y) = 0$ and
$\nu_{A1}(x + y) = \beta$, $\nu_{A1}(x) = 0$, $\nu_{A1}(y) = \beta$ then
$\mu_{A1}(x + y) \geq \min\{\mu_{A1}(x), \mu_{A1}(y)\}$
$\nu_{A1}(x + y) \leq \max\{\nu_{A1}(x), \nu_{A1}(y)\}$.

iv. $x \in H_2$ and $y \notin H_2 \Rightarrow xy \notin H_2$
$\mu_{A2}(xy) = 0$, $\mu_{A2}(x) = \alpha$, $\mu_{A2}(y) = 0$ and
$\nu_{A2}(xy) = \beta$, $\nu_{A2}(x) = 0$, $\nu_{A2}(y) = \beta$ then
$\mu_{A2}(xy) \geq \min\{\mu_{A2}(x), \mu_{A2}(y)\}$
$\nu_{A2}(xy) \leq \max\{\nu_{A2}(x), \nu_{A2}(y)\}$.

Suppose $x, y \notin H$, then

i. $x, y \notin H_1$, then $x + y \in H_1$ or $x + y \notin H_1$
$\mu_{A1}(x + y) = \alpha$ or 0, $\mu_{A1}(x) = 0$, $\mu_{A1}(y) = 0$ and
$\nu_{A1}(x + y) = 0$ or β, $\nu_{A1}(x) = \beta$, $\nu_{A1}(y) = \beta$, then
$\mu_{A1}(x + y) \geq \min\{\mu_{A1}(x), \mu_{A1}(y)\}$
$\nu_{A1}(x + y) \leq \max\{\nu_{A1}(x), \nu_{A1}(y)\}$.

ii. $x, y \notin H_2 \Rightarrow xy \in H_2$ or $xy \notin H_2$
$\mu_{A2}(xy) = \alpha$ or 0, $\mu_{A2}(x) = 0$, $\mu_{A2}(y) = 0$ and
$\nu_{A2}(xy) = 0$ or β, $\nu_{A2}(x) = \beta$, $\nu_{A2}(y) = \beta$, then
$\mu_{A2}(xy) \geq \min\{\mu_{A2}(x), \mu_{A2}(y)\}$
$\nu_{A2}(xy) \leq \max\{\nu_{A2}(x), \nu_{A2}(y)\}$.

Thus in all cases,
$(A_1, +)$ is an IFSG of G_1 and $(A_2, \bullet)$ is an IFSG of G_2.
Clearly $A = (A_1 \cup A_2, +, \bullet)$ is an intuitionistic fuzzy sub-bigroup of G,
Now, we have to prove that A is an intuitionistic M-fuzzy sub-bigroup of G.
Suppose, $m \in M$ and $x \in H_1$, then $m + x \in H_1$.
Then, $\mu_{A1}(m + x) = \alpha$, $\mu_{A1}(x) = \alpha$, and
$\nu_{A1}(m + x) = 0$, $\nu_{A1}(x) = 0$, then

$\mu_{A1}(m + x) \geq \mu_{A1}(x)$,

$\nu_{A1}(m + x) \leq \nu_{A1}(x)$.

Suppose , $m \in M$ and $x \notin H_1$,then $m + x \in H_1$ or $m + x \notin H_1$.

Then , $\mu_{A1}(m + x) = \alpha$ or 0 , $\mu_{A1}(x) = 0$, and

$\nu_{A1}(m + x) = 0$ or β , $\nu_{A1}(x) = \beta$, then

$\mu_{A1}(m + x) \geq \mu_{A1}(x)$,

$\nu_{A1}(m + x) \leq \nu_{A1}(x)$.

Clearly $(A_1, +)$ is an intuitionistic M-fuzzy sub-bigroup of G_1.

Suppose , $m \in M$ and $x \in H_2$,then $m + x \in H_2$.

Then , $\mu_{A2}(mx) = \alpha$, $\mu_{A2}(x) = \alpha$, and

$\nu_{A2}(mx) = 0$, $\nu_{A2}(x) = 0$, then

$\mu_{A2}(mx) \geq \mu_{A2}(x)$,

$\nu_{A2}(mx) \leq \nu_{A2}(x)$.

Suppose , $m \in M$ and $x \notin H_2$,then $m + x \in H_2$ or $m + x \notin H_2$.

Then , $\mu_{A2}(mx) = \alpha$ or 0 , $\mu_{A2}(x) = 0$, and

$\nu_{A2}(mx) = 0$ or β , $\nu_{A2}(x) = \beta$, then

$\mu_{A2}(mx) \geq \mu_{A2}(x)$,

$\nu_{A2}(mx) \leq \nu_{A2}(x)$.

Clearly $(A_2, \bullet)$ is an intuitionistic M-fuzzy sub-bigroup of G_2.

Clearly $A = (A_1 \cup A_2, +, \bullet)$ is an intuitionistic M-fuzzy sub-bigroup of G, where $\mu_A : G \to [0,1]$ and $\nu_A : G \to [0,1]$ are given by

$$\mu_A(x) = \begin{cases} \alpha & \text{for } x \in H \\ 0 & \text{for } x \notin H \end{cases} \qquad \nu_A(x) = \begin{cases} 0 & \text{for } x \in H \\ \beta & \text{for } x \notin H. \end{cases}$$

For this intuitionistic M-fuzzy sub-bigroup, $A_{<\alpha,\beta>} = A_{1<\alpha,\beta>} \cup A_{2<\alpha,\beta>} = H$.

3.3 Theorem

Let G be an M-bigroup and A be an intuitionistic M-fuzzy sub-bigroup of G. Two bi-level M-sub-bigroups $A_{<\alpha,\beta>}$, $A_{<\gamma,\delta>}$ with $\alpha < \gamma$ and $\delta < \beta$ of A are equal iff there is no $x \in G$ such that $\alpha \leq \mu_A(x) < \gamma$ and $\delta < \nu_A(x) \leq \beta$.

Proof

Let $A_{<\alpha,\beta>} = A_{<\gamma,\delta>}$

Suppose that there exists $x \in G$ such that $\alpha < \mu_A(x) < \gamma$ and $\delta < \nu_A(x) < \beta$, then

$$A_{<\alpha,\beta>} \subset A_{<\gamma,\delta>}$$

This implies that $x \in A_{<\alpha,\beta>}$ and $x \notin A_{<\gamma,\delta>}$, which contradicts the hypothesis.

Hence there exists no $x \in G$ such that $\alpha \leq \mu_A(x) < \gamma$ and $\delta < \nu_A(x) \leq \beta$.

Conversely, let there be no $x \in G$ such that $\alpha \leq \mu_A(x) < \gamma$ and $\delta < \nu_A(x) \leq \beta$.

Since $\alpha < \gamma$ and $\delta < \beta$, we have, $A_{<\gamma,\delta>} \subseteq A_{<\alpha,\beta>}$.

Let $x \in A_{<\alpha,\beta>}$,then $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$.

Since there exists no $x \in G$ such that $\alpha \leq \mu_A(x) < \gamma$ and $\delta < \nu_A(x) \leq \beta$, we have

$$\mu_A(x) \geq \gamma \text{ and } \nu_A(x) \leq \delta$$

which implies $A_{<\alpha,\beta>} \subseteq A_{<\gamma,\delta>}$

Hence $A_{<\alpha,\beta>} = A_{<\gamma,\delta>}$

3.4 Theorem

Let G be an M-bigroup. Let A be an intuitionistic fuzzy subset of G such that the bi-level subset $A_{<\alpha,\beta>} = A_{1<\alpha,\beta>} \cup A_{2<\alpha,\beta>}$ is a sub-bigroup of the bigroup G where $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, then A is an intuitionistic M-fuzzy sub-bigroup of G.

Proof

Let $G = (G_1 \cup G_2, +, \bullet)$ be an M-bigroup.

Let the bi-level M-subset $A_{<\alpha,\beta>}$ is an M-sub-bigroup of G.

Then $A_{<\alpha,\beta>} = A_{1<\alpha,\beta>} \cup A_{2<\alpha,\beta>}$ and $(A_{1<\alpha,\beta>}, +)$ is an M-subgroup of G_1 and $(A_{2<\alpha,\beta>}, \bullet)$ is an M-subgroup of G_2

We have to prove that A is an intuitionistic M-fuzzy sub-bigroup of the bigroup G.

Since $(A_{1<\alpha,\beta>}, +)$ is an M-subgroup of $(G_1, +)$,obviously $(A_1, +)$ is an intuitionistic M-fuzzy subgroup of $(G_1, +)$ and $(A_{2<\alpha,\beta>}, \bullet)$ is an M-subgroup of $(G_2, \bullet)$, and hence $(A_2, \bullet)$ is an intuitionistic M-fuzzy subgroup of $(G_2, \bullet)$.

Clearly $A = A_1 \cup A_2$.

This implies that A is an intuitionistic M-fuzzy sub-bigroup of the bigroup G.

Remark

As a consequence of the Theorem 3.3, the bi-level M-sub-bigroups of an intuitionistic M-fuzzy sub-bigroup A of an M-bigroup G form a chain. Since $\mu_A(x) \leq \mu_{A1}(e_1)$ or $\mu_A(x) \leq \mu_{A2}(e_2)$ and $\nu_A(x) \geq \nu_{A1}(e_1)$ or $\nu_A(x) \geq \nu_{A2}(e_2)$ for all x in G. Therefore , $A_{<\alpha_0,\beta_0>}$, $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, where $\min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\} = \alpha_0$ and $\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\} = \beta_0$ is the smallest sub-bigroup and we have the chain :

$\{e_1, e_2\} = A_{<\alpha_0,\beta_0>} \subset A_{<\alpha_1,\beta_1>} \subset A_{<\alpha_2,\beta_2>} \subset \ldots\ldots \subset A_{<\alpha_n,\beta_n>} = G$, where $\alpha_0 > \alpha_1 > \alpha_2 > \ldots > \alpha_n$ and $\beta_0 < \beta_1 < \beta_2 < \ldots\ldots < \beta_n$.

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