

On intuitionistic fuzzy β -supra open set and intuitionistic fuzzy β -supra continuous functions

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Abstract: In this paper, we introduce and investigate a new class of sets and functions between topological space called intuitionistic fuzzy β -supra open set and intuitionistic fuzzy β -supra open continuous functions respectively.

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1 Introduction and Preliminaries

The concept of intuitionistic fuzzy set is defined by Atanassov [2] as a generalization of the concept of fuzzy set given by Zadeh [9]. Using the notation of intuitionistic fuzzy sets, Çoker [3] introduced the notation of intuitionistic fuzzy topological spaces. In 1983, Mashhour et al. [5] introduced the supra topological spaces and studied s -continuous functions and s^* -continuous functions. In 1987, Abd El-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characterizations. In 1996, Keun Min [8] introduced fuzzy s -continuous, fuzzy s -open and fuzzy s -closed maps and established a number of characterizations. In 2008, Devi et al. [4] introduced the concept of supra α -open set, s α -continuous functions and studied some of the basic properties for this class of functions. In 1999, Necla Turan [6] introduced the concept of intuitionistic fuzzy supra topological space. In this paper, we are going to study the basic properties of intuitionistic fuzzy semi-supra open sets and introduce the notation of intuitionistic fuzzy semi-supra continuous functions.

Throughout this paper, by (X, τ) or simply by X we will denote the intuitionistic fuzzy supra topological space (briefly, IFSTS). For a subset A of a space (X, τ) , $\text{cl}(A)$, $\text{int}(A)$ and \bar{A} denote the closure of A the interior of A and the complement of A respectively. Each intuitionistic fuzzy supra set (briefly, IFSS) which belongs to (X, τ) is called an intuitionistic

fuzzy supra open set (briefly, IFSOS) in X . The complement \bar{A} of an IFSOS A in X is called an intuitionistic fuzzy supra closed set (IFSCS) in X .

We introduce some basic notations and results that are used in the sequel.

Definition 1.1 [2]: Let X be a non-empty fixed set and I be the closed interval $[0; 1]$. In intuitionistic fuzzy set (IFSS) A is an object of the following form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

where the mapping $\mu_A : X \rightarrow I$ and $\nu_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) for each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Obviously, every fuzzy set A on a non-empty set X is an IFS of the following form

$$A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}.$$

Definition 1.2 [2] Let A and B are IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$

(ii) $\bar{A} = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$

(iii) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X \}$

(iv) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X \}$

(v) $A = B$ iff $A \subseteq B$ and $B \subseteq A$

(vi) $\square A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}$

(vii) $\diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle \mid x \in X \}$

(viii) $1_- = \{ \langle x, 1, 0 \rangle, x \in X \}$ and $0_- = \{ \langle x, 0, 1 \rangle, x \in X \}$

We will use the notation $A = \langle x, \mu_A, \mu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$.

Definition 1.3. [6] A family τ of IFS's on X is called an intuitionistic fuzzy supra topology (IFST for short) on X if $0_- \in \tau, 1_- \in \tau$ and τ is closed under arbitrary suprema. Then we call the pair (X, τ) intuitionistic fuzzy supra topological space (IFSTS for short). Each member τ is called an intuitionistic fuzzy supra open set and the compliment of an intuitionistic fuzzy supra open set is called an intuitionistic fuzzy supra closed set. The intuitionistic fuzzy supra closure of IFS A is denoted by $s\text{-cl}(A)$. Here $s\text{-cl}(A)$ is the intersection of all intuitionistic fuzzy supra closed sets containing A . The intuitionistic fuzzy supra interior of A will be denoted by $s\text{-int}(A)$. Here, $s\text{-int}(A)$ is the union of all intuitionistic fuzzy supra open sets contained in A .

Definition 1.4. [7] Let (X, τ) be an intuitionistic fuzzy supra topological space. An IFS $A \in \text{IF}(X)$ is called

(a) intuitionistic fuzzy semi-supra open iff $A \subseteq s\text{-cl}(s\text{-int}(A))$,

(b) intuitionistic fuzzy α -supra open iff $A \subseteq s\text{-int}(s\text{-cl}(s\text{-int}(A)))$,

(c) intuitionistic fuzzy pre-supra open iff $A \subseteq s\text{-int}(s\text{-cl}(A))$.

Let f be a mapping from an ordinary set X into an ordinary set Y , if $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle \mid y \in Y \}$ is an IFST in Y , then the inverse image of B under f is an IFST defined by $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle \mid x \in X \}$

The image of IFST $A = \{ \langle y, \mu_A(y), \nu_A(y) \rangle \mid y \in Y \}$ under f is an IFST defined by $f(A) = \{ \langle y, f(\mu_A)(y), f(\nu_A)(y) \rangle \mid y \in Y \}$.

2 Intuitionistic fuzzy β -supra open set

Definition 2.1. Let (X, τ) be an intuitionistic fuzzy supra topological space. An intuitionistic fuzzy set A is called an intuitionistic fuzzy β -supra open set (briefly IF β SOS) if $A \subseteq s\text{-cl}(s\text{-int}(s\text{-cl}(A)))$. The complement of an intuitionistic fuzzy β -supra open set is called an intuitionistic fuzzy β -supra closed set.

Theorem 2.2. Every intuitionistic fuzzy supra open set is an intuitionistic fuzzy β -supra open set.

Proof: Let A be an intuitionistic fuzzy supra open set in (X, τ) . Since $A \subseteq s\text{-int}(A)$, we get $A \subseteq s\text{-cl}(A)(s\text{-int}(s\text{-cl}(A)))$, then $s\text{-int}(A) \subseteq s\text{-cl}(s\text{-int}(s\text{-cl}(A)))$.

Hence $A \subseteq s\text{-cl}(s\text{-int}(s\text{-cl}(A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.3. Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.4 \rangle, \langle 0.5, 0.6 \rangle\}$, $B = \{x, \langle 0.4, 0.2 \rangle, \langle 0.3, 0.4 \rangle\}$ and $C = \{x, \langle 0.4, 0.4 \rangle, \langle 0.4, 0.4 \rangle\}$, $\tau = \{0_-, 1_-, A, B, A \cup B\}$. Then C is called an intuitionistic fuzzy β -supra open but not an intuitionistic fuzzy supra open set.

Theorem 2.4. Every intuitionistic fuzzy α -supra open is an intuitionistic fuzzy β -supra open.

Proof: Let A be an intuitionistic fuzzy α -supra open in (X, τ) , then $A \subseteq s\text{-int}(s\text{-cl}(s\text{-int}(A)))$, it is obvious that $s\text{-int}(s\text{-cl}(s\text{-int}(A))) \subseteq s\text{-cl}(s\text{-int}(s\text{-cl}(A)))$, hence $A \subseteq s\text{-cl}(s\text{-int}(s\text{-cl}(A)))$. □

The converse of the theorem need not be true as shown by the example.

Example 2.5. Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.3 \rangle, \langle 0.5, 0.3 \rangle\}$, $B = \{x, \langle 0.1, 0.2 \rangle, \langle 0.6, 0.5 \rangle\}$ and $C = \{x, \langle 0.2, 0.3 \rangle, \langle 0.2, 0.3 \rangle\}$, $\tau = \{0_-, 1_-, A, B, A \cup B\}$. Then C is called an intuitionistic fuzzy β -supra open but not an intuitionistic fuzzy α -supra open set.

Theorem 2.6. Every intuitionistic pre supra open set is intuitionistic fuzzy β -supra open set.

Proof: Let A be an intuitionistic fuzzy pre supra open set in (X, τ) . Then $A \subseteq s\text{-int}(s\text{-cl}(A))$. Hence $A \subseteq s\text{-cl}(s\text{-int}(s\text{-cl}(A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.7. Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.3 \rangle, \langle 0.5, 0.3 \rangle\}$, $B = \{x, \langle 0.1, 0.2 \rangle, \langle 0.6, 0.5 \rangle\}$ and $C = \{x, \langle 0.2, 0.3 \rangle, \langle 0.2, 0.3 \rangle\}$, $\tau = \{0_-, 1_-, A, B, A \cup B\}$. Then C is an intuitionistic fuzzy β -supra open but not an intuitionistic fuzzy pre - supra open set.

Theorem 2.8

- i) Arbitrary union of intuitionistic fuzzy β -supra open sets are always intuitionistic fuzzy β -supra open set.
- ii) Finite intersection of intuitionistic fuzzy β -supra open sets may fail to be an intuitionistic fuzzy β -supra open set.
- iii) 1_- is an intuitionistic fuzzy β -supra open set.

Proof: (i) Let $\{A_\lambda : \lambda \in \wedge\}$ be a family of an intuitionistic fuzzy β -supra open set in a topological space X . Then for any $\lambda \in \wedge$, we have $A_\lambda \subseteq s\text{-cl}(s\text{-int}(cl(A_\lambda)))$

Hence

$$\begin{aligned} U_{\lambda \in \wedge} A_\lambda &\subseteq U_{\lambda \in \wedge} (s - cl(s - int(s - cl(A_\lambda)))) \\ &\subseteq s - cl(U_{\lambda \in \wedge} (s - int(s - cl(A_\lambda)))) \\ &\subseteq s - cl(s - int(s - cl(U_{\lambda \in \wedge} (A_\lambda)))) \end{aligned}$$

Therefore, $U_{\lambda \in \wedge} A_\lambda$ is an intuitionistic fuzzy β -supra open set.

(ii) Let $X = \{a, b\}$, $A = \{x, \langle 0.4, 0.3 \rangle, \langle 0.2, 0.4 \rangle\}$, $B = \{x, \langle 0.2, 0.3 \rangle, \langle 0.2, 0.3 \rangle\}$ and $\tau = \{0_-, 1_-, A, B, A \cup B\}$ Hence, A and B are intuitionistic fuzzy semi-supra open sets but $A \cap B$ is not an intuitionistic fuzzy β -supra open set.

Theorem 2.8.

- i) Arbitrary intersection of intuitionistic fuzzy β -supra closed sets are always an intuitionistic fuzzy β -supra closed set.
- ii) Finite union of intuitionistic fuzzy β -supra closed sets may fail to be an intuitionistic fuzzy β -supra closed set.
- iii) 0_- is an intuitionistic fuzzy β -supra closed set.

Proof: i) The proof follows immediately from Theorem 2.7.

ii) Let

$$X = \{a, b\}, A = \{x, \langle 0.2, 0.4 \rangle, \langle 0.4, 0.3 \rangle\}, B = \{x, \langle 0.2, 0.3 \rangle, \langle 0.2, 0.3 \rangle\}$$

and $\tau = \{0_-, 1_-, A, B, A \cup B\}$. Hence A and B are intuitionistic fuzzy β -supra closed but $A \cup B$ is not an intuitionistic fuzzy β -supra closed set. □

Definition:2.9. The intuitionistic fuzzy β -supra closure of a set A is denoted by $\beta s\text{-cl}(A) = \cup \{G : G \text{ is an intuitionistic fuzzy } \beta\text{-closed set in } X \text{ and } G \subseteq A\}$ and the intuitionistic fuzzy

β -supra interior of a set A is denoted by $\beta s\text{-int}(A) = \bigcap \{G : G \text{ is an intuitionistic fuzzy } \beta\text{-closed set in } X \text{ and } G \supseteq A\}$.

Remark 2.10. It is clear that $\beta s\text{-int}(A)$ is an intuitionistic fuzzy β -supra open set and $\beta s\text{-cl}(A)$ is an intuitionistic fuzzy β -supra closed set.

Theorem 2.11.

- (i) $X - \beta s\text{-int}(A) = \beta s\text{-cl}(X - A)$
- (ii) $X - \beta s\text{-cl}(A) = \beta s\text{-int}(X - A)$
- (iii) if $A \subseteq B$, then $\beta s\text{-cl}(A) \subseteq \beta s\text{-cl}(B)$ and $\beta s\text{-int}(A) \subseteq \beta s\text{-int}(B)$

Proof: It is obvious from the definition of β -supra open set.

Theorem 2.12.

- (i) $\beta s\text{-int}(A) \cap \beta s\text{-int}(B) \subseteq \beta s\text{-int}(A \cup B)$
- (ii) $\beta s\text{-int}(A \cap B) \subseteq \beta s\text{-int}(A) \cap \beta s\text{-int}(B)$
- (iii) if $A \subseteq B$, then $\beta s\text{-cl}(A) \subseteq \beta s\text{-cl}(B)$ and $\beta s\text{-int}(A) \subseteq \beta s\text{-int}(B)$.

Proof: It is obvious from the definition of β -supra open set.

Theorem 2.13.

- (i) The intersection of an intuitionistic fuzzy supra opens set and an intuitionistic fuzzy β -supra open set is an intuitionistic fuzzy β -supra open set.
- (ii) The intersection of an intuitionistic fuzzy β -supra open set and an intuitionistic fuzzy pre-supra open set is an intuitionistic fuzzy pre-supra open set.

Proof: It is obvious from the definition of β -supra open set. □

3 Intuitionistic fuzzy β -supra continuous mapping

Definition 3.1. Let (X, τ) and (Y, σ) be two intuitionistic fuzzy β -supra open sets and μ be an associated supra topology with τ . A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called intuitionistic fuzzy β -supra continuous mapping if the inverse image of each open set in Y is an intuitionistic fuzzy β -supra open in X .

Theorem 3.2. Every intuitionistic fuzzy supra continuous mapping is an intuitionistic fuzzy β -supra continuous map.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy supra continuous map and A be an open set in Y . Then $f^{-1}(A)$ is an open set in X , since μ is associated with τ . Then $\tau \subseteq \mu$. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy supra open set in X , which is an intuitionistic fuzzy supra open set in X . Hence f is an intuitionistic fuzzy β -supra continuous mapping. □

Remark 3.2. Every intuitionistic fuzzy β -supra continuous map need not be an intuitionistic fuzzy supra continuous map.

Theorem 3.3. Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ . Let f be a map from X into Y . Then the following are equivalent.

- (i) f is an intuitionistic fuzzy β -supra continuous mapping;
- (ii) The inverse image of closed sets in Y is an intuitionistic fuzzy β -closed set in X ;
- (iii) $\beta s\text{-cl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$ for every set A in Y ;
- (iv) $f(\beta s\text{-cl}(A)) \subseteq \text{cl}(f(A))$ for every set A in X ;
- (v) $f^{-1}(\text{int}(B)) \subseteq \beta s\text{-int}(f^{-1}(B))$ for every set B in Y .

Proof: (i) \Rightarrow (ii): Let A be a closed set in Y . Then $Y - A$ is open in Y , Thus $f^{-1}(Y - A) = X - f^{-1}(A)$ is βs open in X . It follows that $f^{-1}(A)$ is a βs -closed set of X .

(ii) \Rightarrow (iii): Let A be any subset of X . Since $\text{cl}(A)$ is closed in Y , then it follows that $f^{-1}(\text{cl}(A))$ is βs -closed in X . Therefore $f^{-1}(\text{cl}(A)) = \beta s\text{-cl}(f^{-1}(\text{cl}(A))) \supseteq \beta s\text{-cl}(f^{-1}(A))$.

(iii) \Rightarrow (iv): Let A be any subset of X . By (iii) we obtain $f^{-1}(\text{cl}(f(A))) \supseteq \beta s\text{-cl}(f^{-1}(f(A))) \supseteq \beta s\text{-cl}(A)$ and hence $f(\beta s\text{-cl}(A)) \subseteq \text{cl}(f(A))$.

(iv) \Rightarrow (v): Let $f(\beta s\text{-cl}(A)) \subseteq \text{cl}(f(A))$ for every set A in X . Then $\beta s\text{-cl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$, $X - \beta s\text{-cl}(A) \supseteq X - f^{-1}(\text{cl}(f(A)))$ and $\beta s\text{-int}(X - A) \supseteq f^{-1}(\text{int}(Y - f(A)))$. Then $\beta s\text{-int}(f^{-1}(B)) \supseteq f^{-1}(\text{int}(B))$. Therefore $f^{-1}(\text{int}(B)) \subseteq \beta s\text{-int}(f^{-1}(B))$, for every B in Y .

(v) \Rightarrow (i): Let A be an open set in Y . Therefore $f^{-1}(\text{int}(A)) \subseteq \beta s\text{-int}(f^{-1}(A))$, hence $f^{-1}(A) \subseteq \beta s\text{-int}(f^{-1}(A))$. But by other hand, we know that, $\beta s\text{-int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Then $f^{-1}(A) = \beta s\text{-int}(f^{-1}(A))$. Therefore, $f^{-1}(A)$ is a βs -open set. \square

Theorem 3.4. If a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a β -supra continuous and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is continuous. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is β -supra continuous.

Proof: Let A be an open set in Y , Then $f^{-1}(A)$ is an open set in X . Since $g \circ f$ is a β -supra open mapping, then $(g \circ f)(A) = f^{-1}(g^{-1}(A)) = f^{-1}(A) = \beta$ -supra open set in z . Therefore, g is a β -supra continuous mapping. \square

Theorem 3.5. Let a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy β -supra continuous mapping, if one of the following holds.

- (i) $f^{-1}(\beta s\text{-int}(A)) \subseteq \text{int}(f^{-1}(A))$ for every set A in Y .
- (ii) $\text{cl}(f^{-1}(A)) \subseteq f^{-1}(\beta s\text{-cl}(A))$ for every set A in Y .
- (iii) $f(\text{cl}(B)) \subseteq \beta s\text{-cl}(f(B))$ for every set B in X .

Proof: Let A be any open set of Y , if condition (i) is satisfied, then

$$f^{-1}(\beta\text{s-int}(A)) \subseteq \text{int}(f^{-1}(A)).$$

We get, $f^{-1}(A) \subseteq \text{int}(f^{-1}(A))$. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy supra open set. Every intuitionistic fuzzy supra open set is an intuitionistic fuzzy β -supra open set. Hence, f is an intuitionistic fuzzy β -supra continuous function.

If condition (ii) is satisfied, then we can easily prove that f is an intuitionistic fuzzy β -supra continuous function. If condition (iii) is satisfied and A is any open set of Y . Then, $f^{-1}(A)$ is a set in X and $f(\text{cl}(f^{-1}(A))) \subseteq \beta\text{s-cl}(f(f^{-1}(A)))$. This implies $f(\text{cl}(f^{-1}(A))) \subseteq \beta\text{s-cl}(A)$. This is nothing but condition (ii).

Hence f is an intuitionistic fuzzy β -supra continuous function. \square

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