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Further remarks about the unconscientious experts' evaluations in the intuitionistic fuzzy environment

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Abstract: In the intuitionistic fuzzy environment, unconscientious opinions may cause problems in the data processing. In this paper, new ways of correction of the unconscientious experts' evaluations are proposed.

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In the Notes on Intuitionistic Fuzzy Sets, Vol. 18, 2012, No. 3, is included the paper on the unconscientious experts' evaluations. In it, I have proposed some ways of correction of the unconscientious intuitionistic fuzzy evaluations. The present paper contains further remarks on this subject. The introduction, given below, is the same as in [2], but I decided to leave it for consistency of this paper.

1 Introduction

Intuitionistic fuzzy (IF) sets are first introduced by Krassimir T. Atanassov in 1983. The latest developments of the theory are presented in the monograph [1]. In one of the subsections are discussed the issues regarding the use of experts' opinions to determination of the membership degree and the non-membership degree, with which the evaluated variant belong/not belong to the IF set of variants satisfying certain criterion.

The problem arises if an expert is *more than 100% sure* that the variant belongs either to the set or to the complement of this set.

More precisely, we can describe this fact in terms of membership, and non-membership functions as follows.

Let E_i , I = 1, …, n , be an $i^{\text{-th}}$ expert from the group of n experts. Following Atanassov ([1], p.12) we call the expert E_i *unconscientious*, if among his estimations $\{\langle \mu_{i,j}, \nu_{i,j}\rangle \mid j \in J_i\}$, where $J = \bigcup_{i=1}^{n} J_i$ is an index set (related to the evaluated variants), there exists an estimation for which $\mu_{i,j} \le 1$ and $\nu_{i,j} \le 1$, but $\mu_{i,j} + \nu_{i,j} > 1$.

Let us call the IF value $\langle \mu_{i,j}, \nu_{i,j}\rangle$ for which $\mu_{i,j} \le 1$ and $\nu_{i,j} \le 1$, but $\mu_{i,j} + \nu_{i,j} > 1$ an *unconscientious evaluation* (UE) of $j^{\text{-th}}$ variant (feature, event) by the $i^{\text{-th}}$ expert.

From now on, the UE $\langle \mu_{i,j} , \nu_{i,j}\rangle$ we denote, for shortly, as UE $\langle \mu, \nu\rangle$.

To apply the intuitionistic fuzzy sets theory to the processing of evaluations, the UE $\langle \mu, \nu\rangle$ must be adjusted (convert) to the correct IF value $\langle \overline{\mu}, \overline{\nu} \rangle$ where $\overline{\mu}, \overline{\nu}, \overline{\pi} \in [0, 1]$ and $\overline{\mu} + \overline{\nu} \in [0, 1]$, with hesitation margin $\overline{\pi} = 1 - \overline{\mu} - \overline{\nu}$.

Atanassov notes that the fact of existence of this kind of problems by the evaluation of events distinguishes the decision aid in the intuitionistic fuzzy environment from the decision aid in the (classical) fuzzy environment, where such unconscientious evaluations do not exist (or are easy to correction).

No general condition has been given that should be fulfilled in order to the conversion can being considered as proper.

I think that the conversion's mapping should fulfill at least the property given below.

Property (P1):

a) if $\mu \ge \nu$, then $\overline{\mu} \ge \overline{\nu}$;

b) if $\mu \le \nu$, then $\overline{\mu} \le \overline{\nu}$.

In the terms of intuitionistic fuzzy logic, when we consider the UE $\langle \mu, \nu\rangle$ as the logical truth-value of the hadjudicate about the possession of a given attribute by the object, the property (P1) can be written as:

Property (P1′):

a) if $\langle \mu, \nu\rangle$ is an IFT, then $\langle \overline{\mu}, \overline{\nu} \rangle$ is an IFT;

b) if $\langle \mu, \nu\rangle$ is an IFcT, then $\langle \overline{\mu}, \overline{\nu} \rangle$ is an IFcT.

Let us recall, that we call the IF value $\langle a, b\rangle$ an Intuitionistic Fuzzy Tautology (IFT) *iff* $a \ge b$, and, similarly, an Intuitionistic Fuzzy co-Tautology (IFcT) *iff* $a \le b$.

I think that in the case of unconscientious experts' evaluations another problem should be considered.

Problem 1: If $\langle \mu, \nu\rangle$ is an UE, then, for the corrected value $\langle \overline{\mu}, \overline{\nu} \rangle$, should be:

a) $\overline{\pi} = 0$;

b) $\overline{\pi} > 0$;

c) $\overline{\pi}$ does not have to meet any conditions.

I am not able to solve Problem 1.

On the one hand, it can be concluded that an *expert* is a serious man, and he does not specify that is more than 100% sure. The *expert* is, at most, 100% sure of his opinion, and the surplus of more than 100% is irrelevant. It seems to be rational because this type of *expert's* mistake can happen just by accident.

On the other hand, it is reasonable that the *unconscientious* expert is, in fact, *unsure* and his estimation should be considered as *uncertain* with the hesitation degree greater than 0. In this case the degree $\overline{\pi}$ should be an increasing (non-decreasing?) function of the sum $\mu + \nu$. It seems to be rational too, because the greater sum $\mu + \nu$ means the greater *un-precision* of the evaluation of the variant by the expert.

Problem 1 can be described also in terms of *accuracy* of the IF value $\langle \overline{\mu}, \overline{\nu} \rangle$. The accuracy is defined as: *accuracy*($\langle \overline{\mu}, \overline{\nu} \rangle$) = $\overline{\mu} + \overline{\nu}$. In the problem 1 we would consider the question: Should *accuracy*($\langle \overline{\mu}, \overline{\nu} \rangle$) be equivalent to 1, or should it be less than 1, or whether does not have to meet any conditions.

In the cited monograph [1], Atanassov proposed five ways for the adjustment of the values in the *unconscientious experts' case*. In the [2], are proposed another four. Further ways are proposed as follows.

2 A note on the fuzzy negation

A fuzzy negation is defined (see eg [3], p. 52) as a function N: $[0, 1] \to [0,1]$, fulfilling at least the conditions:

(C1) $N(0) = 1$ and $N(1) = 0$;

(C2) if $a \le b$ then $N(a) \ge N(b)$, for any $a, b \in [0, 1]$.

Besides the above conditions another two are often given:

(C3) $N(N(a)) = a$, for any $a \in [0, 1]$;

(C4) N is a continuous function.

The conditions (C1) – (C4) do not allow for unequivocal determination of the function N.

In the literature, we can distinguish several classes of fuzzy negation. The negation N_λ^S in form:

$$N_\lambda^S(a) = \frac{1-a}{1+\lambda a},$$

where $\lambda > -1$, is called a *Sugeno negation*.

The negation N_w^Y in form:

$$N_w^Y(a) = (1-a^w)^{\frac{1}{w}},$$

where $w > 0$, is called a *Yager negation*.

These negations, with a simple modification of the parameter range, can be used as a basis of the correction of unconscientious evaluations in the intuitionistic fuzzy environment

3 New ways of the correction of unconscientious evaluations

Two ways of correction of unconscientious evaluations are given below. Let $\langle\mu, \nu\rangle$ be an UE.

Way 1: We calculate the corrected degrees as

$$\overline{\mu} = N_\lambda^S(\nu) = \frac{1-\nu}{1+\lambda\nu},$$

$$\overline{\nu} = N_\lambda^S(\mu) = \frac{1-\mu}{1+\lambda\mu},$$

where N_λ^S is Sugeno negation with parameter $\lambda \geq 0$.

The correction is well-defined.

For the UE $\langle\mu, \nu\rangle$ we have $\mu, \nu \in [0, 1]$ and $\mu + \nu > 1$, therefore $\overline{\mu}, \overline{\nu} \in [0, 1]$. The sum $\overline{\mu} + \overline{\nu}$ equals

$$N_\lambda^S(\mu) + N_\lambda^S(\nu) = \frac{1-\nu}{1+\lambda\nu} + \frac{1-\mu}{1+\lambda\mu} = \frac{\lambda(\mu+\nu-2\mu\nu)+2-\mu-\nu}{(1+\lambda\mu)(1+\lambda\nu)},$$

and for $\lambda \geq 0$ it is

$$\overline{\mu} + \overline{\nu} \leq 1 \;\; iff \;\; \frac{\lambda(\mu+\nu-2\mu\nu)+2-\mu-\nu}{(1+\lambda\mu)(1+\lambda\nu)} \leq 1 \;\; iff \;\; \lambda(\lambda+2)\mu\nu + \mu + \nu - 1 \geq 0,$$

and this holds.

It can be shown that for $\lambda \in (-1, 0)$, in general, could be $\overline{\mu} + \overline{\nu} > 1$, which does not cause the correction of the unconscientious evaluation.

Furthermore,

$$\overline{\pi} = 1 - \overline{\mu} - \overline{\nu} = \frac{\lambda(\lambda+2)\mu\nu+\mu+\nu-1}{(1+\lambda\mu)(1+\lambda\nu)} \in (0, 1],$$

what means that there always exists a positive hesitation margin of corrected degree.

The property (P1) is fulfilled.

The special case of this correction, for $\lambda = 0$, is presented in [2].

Way 2: We calculate the corrected degrees as

$$\overline{\mu} = N_w^Y(\nu) = (1-\nu^w)^{\frac{1}{w}},$$

$$\overline{\nu} = N_w^Y(\mu) = (1-\mu^w)^{\frac{1}{w}},$$

where N_w^Y is Yager's negation with parameter $w \in (0, 1]$.

The correction is well-defined.

For the UE $\langle\mu, \nu\rangle$ we have $\overline{\mu}, \overline{\nu} \in [0, 1]$.

The sum $\overline{\mu} + \overline{\nu}$ equals

$$N_w^Y(\mu) + N_w^Y(\nu) = (1-\nu^w)^{\frac{1}{w}} + (1-\mu^w)^{\frac{1}{w}},$$

and it is $\overline{\mu} + \overline{\nu} \leq 1$, because

$$(1-\nu^w)^{\frac{1}{w}} + (1-\mu^w)^{\frac{1}{w}} \leq 1 - \nu + 1 - \mu = 2 - (\mu + \nu) < 1.$$

The above inequality holds because for any $x \in [0, 1]$ and $w \in (0, 1]$, there is $x^w + (1-x)^w \geq 1$, what is equivalent to $1-x^w \leq (1-x)^w$ and also $(1-x^w)^{\frac{1}{w}} \leq 1-x$.

For $w > 1$, in general, could be $\overline{\mu} + \overline{\nu} > 1$, which does not cause the correction of the unconscientious evaluation.

Furthermore,

$$\overline{\pi} = 1 - \overline{\mu} - \overline{\nu} = 1 - (1-\nu^w)^{\frac{1}{w}} - (1-\mu^w)^{\frac{1}{w}} \in (0, 1],$$

what means that there exist always a positive hesitation margin of corrected degree.

The property (P1) is fulfilled.

The special case of Way 2, for $w = 1$, is the same as the special case of Way 1 (for $\lambda = 0$).

4 Conclusion

In the intuitionistic fuzzy environment unconscientious opinions may cause problems in the data processing. In this paper new ways of correction of the unconscientious experts evaluations are proposed. The basic property, which should be fulfilled in order to the conversion can being considered as proper, is given.

References

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