Generalized intuitionistic fuzzy contra continuity

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Abstract: In this paper the concept of generalized intuitionistic fuzzy contra continuous function, strongly generalized intuitionistic fuzzy contra continuous function and generalized intuitionistic fuzzy contra irresolute are studied. The concepts of generalized intuitionistic fuzzy S-closed and strongly generalized intuitionistic fuzzy S-closed are studied. The concepts of generalized intuitionistic fuzzy compact spaces and generalized intuitionistic fuzzy almost compact spaces are established. The concepts of generalized intuitionistic fuzzy filter and intuitionistic fuzzy C-convergent are established. Some interesting properties are investigated besides giving several examples.

Keywords: Generalized intuitionistic fuzzy contra continuity, strongly generalized intuitionistic fuzzy contra continuity, generalized intuitionistic fuzzy contra irresolute, generalized intuitionistic fuzzy S-closed, generalized intuitionistic fuzzy compact spaces, generalized intuitionistic fuzzy almost compact spaces, generalized intuitionistic fuzzy filter and intuitionistic fuzzy C-convergent.

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1   Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L. A. Zadeh [16]. Fuzzy sets have applications in many field such as information [14] and control [15]. The theory of fuzzy topological space was introduced and developed by C. L. Chang [7] and since then various notions in classical topology have been extended to fuzzy topological space. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2 - 4]. Later, this concept was generalized to "intuitionistic L - fuzzy sets"by Atanassov and Stoeva [5]. The concepts of on some generalizations of fuzzy continuous functions was introduced by G. Bala-subramanian and P. Sundaram [6]. The concepts of Generalized intuitionistic fuzzy closed sets was introduced by R. Dhavaseelan, E. Roja and M. K. Uma [9]. The concepts of fuzzy contra

continuous was introduced by E. Ekici and E. Kerre [10]. In this paper, the concept of generalized intuitionistic fuzzy contra continuous function, strongly generalized intuitionistic fuzzy contra continuous function and generalized intuitionistic fuzzy contra irresolute are studied. The concepts of generalized intuitionistic fuzzy compact spaces and almost generalized intuitionistic fuzzy compact spaces are established. The concepts of generalized intuitionistic fuzzy filter and almost generalized intuitionistic fuzzy convergent are established. Some interesting properties are investigated besides giving several examples.

2 Preliminaries

Definition: 2.1. *[3] Let X be a nonempty fixed set. An intuitionistic fuzzy set(IFS for short) A is an object having the form $A = \{\langle x, \mu_A(x), \delta_A(x) \rangle : x \in X\}$ where the function $\mu_A : X \to I$ and $\delta_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$)and the degree of nonmembership($\delta_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \delta_A(x) \leq 1$ for each $x \in X$.*

Definition: 2.2. *[3] Let X be a nonempty set and the intuitionistic fuzzy sets A and B in the form $A = \{\langle x, \mu_A(x), \delta_A(x) \rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \delta_B(x) \rangle : x \in X\}$. Then*

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\delta_A(x) \geq \delta_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \delta_A(x), \mu_A(x) \rangle : x \in X\}$;

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \delta_A(x) \vee \delta_B(x) \rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \delta_A(x) \wedge \delta_B(x) \rangle : x \in X\}$;

(f) $[\ ]A = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X\}$;

(g) $\langle \rangle A = \{\langle x, 1 - \delta_A(x), \delta_A(x) \rangle : x \in X\}$.

Definition: 2.3. *[3] Let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in X. Then*

(a) $\bigcap A_i = \{\langle x, \wedge \mu_{A_i}(x), \vee \delta_{A_i}(x) \rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee \mu_{A_i}(x), \wedge \delta_{A_i}(x) \rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing intuitionistic fuzzy topological spaces, we must introduce the intuitionistic fuzzy sets $0_\sim$ and $1_\sim$ in X as follows:

Definition: 2.4. *[3] $0_\sim = \{\langle x, 0, 1 \rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle : x \in X\}$.*

Here are the basic properties of inclusion and complementation:

Corollary: 2.1. *[3] Let A, B, C be intuitionistic fuzzy sets in X. Then,*

(a) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cup C \subseteq B \cup D$ and $A \cap C \subseteq B \cap D$,

(b) $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(c) $A \subseteq C$ and $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(d) $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$,

(e) $\overline{A \cup B} = \overline{A} \cap \overline{B}$,

(f) $\overline{A \cap B} = \overline{A} \cup \overline{B}$,

(g) $A \subseteq B \Rightarrow \overline{B} \subseteq \overline{A}$,

(h) $\overline{\overline{(A)}} = A$,

(i) $\overline{1_\sim} = 0_\sim$,

(j) $\overline{0_\sim} = 1_\sim$.

Now we shall define the image and preimage of intuitionistic fuzzy sets. Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

Definition: 2.5. *[3] (a) If $B = \{\langle y, \mu_B(y), \delta_B(y)\rangle : y \in Y\}$ is an intuitionistic fuzzy set in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\delta_B)(x)\rangle : x \in X\}$.*
(b) If $A = \{\langle x, \lambda_A(x), \vartheta_A(x)\rangle : x \in X\}$ is an intuitionistic fuzzy set in X, then the image of A under f, denoted by $f(A)$, is the intuitionistic fuzzy set in Y defined by $f(A) = \{\langle y, f(\lambda_A)(y), (1 - f(1 - \vartheta_A))(y)\rangle : y \in Y\}$.
where

$$f(\lambda_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

$$(1 - f(1 - \vartheta_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \vartheta_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise}, \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\vartheta_A)$ for $1 - f(1 - \vartheta_A)$.

Corollary: 2.2. *[3] Let A, $A_i(i \in J)$ be intuitionistic fuzzy sets in X, B, $B_i(i \in K)$ be intuitionistic fuzzy sets in Y and $f : X \to Y$ a function. Then,*

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(c) $A \subseteq f^{-1}(f(A))$ { *If f is injective, then* $A = f^{-1}(f(A))$ },

(d) $f(f^{-1}(B)) \subseteq B$ *{ If f is surjective, then $f(f^{-1}(B)) = B$ },*

(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,

(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,

(g) $f(\bigcup A_i) = \bigcup f(A_i)$,

(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$ *{ If f is injective, then $f(\bigcap A_i) = \bigcap f(A_i)$ },*

(i) $(f^{-1}(1_\sim) = 1_\sim$,

(j) $f^{-1}(0_\sim) = 0_\sim$,

(k) $f(1_\sim) = 1_\sim$, *if f is surjective*

(l) $f(0_\sim) = 0_\sim$,

(m) $\overline{f(A)} \subseteq f(\overline{A})$, *if f is surjective,*

(n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Definition: 2.6. *[13] An intuitionistic fuzzy set A of intuitionistic fuzzy topological space X is called an intuitionistic fuzzy regular closed set if $IFcl(IFint(A)) = A$.*

Equivalently An intuitionistic fuzzy set A of intuitionistic fuzzy topological space X is called an intuitionistic fuzzy regular open set if $IFint(IFcl(A)) = A$.

Definition: 2.7. *[9] Let (X,T) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in (X,T) is said to be generalized intuitionistic fuzzy closed (in shortly GIF closed) if $IFcl(A) \subseteq G$ whenever $A \subseteq G$ and G is intuitionistic fuzzy open. The complement of a GIF-closed set is GIF open.*

Definition: 2.8. *[9] Let (X,T) be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in X. Then intuitionistic fuzzy generalized closure (in short IFGcl) and intuitionistic fuzzy generalized interior (in short IFGint) of A are defined by*

(a) $IFGcl(A) = \bigcap\{G$: *G is a GIF closed set in X and $A \subseteq G$ }.*

(b) $IFGint(A) = \bigcup\{G$: *G is a GIF open set in X and $A \supseteq G\}$.*

Definition: 2.9. *[11] A nonvoid family $\mathbb{F}$ of GIF sets on X is said to be generalized intuitionistic fuzzy filter (in short GIF filter) if*

(1) $0_\sim \notin \mathbb{F}$

(2) If A, $B \in \mathbb{F}$ then $A \cap B \in \mathbb{F}$

(3) If $A \in \mathbb{F}$ and $A \subset B$ then $B \in \mathbb{F}$

3 Generalized intuitionistic fuzzy contra continuous function

Definition: 3.1. *Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces.*

(a) *A map $f : (X, T) \rightarrow (Y, S)$ is called intuitionistic fuzzy contra continuous(in short IF contra continuous)if the inverse image of every open set in (Y, S) is intuitionistic fuzzy closed in (X, T).*

   Equivalently if the inverse image of every closed set in (Y, S) is intuitionistic fuzzy open in (X, T).

(b) *A map $f : (X, T) \rightarrow (Y, S)$ is called generalized intuitionistic fuzzy contra continuous(in short GIF contra continuous)if the inverse image of every open set in (Y, S) is GIF closed in (X, T).*

   Equivalently if the inverse image of every closed set in (Y, S) is GIF open in (X, T).

(c) *A map $f : (X, T) \rightarrow (Y, S)$ is called GIF contra irresolute if the inverse image of every GIF closed set in (Y, S) is GIF open in (X, T).*

   Equivalently if the inverse image of every GIF open set in (Y, S) is GIF closed in (X, T).

(d) *A map $f : (X, T) \rightarrow (Y, S)$ is said to be strongly GIF contra continuous if the inverse image of every GIF open set in (Y, S) is intuitionistic fuzzy closed in (X, T).*

   Equivalently if the inverse image of every GIF closed set in (Y, S) is intuitionistic fuzzy open in (X, T).

Proposition: 3.1. Let $f : (X, T) \rightarrow (Y, S)$ be a bijective map. Then f is GIF contra continuous mapping if $IFcl(f(A)) \subseteq f(IFGint(A))$ for every intuitionistic fuzzy set A in (X, T).

Proposition: 3.2. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a map. Suppose that one of the following properties hold.

(a) $f(IFGcl(A)) \subseteq IFint(f(A))$ for each intuitionistic fuzzy set A in (X, T).

(b) $IFGcl(f^{-1}(B)) \subseteq f^{-1}(IFint(B))$ for each intuitionistic fuzzy set B in (Y, S).

(c) $f^{-1}(IFcl(B)) \subseteq IFGint(f^{-1}(B))$ for each intuitionistic fuzzy set B in (Y, S).
   Then f is GIF contra continuous mapping.

Proposition: 3.3. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a map. Suppose that one of the following properties hold.

(a) $f^{-1}(IFGcl(A)) \subseteq IFGint(IFGcl(f^{-1}(A))$ for each intuitionistic fuzzy set A in (Y, S).

(b) $IFGcl(IFGint(f^{-1}(A))) \subseteq f^{-1}(IFGint(A))$ for each intuitionistic fuzzy set A in (Y, S).

(c) $f(IFGcl(IFGint(A))) \subseteq IFGint(f(A))$ for each intuitionistic fuzzy set A in (X, T).

(d) $f(IFGcl(A)) \subseteq IFGint(f(A))$ for each intuitionistic fuzzy set A in (X, T).
   Then f is GIF contra continuous mapping.

Proposition: 3.4. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is intuitionistic fuzzy contra continuous then it is GIF contra continuous.

The converse of Proposition 3.4 is not true. See Example 3.1

Example: 3.1. Let $X = \{a, b, c\}$. Define intuitionistic fuzzy sets A and B as follows

$$A = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}) \rangle,$$

and

$$B = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.4}, \tfrac{c}{0.3}), (\tfrac{a}{0.5}, \tfrac{b}{0.6}, \tfrac{c}{0.7}) \rangle.$$

Then $T = \{0_\sim, 1_\sim, A\}$ and $S = \{0_\sim, 1_\sim, B\}$ are intuitionistic fuzzy topologies on X. Thus (X, T) and (X, S) are intuitionistic fuzzy topological spaces. Define $f : (X, T) \to (X, S)$ as $f(a) = b, f(b) = a, f(c) = c$. Clearly f is GIF contra continuous. But f is not intuitionistic fuzzy contra continuous. Since, $f^{-1}(B) \notin T$ for $B \in S$.

Proposition: 3.5. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is GIF contra irresolute then it is GIF contra continuous.

The converse of Proposition 3.5 is not true. See Example 3.2.

Example: 3.2. Let $X = \{a, b, c\}$. Define the intuitionistic fuzzy sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5}) \rangle,$$
$$B = \langle x, (\tfrac{a}{0.7}, \tfrac{b}{0.6}, \tfrac{c}{0.5}), (\tfrac{a}{0.3}, \tfrac{b}{0.4}, \tfrac{c}{0.5}) \rangle$$

and

$$C = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}) \rangle.$$

$T = \{0_\sim, 1_\sim, A, B\}$ and $S = \{0_\sim, 1_\sim, C\}$ are intuitionistic fuzzy topologies on X. Thus (X, T) and (X, S) are intuitionistic fuzzy topological spaces. Define $f : (X, T) \to (X, S)$ as follows $f(a) = b, f(b) = a, f(c) = c$. Clearly f is GIF contra continuous. But f is not GIF contra irresolute. Since $D = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5}) \rangle$ is GIF closed in (X, S), $f^{-1}(D)$ is not GIF-open in (X, T).

Proposition: 3.6. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is strongly GIF contra continuous then f is intuitionistic fuzzy contra continuous

The converse Proposition 3.6 is not true. see Example 3.3.

Example: 3.3. Let $X = \{a, b, c\}$. Define the intuitionistic fuzzy sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.2}, \tfrac{c}{0.2}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}) \rangle,$$
$$B = \langle x, (\tfrac{a}{0.1}, \tfrac{b}{0.1}, \tfrac{c}{0.1}), (\tfrac{a}{0.9}, \tfrac{b}{0.9}, \tfrac{c}{0.9}) \rangle$$

and

$$C = \langle x, (\tfrac{a}{0.9}, \tfrac{b}{0.9}, \tfrac{c}{0.9}), (\tfrac{a}{0.1}, \tfrac{b}{0.1}, \tfrac{c}{0.1}) \rangle.$$

$T = \{0_\sim, 1_\sim, A, B\}$ and $S = \{0_\sim, 1_\sim, C\}$ are intuitionistic fuzzy topologies on X. Thus (X, T) and (X, S) are intuitionistic fuzzy topological spaces. Define $f : (X, T) \to (X, S)$ as follows $f(a) = a, f(b) = b, f(c) = b$. Clearly f is intuitionistic fuzzy contra continuous. But f is not strongly GIF contra continuous. Since $D = \langle x, (\tfrac{a}{0.9}, \tfrac{b}{0.99}, \tfrac{c}{0.9}), (\tfrac{a}{0.1}, \tfrac{b}{0}, \tfrac{c}{0.1}) \rangle$ is GIF open in (X, S), $f^{-1}(D)$ is not intuitionistic fuzzy closed in (X, T).

Proposition: 3.7. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is strongly GIF contra continuous then f is GIF contra continuous.

The converse Proposition 3.7 is not true. See Example 3.4.

Example: 3.4. Let $X = \{a, b, c\}$. Define the intuitionistic fuzzy sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5}) \rangle,$$
$$B = \langle x, (\tfrac{a}{0.6}, \tfrac{b}{0.7}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.3}, \tfrac{c}{0.5}) \rangle$$

and

$$C = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}) \rangle.$$

$T = \{0_\sim, 1_\sim, A, B\}$ and $S = \{0_\sim, 1_\sim, C\}$ are intuitionistic fuzzy topologies on X . Thus (X, T) and (X, S) are intuitionistic fuzzy topological spaces. Define $f : (X, T) \to (X, S)$ as follows $f(a) = c, f(b) = c, f(c) = c$. Clearly f is GIF contra continuous. But f is not strongly GIF contra continuous. Since $D = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.3}) \rangle$ is a GIF open set in (X, S), $f^{-1}(D)$ is not intuitionistic fuzzy closed in (X, T).

Proposition: 3.8. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If $f : (X, T) \to (Y, S)$ is strongly GIF contra continuous then f is GIF contra irresolute.

The converse Proposition 3.8 is not true. See Example 3.5.

Example: 3.5. Let $X = \{a, b, c\}$. Define the intuitionistic fuzzy sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.9}, \tfrac{b}{0.9}, \tfrac{c}{0.9}), (\tfrac{a}{0.1}, \tfrac{b}{0.1}, \tfrac{c}{0.1}) \rangle,$$
$$B = \langle x, (\tfrac{a}{0.99}, \tfrac{b}{0.99}, \tfrac{c}{0.99}), (\tfrac{a}{0}, \tfrac{b}{0.01}, \tfrac{c}{0}) \rangle$$

and

$$C = \langle x, (\tfrac{a}{0.9}, \tfrac{b}{0.9}, \tfrac{c}{0.9}), (\tfrac{a}{0.1}, \tfrac{b}{0}, \tfrac{c}{0.05}) \rangle.$$

$T = \{0_\sim, 1_\sim, A, B\}$ and $S = \{0_\sim, 1_\sim, C\}$ are intuitionistic fuzzy topologies on X. Thus (X, T) and (X, S) are intuitionistic fuzzy topological spaces. Define $f : (X, T) \to (X, S)$ as follows $f(a) = a, f(b) = c, f(c) = b$. Clearly f is GIF contra irresolute. But f is not strongly GIF contra continuous. Since $D = \langle x, (\tfrac{a}{0.99}, \tfrac{b}{0.99}, \tfrac{c}{0.99}), (\tfrac{a}{0.01}, \tfrac{b}{0}, \tfrac{c}{0.01}) \rangle$ is a GIF open set in (X, S). $f^{-1}(D)$ is not intuitionistic fuzzy closed in (X, T).

Proposition: 3.9. Let (X, T), (Y, S) and (Z, R) be any three intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be maps.

(i) If f is GIF contra irresolute and g is GIF contra continuous then $g \circ f$ is GIF continuous.

(ii) If f is GIF contra irresolute and g is GIF continuous then $g \circ f$ is GIF contra continuous.

(iii) If f is GIF irresolute and g is GIF contra continuous then $g \circ f$ is GIF contra continuous.

(iv) If f is strongly GIF contra continuous and g is GIF contra continuous then $g \circ f$ is intuitionistic fuzzy continuous.

(v) If f is strongly GIF contra continuous and g is GIF continuous then $g \circ f$ is intuitionistic fuzzy contra continuous.

(vi) If f is strongly GIF continuous and g is GIF contra continuous then $g \circ f$ is intuitionistic fuzzy contra continuous.

Definition: 3.2. *An intuitionistic fuzzy topological space (X, T) is said to be intuitionistic fuzzy $T_{1/2}$ if every GIF closed set in (X, T) is intuitionistic fuzzy closed in (X, T).*

Proposition: 3.10. Let (X, T), (Y, S) and (Z, R) be any three intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be mapping and (Y, S) be intuitionistic fuzzy $T_{1/2}$ if f and g are GIF contra continuous then $g \circ f$ is GIF continuous.

The Proposition 3.10 is not valid if (Y, S) is not intuitionistic fuzzy $T_{1/2}$. See Example 3.6.

Example: 3.6. Let $X = \{a, b, c\}$. Define the intuitionistic fuzzy sets A, B, C and D as follows.

$$A = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}) \rangle,$$
$$B = \langle x, (\tfrac{a}{0.6}, \tfrac{b}{0.6}, \tfrac{c}{0.6}), (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.3}) \rangle,$$
$$C = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}) \rangle$$

and

$$D = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.4}, \tfrac{c}{0.3}), (\tfrac{a}{0.5}, \tfrac{b}{0.6}, \tfrac{c}{0.7}) \rangle.$$

Then the family $T = \{0_\sim, 1_\sim, A, B\}$, $S = \{0_\sim, 1_\sim, C\}$ and $R = \{0_\sim, 1_\sim, D\}$ are intuitionistic fuzzy topologics on X. Thus (X, T), (X, S) and (X, R) are intuitionistic fuzzy topological spaces. Also define $f : (X, T) \to (X, S)$ as $f(a) = a, f(b) = b, f(c) = b$ and $g : (X, S) \to (X, R)$ as $g(a) = b, g(b) = a, g(c) = c$. Then f and g are GIF contra continuous functions. But $g \circ f$ is not GIF continuous. For D is intuitionistic fuzzy open in (X, R). $f^{-1}(g^{-1}(D))$ is not GIF open in (X, T). $g \circ f$ is not GIF continuous. Further (X, S) is not intuitionistic fuzzy $T_{1/2}$.

Proposition: 3.11. Let (X, T), (Y, S) and (Z, R) be any three intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be mapping and (Y, S) be intuitionistic fuzzy $T_{1/2}$ if f is intuitionistic fuzzy contra continuous and g is GIF contra irresolute then $g \circ f$ is strongly GIF continuous.

The Proposition 3.11 is not valid if (Y, S) is not intuitionistic fuzzy $T_{1/2}$. See Example 3.7.

Example: 3.7. Let $X = \{a, b, c\}$. Define the intuitionistic fuzzy sets A, B, C and D as follows.

$$A = \langle x, (\tfrac{a}{0.9}, \tfrac{b}{0.9}, \tfrac{c}{0.9}), (\tfrac{a}{0.1}, \tfrac{b}{0.1}, \tfrac{c}{0.1}) \rangle,$$

$$B = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}) \rangle,$$

$$C = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}) \rangle$$

and

$$D = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.2}, \tfrac{c}{0.3}), (\tfrac{a}{0.6}, \tfrac{b}{0.8}, \tfrac{c}{0.7}) \rangle.$$

Then the family $T = \{0_\sim, 1_\sim, A, B\}$, $S = \{0_\sim, 1_\sim, C\}$ and $R = \{0_\sim, 1_\sim, D\}$ are intuitionistic fuzzy topologics on X. Thus (X, T) , (X, S) and (X, R) are intuitionistic fuzzy topological spaces. Also define $f : (X, T) \to (X, S)$ as $f(a) = a, f(b) = a, f(c) = b$ and $g : (X, S) \to (X, R)$ as $g(a) = c, g(b) = a, g(c) = b$. Then f is intuitionistic fuzzy contra continuous and g is GIF contra irresolute . But $g \circ f$ is not strongly GIF continuous. For D is GIF open in (X, R). $f^{-1}(g^{-1}(D))$ is not intuitionistic fuzzy open in (X, T). $g \circ f$ is not strongly GIF continuous. Further (X, S) is not intuitionistic fuzzy $T_{1/2}$.

Definition: 3.3. *Let X be a non-empty set and $x \in X$ a fixed element in X. If $r \in I_0$, $s \in I_1$ are fixed real number such that $r + s \leq 1$ then the intuitionistic fuzzy set $x_{r,s} = \langle y, x_r, x_s \rangle$ is called an intuitionistic fuzzy point in X, where r denotes the degree of membership of $x_{r,s}$, s denotes the degree of nonmembership of $x_{r,s}$. The intuitionistic fuzzy point $x_{r,s}$ is contained in the intuitionistic fuzzy set A iff $r < \mu_A(x)$, $s > \delta_A(x)$.*

Proposition: 3.12. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. For a function $f : (X, T) \to (Y, S)$ the following statements are equivalents

  (a) f is GIF contra continuous mapping

  (b) For each intuitionistic fuzzy point $x_{r,s}$ of X and for each intuitionistic fuzzy closed set B of (Y, S) containing $f(x_{r,s})$, there exists a GIF open set A of (X, T) containing $x_{r,s}$, such that $A \subseteq f^{-1}(B)$.

  (c) For each intuitionistic fuzzy point $x_{r,s}$ of X and for each intuitionistic fuzzy closed set B of (Y, S) containing $f(x_{r,s})$, there exists a GIF open set A of (X, T) containing $x_{r,s}$, such that $f(A) \subseteq B$.

Proof. $(a) \Rightarrow (b)$ Let f is GIF contra continuous mapping. Let B be an intuitionistic fuzzy closed set in (Y, S) and let $x_{r,s}$ be an intuitionistic fuzzy point of X, such that $f(x_{r,s}) \in B$. Then $x_{r,s} \in f^{-1}(B)$, $x_{r,s} \in f^{-1}(B) = IFGint(f^{-1}(B))$. Let $A = IFGint(f^{-1}(B))$, then A is a GIF open set and $A = IFGint(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $A \subseteq f^{-1}(B)$.
$(b) \Rightarrow (c)$ Let B be an intuitionistic fuzzy closed set in (Y, S) and let $x_{r,s}$ be an intuitionistic fuzzy point in X, such that $f(x_{r,s}) \in B$. Then $x_{r,s} \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is a GIF open set in (X, T) and $A \subseteq f^{-1}(B)$. This implies $f(A) \subseteq f(f^{-1}(B)) \subseteq B$.
$(c) \Rightarrow (a)$ Let B be an intuitionistic fuzzy closed set in (Y, S) and let $x_{r,s}$ be an intuitionistic fuzzy

point in X, such that $f(x_{r,s}) \in B$. Then $x_{r,s} \in f^{-1}(B)$. By hypothesis there exists a GIF open set A of (X, T), such that $x_{r,s} \in A$ and $f(A) \subseteq B$. This implies $x_{r,s} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Since A is GIF open, $A = IFGint(A) \subseteq IFGint(f^{-1}(B))$. Therefore $x_{r,s} \in IFGint(f^{-1}(B))$, $f^{-1}(B) = \bigcup_{x_{r,s} \in f^{-1}(B)}(x_{r,s}) \subseteq IFGint(f^{-1}(B) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is a GIF open set in (X, T). Thus f is GIF contra continuous mapping.

Definition: 3.4. *Let (X, T) and (Y, S) be any intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be a mapping. The graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$, $\forall x \in X$.*

Proposition: 3.13. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be any mapping. If the graph $g : X \to X \times Y$ of f is GIF contra continuous then f is also GIF contra continuous.

Proof. Let A be an intuitionistic fuzzy open set in (Y, S). By definition $f^{-1}(A) = 1_{\sim} \bigcap f^{-1}(A) = g^{-1}(1_{\sim} \times A)$. Since g is GIF contra continuous, $g^{-1}(1_{\sim} \times A)$ is GIF closed in (X, T). Now $f^{-1}(A)$ is a GIF closed set in (X, T). Thus f is GIF contra continuous.

Proposition: 3.14. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be any mapping. If the graph $g : X \to X \times Y$ of f is strongly GIF contra continuous then f is also strongly GIF contra continuous.

Proof. Let A be an GIF open set in (Y, S). By definition $f^{-1}(A) = 1_{\sim} \bigcap f^{-1}(A) = g^{-1}(1_{\sim} \times A)$. Since g is strongly GIF contra continuous, $g^{-1}(1_{\sim} \times A)$ is intuitionistic fuzzy closed in (X, T). Now $f^{-1}(A)$ is a intuitionistic fuzzy closed set in (X, T). Thus f is strongly GIF contra continuous.

Proposition: 3.15. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be any mapping. If the graph $g : X \to X \times Y$ of f is GIF contra irresolute then f is also GIF contra irresolute.

Proof. Let A be an GIF open set in (Y, S). By definition $f^{-1}(A) = 1_{\sim} \bigcap f^{-1}(A) = g^{-1}(1_{\sim} \times A)$. Since g is GIF contra irresolute, $g^{-1}(1_{\sim} \times A)$ is GIF closed in (X, T). Now $f^{-1}(A)$ is a GIF closed set in (X, T). Thus f is GIF contra irresolute.

4 Generalized intuitionistic fuzzy compact spaces, Generalized intuitionistic fuzzy S-closed, Generalized intuitionistic fuzzy almost compact spaces and Generalized intuitionistic fuzzy convergent

Definition: 4.1. *Let (X, T) be an intuitionistic fuzzy topological space. If a family*

$$\{\langle x, \mu_{Gi}, \delta_{Gi} \rangle : i \in J\}$$

of GIF open sets in (X, T) satisfies the condition $\bigcup\{\langle x, \mu_{Gi}, \delta_{Gi} \rangle : i \in J\} = 1_{\sim}$ then it is called a GIF open cover of (X, T).

Definition: 4.2. *A finite subfamily of a GIF open cover $\{\langle x, \mu_{Gi}, \delta_{Gi}\rangle : i \in J\}$ of (X,T) which is also a GIF open cover of (X,T) is called a finite subcover of $\{\langle x, \mu_{Gi}, \delta_{Gi}\rangle : i \in J\}$.*

Definition: 4.3. *An intuitionistic fuzzy topological space (X,T) is called GIF compact iff every GIF open cover of (X,T) has a finite subcover.*

Proposition: 4.1. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces and $f : (X,T) \to (Y,S)$ be GIF contra continuous surjection. If (X,T) is GIF compact then so is (Y,S)

Proof. Let $G_i = \{\langle y, \mu_{Gi}, \delta_{Gi}\rangle : i \in J\}$ be an intuitionistic fuzzy closed in (Y,S). $1 - G_i$ be an intuitionistic fuzzy open cover in (Y,S) with $\bigcup\{\langle 1-y, 1-\mu_{Gi}, 1-\delta_{Gi}\rangle : i \in J\} = \bigcup_{i \in J} 1 - G_i = 1_\sim$. Since f is GIF contra continuous, $f^{-1}(G_i) = \{\langle x, \mu_{f^{-1}(Gi)}, \delta_{f^{-1}(Gi)}\rangle : i \in J\}$ is GIF open cover of (X,T). Now $\bigcup_{i \in J} f^{-1}(G_i) = f^{-1}(\bigcup_{i \in J} G_i) = 1_\sim$. Since (X,T) is GIF compact, there exists a finite subcover $J_0 \subset J$ such that $\bigcup_{i \in J_0} f^{-1}(G_i) = 1_\sim$. Hence $f(\bigcup_{i \in J_0} f^{-1}(G_i)) = 1_\sim$, $ff^{-1}(\bigcup_{i \in J_0}(G_i)) = \bigcup_{i \in J_0}(1 - G_i) = 1_\sim$. That is, $\bigcup_{i \in J_0}(1 - G_i) = 1_\sim$. Therefore (Y,S) is intuitionistic fuzzy compact.

Definition: 4.4.   (i) *An intuitionistic fuzzy set A of intuitionistic fuzzy topological space X is called a GIF regular closed set if $IFGcl(IFGint(A)) = A$.*

   Equivalently An intuitionistic fuzzy set A of intuitionistic fuzzy topological space X is called a GIF regular open set if $IFGint(IFGcl(A)) = A$.

(ii) *An intuitionistic fuzzy topological space (X,T) is called an intuitionistic fuzzy S-closed space if each intuitionistic fuzzy regular closed cover of X has a finite subcover for X.*

(iii) *An intuitionistic fuzzy topological space (X,T) is called a GIF S-closed space if each GIF regular closed cover of X has a finite subcover for X.*

(iv) *An intuitionistic fuzzy topological space (X,T) is called a strongly intuitionistic fuzzy S-closed space if each intuitionistic fuzzy closed cover of X has a finite subcover for X.*

(v) *An intuitionistic fuzzy topological space (X,T) is called a strongly GIF S-closed space if each GIF closed cover of X has a finite subcover for X.*

Proposition: 4.2. Every strongly GIF S-closed space of (X,T) is GIF S-closed.

Proof. Let (X,T) be strongly GIF S-closed space and let $\bigcup_{i \in J}(G_i) = 1_\sim$. Where $\{G_i\}_{i \in J}$ is a family of GIF regular closed sets in (X,T). Since every GIF regular closed is a GIF closed set, $\bigcup_{i \in J}(G_i) = 1_\sim$ and (X,T) is strongly GIF S-closed implies that there exists a finite subfamily $\{G_i\}_{i \in J_0 \subset J}$, such that $\bigcup_{i \in J_0}(G_i) = 1_\sim$. Here the finite cover of X by GIF regular closed sets has a finite subcover. Therefore (X,T) is GIF S-closed.

Proposition: 4.3. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces and let $f : (X,T) \to (Y,S)$ be GIF contra continuous function. If (X,T) is strongly GIF S-closed space then (Y,S) is intuitionistic fuzzy compact.

Proof. Let $G_i = \{\langle y, \mu_{G_i}(y), \delta_{G_i}(y)\rangle : i \in J\}$ be intuitionistic fuzzy open cover of (Y,S) and let $\bigcup_{i \in J}(G_i) = 1_\sim$. Since f is GIF contra continuous, $f^{-1}(G_i) = \{\langle x, \mu_{f^{-1}(G_i)}(x), \delta_{f^{-1}(G_i)}(x)\rangle\}$ is

GIF closed cover of (X, T) and $\bigcup_{i \in J} f^{-1}(G_i) = f^{-1}(\bigcup_{i \in J}(G_i)) = 1_\sim$. Since (X, T) is strongly GIF S-closed , there exists a finite subcover $J_0 \subset J$, such that $\bigcup_{i \in J_0} f^{-1}(G_i) = 1_\sim$. Hence $f(\bigcup_{i \in J_0} f^{-1}(G_i)) = ff^{-1}(\bigcup_{i \in J_0}(G_i)) = \bigcup_{i \in J_0}(G_i) = 1_\sim$. Therefore (Y, S) is intuitionistic fuzzy compact.

Proposition: 4.4. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces and let $f : (X, T) \to (Y, S)$ be Strongly GIF contra continuous function. If (X, T) is intuitionistic fuzzy compact space then (Y, S) is GIF S-closed.

Proof. Let $G_i = \{\langle y, \mu_{G_i}(y), \delta_{G_i}(y) \rangle : i \in J\}$ be GIF regular closed cover of (Y, S). Every GIF regular closed set is GIF closed set, let $\bigcup_{i \in J}(G_i) = 1_\sim$. Since f is strongly GIF contra continuous, $f^{-1}(G_i) = \{\langle x, \mu_{f^{-1}(G_i)}(x), \delta_{f^{-1}(G_i)}(x) \rangle\}$ is intuitionistic fuzzy open cover of (X, T) and $\bigcup_{i \in J} f^{-1}(G_i) = f^{-1}(\bigcup_{i \in J}(G_i)) = 1_\sim$. Since (X, T) is intuitionistic fuzzy compact space, there exists a finite subcover $J_0 \subset J$, such that $\bigcup_{i \in J_0} f^{-1}(G_i) = 1_\sim$. Hence $f(\bigcup_{i \in J_0} f^{-1}(G_i)) = ff^{-1}(\bigcup_{i \in J_0}(G_i)) = \bigcup_{i \in J_0}(G_i) = 1_\sim$. Therefore (Y, S) is GIF S-closed.

Proposition: 4.5. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces and let $f : (X, T) \to (Y, S)$ be GIF contra irresolute. If (X, T) is GIF compact space then (Y, S) is strongly GIF S-closed.

Proof. Let $G_i = \{\langle y, \mu_{G_i}(y), \delta_{G_i}(y) \rangle : i \in J\}$ be GIF closed cover of (Y, S) and let $\bigcup_{i \in J}(G_i) = 1_\sim$. Since f is GIF contra irresolute, $f^{-1}(G_i) = \{\langle x, \mu_{f^{-1}(G_i)}(x), \delta_{f^{-1}(G_i)}(x) \rangle\}$ is GIF open cover of (X, T) and $\bigcup_{i \in J} f^{-1}(G_i) = f^{-1}(\bigcup_{i \in J}(G_i)) = 1_\sim$. Since (X, T) is GIF compact space , there exists a finite subcover $J_0 \subset J$, such that $\bigcup_{i \in J_0} f^{-1}(G_i) = 1_\sim$. Hence $f(\bigcup_{i \in J_0} f^{-1}(G_i)) = ff^{-1}(\bigcup_{i \in J_0}(G_i)) = \bigcup_{i \in J_0}(G_i) = 1_\sim$. Therefore (Y, S) is strongly GIF S-closed.

Definition: 4.5. *An intuitionistic fuzzy topological space X is called almost compact space if each intuitionistic fuzzy open cover of X has finite subcover, the intuitionistic fuzzy closure of whose members cover X.*

Definition: 4.6. *An intuitionistic fuzzy topological space X is called GIF almost compact space if each GIF open cover of X has finite subcover, the GIF closure of whose members cover X.*

Proposition: 4.6. *Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces and let $f : (X, T) \to (Y, S)$ be GIF contra irresolute and onto mapping. If (X, T) is GIF compact space then (Y, S) is GIF almost compact.*

Proof. *Let $G_i = \{\langle y, \mu_{G_i}(y), \delta_{G_i}(y) \rangle : i \in J\}$ be GIF open cover of (Y, S). Then $1_\sim = \bigcup_{i \in J} G_i \subseteq \bigcup_{i \in J}(IFGcl(G_i))$. Since f is GIF contra irresolute, $f^{-1}(IFGcl(G_i))$ is GIF open cover of (X, T) and $\bigcup_{i \in J} f^{-1}(IFGcl(G_i)) = 1_\sim$. Since (X, T) is GIF compact, there exists a finite subcover $J_0 \subset J$, such that $\bigcup_{i \in J_0} f^{-1}(IFGcl(G_i)) = 1_\sim$. Hence $1_\sim = f(\bigcup_{i \in J_0} f^{-1}(IFGcl(G_i))) = ff^{-1}(\bigcup_{i \in J_0}(IFGcl(G_i))) = \bigcup_{i \in J_0}(IFGcl(G_i)$. Therefore (Y, S) is GIF almost compact.*

Definition: 4.7. *(i) A nonempty family $\mathbb{F}$ of GIF open sets on (X, T) is said to be a GIF filter if*

(1) $0_\sim \notin \mathbb{F}$

(2) If A, B $\in \mathbb{F}$ then $A \cap B \in \mathbb{F}$

(3) If $A \in \mathbb{F}$ and $A \subset B$ then $B \in \mathbb{F}$

(ii) A nonempty family $\mathbb{B}$ of GIF open sets on $\mathbb{F}$ is said to be a GIF filter base if

(1) $0_\sim \notin \mathbb{B}$

(2) If $B_1, B_2 \in \mathbb{B}$ then $B_3 \subset B_1 \cap B_2$ for some $B_3 \in \mathbb{B}$

(iii) A GIF filter $\mathbb{F}$ is called GIF convergent to a intuitionistic fuzzy point $x_{r,s}$ of a intuitionistic fuzzy topological space (X, T) if for each GIF open set A of (X, T) containing $x_{r,s}$, there exists a intuitionistic fuzzy set $B \in \mathbb{F}$ such that $B \subseteq A$.

(iv) A intuitionistic fuzzy filter $\mathbb{F}$ is said to be intuitionistic fuzzy C-convergent to a intuitionistic fuzzy point $x_{r,s}$ of a intuitionistic fuzzy topological space (X, T) if for each intuitionistic fuzzy closed set A of (X, T) containing $x_{r,s}$, there exists a intuitionistic fuzzy set $B \in \mathbb{F}$ such that $B \subseteq A$.

Proposition: 4.7. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is GIF contra continuous. If for each intuitionistic fuzzy point $x_{r,s}$ and each GIF filter $\mathbb{F}$ in (X, T) is GIF convergent to $x_{r,s}$ then intuitionistic fuzzy filter $f(\mathbb{F})$ is intuitionistic fuzzy C-convergent to $f(x_{r,s})$.

Proof. Let $x_{r,s} = \langle x, x_r, x_s \rangle$ be a intuitionistic fuzzy point and $\mathbb{F}$ be any GIF filter in (X, T) is GIF convergent to $x_{r,s}$. Since f is GIF contra continuous mapping, for each intuitionistic fuzzy closed set A containing $f(x_{r,s})$, there exists a GIF open set B of (X, T) containing $x_{r,s}$, such that $f(B) \subseteq A$. Since $\mathbb{F}$ is GIF convergent to $x_{r,s}$, there exists intuitionistic fuzzy set $C \in \mathbb{F}$ such that $C \subseteq B$. Then $f(C) \subseteq f(B) \subseteq A$. That is $f(C) \subseteq A$. Hence the intuitionistic fuzzy filter $f(\mathbb{F})$ is intuitionistic fuzzy C-convergent to $f(x_{r,s})$.

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