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Some properties of intuitionistic fuzzy primary and semiprimary ideals

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Abstract: In this paper, some properties of intuitionistic fuzzy primary ideals as well as intuitionistic semiprimary ideal were defined. We also proved some results based on the properties of intuitionistic fuzzy primary and semiprimary ideals.

Keywords: Intuitionistic fuzzy set, intuitionistic fuzzy primary ideal, intuitionistic fuzzy semiprimary ideal.

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1 Introduction

Ever since an introduction of fuzzy sets by Zadeh [12], the fuzzy concept has invaded almost all branches of mathematics. The concept of intuitionistic fuzzy set and its operations was introduced by Atanassov [1, 2], as a generalization of the notion of fuzzy set. Kumbhojkar and Bapat [5] discussed on correspondence theorem for fuzzy ideals. Palanivelrajan and Nandakumar [7, 8] introduced the definition and some operations of intuitionistic fuzzy primary and semiprimary ideal.

In this paper, some properties of intuitionistic fuzzy primary and semiprimary ideal are discussed.

2 Preliminaries

Definition 1: Let S be any nonempty set. A mapping $\mu : S \to [0, 1]$ is called a fuzzy subset of S.

Definition 2: A fuzzy ideal μ of a ring R is called fuzzy primary ideal, if for all $a, b \in R$ either $\mu(ab) = \mu(a)$ or else $\mu(ab) \leq \mu(b^m)$ for some $m \in Z_+$.

Definition 3: A fuzzy ideal μ of a ring R is called fuzzy semiprimary ideal, if for all $a, b \in R$ either $\mu(ab) \leq \mu(a^n)$, for some $n \in Z_+$, or else $\mu(ab) \leq \mu(b^m)$ for some $m \in Z_+$.

Definition 4: An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 5: A fuzzy ideal A of a ring R is called Intuitionistic fuzzy primary ideal if for all a, $b \in R$ either $\mu(ab) = \mu(a)$ and $\nu_A(ab) = \nu_A(a)$, or $\mu(ab) \leq \mu_A(b^m)$ and $\nu_A(ab) \geq \nu_A(b^m)$, for some $m \in Z_+$.

Example 1: Consider

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x = 0, \\ 0.8 & \text{if} \quad x \in \langle 4 \rangle \sim \langle 0 \rangle \\ 0.6 & \text{if} \quad x \in z \sim \langle 4 \rangle \end{cases}$$

$$\nu_A(x) = \begin{cases} 0 & \text{if} \quad x = 0, \\ 0.1 & \text{if} \quad x \in \langle 4 \rangle \sim \langle 0 \rangle \\ 0.3 & \text{if} \quad x \in z \sim \langle 4 \rangle \end{cases}$$

Definition 6: A fuzzy ideal A of a ring R is called intuitionistic fuzzy semiprimary ideal if for all $a, b \in R$ either $\mu(ab) \leq \mu_A(a^n)$ and $\nu_A(ab) \geq \nu_A(a^n)$, for some $n \in Z_+$ or else $\mu(ab) \leq \mu_A(b^m)$ and $\nu_A(ab) \geq \nu_A(b^m)$ for some $m \in Z_+$.

3 Some properties of intuitionistic fuzzy primary and semiprimary ideal

Theorem 1: If A and B are intuitionistic fuzzy primary ideal of a ring R then $A \times B$ is also an intuitionistic fuzzy primary ideal of R.

Proof: Let $(x_1, y_1), (x_2, y_2) \in A \times B$, where $x_1, x_2 \in A$ and $y_1, y_2 \in B$. Consider

$$\begin{aligned} \mu_{A \times B}((x_1, y_1).(x_2, y_2)) &= \mu_{A \times B}((x_1 x_2, y_1 y_2)) \\ &= \min(\mu_A(x_1, x_2), \mu_B(y_1, y_2)) \\ &= \min(\mu_A(x_1), \mu_B(y_1)). \end{aligned}$$

Therefore, $\mu_{A \times B}(((x_1, y_1).(x_2, y_2)) = \mu_{A \times B}(x_1, y_1)$. Consider

$$\begin{aligned} \nu_{A \times B}((x_1, y_1).(x_2, y_2)) &= \nu_{A \times B}((x_1 x_2, y_1 y_2)) \\ &= \max(\nu_A(x_1 x_2), \nu_B(y_1 y_2) \\ &= \max(\nu_A(x_1), \nu_B(y_1)). \end{aligned}$$

Therefore, $\nu_{A \times B}((x_1, y_1).(x_2, y_2)) = \nu_{A \times B}(x_1, y_1)$.

Therefore, $A \times B$ is an intuitionistic fuzzy primary ideal of R. ☐

Theorem 2: If *A, B* and *C* are intuitionistic fuzzy primary ideal of a ring *R* then

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

is also an intuitionistic fuzzy primary ideal of *R*.

Proof: Consider

$$\begin{aligned} \mu_{A \cup (B \cap C)}(xy) &= \max(\mu_A(xy), \mu_{B \cap C}(xy)) \\ &= \max(\mu_A(x), \mu_{B \cap C}(x)) \\ &= \max(\mu_A(x), \min(\mu_B(x), \mu_C(x))) \\ &= \min(\max(\mu_A(x), \mu_B(x)), \max(\mu_A(x), (\mu_C(x)) \\ &= \min(\mu_{A \cup B}(x), \mu_{A \cup C}(x)). \end{aligned}$$

Therefore,

$$\mu_{A \cup (B \cap C)}(xy) = \mu_{(A \cup B) \cap (A \cup C)}(x).$$

Consider

$$\begin{aligned} \nu_{A \cup (B \cap C)}(xy) &= \min(\nu_A(xy), \nu_{B \cap C}(xy)) \\ &= \min(\nu_A(x), \nu_{B \cap C}(x)) \\ &= \min(\nu_A(x), \max(\nu_B(x), \nu_C(x))) \\ &= \max(\min(\nu_A(x), \nu_B(x)), \min(\nu_A(x), \nu_C(x)) \\ &= \max(\nu_{A \cap B}(x), \nu_{A \cap C}(x)) \\ &= \nu_{(A \cap B) \cap (A \cap C)}(x). \end{aligned}$$

Therefore,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

is an intuitionistic fuzzy primary ideal of *R*. ☐

Theorem 3: If *A* and *B* are intuitionistic fuzzy primary ideal of a ring *R* then $A \cup (A \cap B) = A$ is also an intuitionistic fuzzy primary ideal of *R*.

Proof: Consider

$$\begin{aligned} \mu_{A \cup (A \cap B)}(xy) &= \max(\mu_A(xy), \mu_{A \cap B}(xy)) \\ &= \max(\mu_A(x), \mu_{A \cap B}(x)) \\ &= \max(\mu_A(x), \min(\mu_A(x), \mu_B(x))) \\ &= \max(\mu_A(x), \mu_B(x)). \end{aligned}$$

Therefore, $\mu_{A \cup (A \cap B)}(xy) = \mu_A(x)$.

Consider

$$\begin{aligned} \nu_{A \cup (A \cap B)}(xy) &= \min(\nu_A(xy), \nu_{A \cap B}(xy)) \\ &= \min(\nu_A(x), \nu_{A \cap B}(x)) \\ &= \min(\nu_A(x), \max(\nu_A(x), \nu_B(x))) \\ &= \min(\nu_A(x), \nu_B(x)) \\ &= \nu_A(x). \end{aligned}$$

Therefore, $\nu_{A \cup (A \cap B)}(xy) = \nu_A(x)$. Therefore, $A \cup (A \cap B) = A$ is an intuitionistic fuzzy primary ideal of *R*. ☐

Theorem 4: If A and B are intuitionistic fuzzy primary ideal of a ring R then $A \cap (A \cup B) = A$ is also an intuitionistic fuzzy primary ideal of R.

Proof: Consider

$$\begin{aligned}\mu_{A \cap (A \cup B)}(xy) &= \min(\mu_A(xy), \mu_{A \cup B}(xy)) \\ &= \min(\mu_A(x), \mu_{A \cup B}(x)) \\ &= \min(\mu_A(x), \max(\mu_A(x), \mu_B(x))) \\ &= \min(\mu_A(x), \mu_B(x)).\end{aligned}$$

Therefore, $\mu_{A \cap (A \cup B)}(xy) = \mu_A(x)$.

Consider

$$\begin{aligned}\nu_{A \cap (A \cup B)}(xy) &= \max(\nu_A(xy), \nu_{A \cup B}(xy)) \\ &= \max(\nu_A(x), \nu_{A \cup B}(x)) \\ &= \max(\nu_A(x), \max(\nu_A(x), \nu_B(x))) \\ &= \max(\nu_A(x), \nu_B(x)) \\ &= \nu_A(x).\end{aligned}$$

Therefore, $\nu_{A \cap (A \cup B)}(xy) = \nu_A(x)$. Therefore, $A \cap (A \cup B) = A$ is an intuitionistic fuzzy primary ideal of R. □

Theorem 5: If A, B and C are intuitionistic fuzzy primary ideal of a ring R then

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

is also an intuitionistic fuzzy primary ideal of R.

Proof: Consider

$$\begin{aligned}\mu_{A \cap (B \cup C)}(xy) &= \min(\mu_A(xy), \mu_{B \cup C}(xy)) \\ &= \min(\mu_A(x), \mu_{B \cup C}(x)) \\ &= \min(\mu_A(x), \max(\mu_B(x), \mu_C(x))) \\ &= \max(\min(\mu_A(x), \mu_B(x)), \min(\mu_A(x), (\mu_C(x)) \\ &= \max(\mu_{A \cap B}(x), \mu_{A \cap C}(x)).\end{aligned}$$

Therefore, $\mu_{A \cap (B \cup C)}(xy) = \mu_{(A \cap B) \cup (A \cap C)}(x)$.

Consider

$$\begin{aligned}\nu_{A \cap (B \cup C)}(xy) &= \max(\nu_A(xy), \nu_{B \cup C}(xy)) \\ &= \max(\nu_A(x), \nu_{B \cup C}(x)) \\ &= \max(\nu_A(x), \min(\nu_B(x), \nu_C(x))) \\ &= \min(\max(\nu_A(x), \nu_B(x)), \max(\nu_A(x), \nu_C(x)) \\ &= \min(\nu_{A \cup B}(x), \nu_{A \cup C}(x)).\end{aligned}$$

Therefore, $\nu_{A \cap (B \cup C)}(xy) = \nu_{(A \cup B) \cap (A \cup C)}(x)$.

Therefore,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

is an intuitionistic fuzzy primary ideal of R. □

Theorem 6: If A is an intuitionistic fuzzy primary ideal of a ring R then $\overline{\overline{A}}$ is also an intuitionistic fuzzy primary ideal of R.

Proof: Consider $\mu_{\overline{\overline{A}}}(xy) = \nu_{\overline{A}}(xy) = \mu_A(xy)$. Therefore, $\mu_{\overline{\overline{A}}}(xy) = \mu_A(x)$.

Consider $\nu_{\overline{\overline{A}}}(xy) = \mu_{\overline{A}}(xy) = \nu_A(xy)$. Therefore, $\nu_{\overline{\overline{A}}}(xy) = \nu_A(xy)$.

Therefore, $\overline{\overline{A}} = A$ is an intuitionistic fuzzy primary ideal of a ring R. □

Theorem 7: Let f be a homomorphism from a ring R onto a ring R'. Let A and A' are intuitionistic fuzzy primary ideals of R and R' respectively then the following statements are true.

i. $f(A)$ is a fuzzy primary ideal of R' if A is f-invariant.

ii. $f^{-1}(A')$ is an intuitionistic fuzzy primary ideal of R.

Proof: i. Let $a', b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$. Now if

$$(f(\mu_A))(a'b') > (f(\mu_A))((a')^n),$$

for all $n \in Z_+$ then $(f(\mu_A))(f(ab)) > (f(\mu_A))(f(a^n)) = f^{-1}(f(\mu_A))(a^n)$. Which implies that

$$f^{-1}(f(\mu_A))(ab) = \mu_A(ab) > \mu_A(a^n).$$

That is $\mu_A(ab) \le \mu_A(b^m)$ for some $m \in Z_+$, as A is an intuitionistic fuzzy primary ideal. Therefore, $(f(\mu_A))(a'b') \le (f(\mu_A))((b')^m)$.

Similarly, let $a', b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$. Now if $(f(\nu_A))(a'b') < (f(\nu_A))((a')^n)$ for all $n \in Z_+$, then

$$(f(\nu_A))(f(ab)) < (f(\nu_A))(f(a^n)).$$

Which implies that $f^{-1}(f(\nu_A))(ab) = \nu_A(ab) < \nu_A(a^n)$. That is $\nu_A(b^m) \le \nu_A(ab) < \nu_A(a^n)$ for some $m \in Z_+$. That is $\nu_A(ab) \ge \nu_A(b^m)$. Therefore, $(f(\nu_A))(a'b') \ge (f(\nu_A))((b')^m)$. Which results that $f(A)$ is an intuitionistic fuzzy primary ideal of R'.

ii. Let $a, b \in R$, $f(a) = a'$ and $f(b) = b'$ then $f^{-1}(A')$ is an intuitionistic fuzzy primary ideal because $(f^{-1}(\mu_{A'}))(ab) > (f^{-1}(\mu_{A'}))(a^n)$, for all $n \in Z_+$. Which implies that

$$\mu_{A'}(f(ab)) > \mu_{A'}(f(a^n)).$$

That is $\mu_{A'}(a'b') > \mu_{A'}((a')^n)$. Therefore $\mu_{A'}(a'b') \le \mu_{A'}((b')^m)$ for some $m \in Z_+$, since A' is an intuitionistic fuzzy primary ideal.

Also, $\mu_{A'}(f(ab)) \le \mu_{A'}((b)^m)$. Which implies that $(f^{-1}(\mu_{A'}))(ab) \le (f^{-1}(\mu_{A'}))(b^m)$.

Let $a, b \in R$, $f(a) = a'$ and $f(b) = b'$, $(f^{-1}(\nu_{A'}))(ab) < (f^{-1}(\nu_{A'}))(a^n)$, for all $n \in Z_+$.

Consider $\nu_{A'}(f(ab)) < \nu_{A'}(f(a^n))$. That is $\nu_{A'}(a'b') < \nu_{A'}((a')^n)$. Which implies that

$$\nu_{A'}((b')^m) \le \nu_{A'}(a'b').$$

Also, $\nu_{A'}(f(b^m)) \le \nu_{A'}(f(ab))$. That is $(f^{-1}(\nu_{A'}))(b^m) \le (f^{-1}(\nu_{A'}))(ab)$. Therefore,

$$(f^{-1}(\nu_{A'}))(ab) \ge f^{-1}(\nu_{A'}))(b^m).$$

Which results $f^{-1}(A')$ is an intuitionistic fuzzy primary ideal. □

Theorem 8: Let f be a homomorphism from a ring R onto a ring R'. Let A and A' be an intuitionistic fuzzy semiprimary ideal of R and R' respectively then the following statements are true:

i. $f(A)$ is an intuitionistic fuzzy semiprimary ideal of R' provided that A is f-invariant;
ii. $f^{-1}(A')$ is an intuitionistic fuzzy semiprimary ideal of R.

Proof: i. Let $a', b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$. Now if

$$(f(\mu_A))(a'b') > (f(\mu_A))((a')^n)$$

for all $n \in Z_+$ then $(f(\mu_A))(f(ab)) > (f(\mu_A))(f(a^n)) = f^{-1}(f(\mu_A))(a^n)$. Which implies that

$$f^{-1}(f(\mu_A))(ab) = \mu_A(ab) > \mu_A(a^n).$$

That is $\mu_A(ab) \le \mu_A(b^m)$ for some $m \in Z_+$, as A is an intuitionistic fuzzy semiprimary ideal. Therefore, $(f(\mu_A))(a'b') \le (f(\mu_A))((b')^m)$.

Similarly let $a', b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$. Now if

$$(f(\nu_A))(a'b') < (f(\nu_A))((a')^n)$$

for all $n \in Z_+$, then $(f(\nu_A))(f(ab)) < (f(\nu_A))(f(a^n))$. Which implies that

$$f^{-1}(f(\nu_A))(ab) = \nu_A(ab) < \nu_A(a^n).$$

That is $\nu_A(b^m) \le \nu_A(ab) < \nu_A(a^n)$ for some $m \in Z_+$. Also, $\nu_A(ab) \ge \nu_A(b^m)$. Therefore,

$$(f(\nu_A))(a'b') \ge (f(\nu_A))((b')^m).$$

Which results $f(A)$ is an intuitionistic fuzzy semiprimary ideal of R.

ii. Let $a, b \in R$, $f(a) = a'$ and $f(b) = b'$, then $f^{-1}(A')$ is an intuitionistic fuzzy semiprimary ideal because $(f^{-1}(\mu_{A'}))(ab) > (f^{-1}(\mu_{A'}))(a^n)$ for all $n \in Z_+$. Which implies that

$$\mu_{A'}(f(ab)) > \mu_{A'}(f(a^n)).$$

That is $\mu_{A'}(a'b') > \mu_{A'}((a')^n)$. Therefore, $\mu_{A'}(a'b') \le \mu_{A'}((b')^m)$ for some $m \in Z_+$, since A' is an intuitionistic fuzzy semiprimary ideal.

Also $\mu_{A'}(f(ab)) \le \mu_{A'}((b)^m)$. Therefore $(f^{-1}(\mu_{A'}))(ab) \le (f^{-1}(\mu_{A'}))(b^m)$.

Let $a, b \in R$, $f(a) = a'$ and $f(b) = b'$, $(f^{-1}(\nu_{A'}))(ab) < f^{-1}(\nu_{A'})(a^n)$, for all $n \in Z_+$. Consider $\nu_{A'}(f(ab)) < \nu_{A'}(f(a^n))$. That is $\nu_{A'}(a'b') < \nu_{A'}((a')^n)$. Which implies that $\nu_{A'}((b')^m) \le \nu_{A'}(a'b')$.

Also $\nu_{A'}(f(b)^m) \le \nu_{A'}(f(ab))$. That is $(f^{-1}(\nu_{A'}))(b^m) \le (f^{-1}(\nu_{A'}))(ab)$. Therefore,

$$(f^{-1}(\nu_{A'}))(ab) \ge (f^{-1}(\nu_{A'}))(b^m)$$

for some $m \in Z_+$. Which results $f^{-1}(A')$ is an intuitionistic fuzzy semiprimary ideal. □

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