Notes on Intuitionistic Fuzzy Sets
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Defuzzification of intuitionistic fuzzy sets

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Abstract: Defuzzification is the process of converting a fuzzy quantity to precise quantity, just as fuzzification is the conversion of a precise quantity to a fuzzy quantity. Various types of defuzzification methods, are available for conversion of fuzzy to non-fuzzy. In this paper, defuzzification functions in intuitionistic fuzzy environment such as triangular, trapezoidal, L-trapezoidal, R-trapezoidal, Gaussian, S-shaped, Z-shaped functions are defined. The proposed defuzzification techniques are useful to develop intuitionistic fuzzy logic controller.
Keywords: Intuitionistic fuzzy sets, Membership and non-membership functions, Intuitionistic fuzzy index, Intuitionistic fuzzification functions, Defuzzification functions.
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1 Introduction

Uncertainty prevails in every aspects of life due to partial information about the problem and it is important to apply the fuzzy results to the world around us. One major feature of fuzzy logic is its ability to express the amount of uncertainty. Therefore it is easy to model the real world problem using fuzzy logic.

Fuzzy set (Zadeh, 1965) [14] considers the uncertainty in the form of a membership function which refers to the degree of belongingness. Further this theory was generalized into many-valued logic with the efforts of Łukasiewicz, Gottwald, Post, Gödel and so on. Therefore, during the past several years fuzzy logic control has emerged as one of the most active and fruitful areas of research in the application of decision making, image processing, control theory [3].

Atanassov (1983), further generalized fuzzy sets into intuitionistic fuzzy (IF) sets where he considered a "hesitation degree" while defining the membership function. This hesitation is due to the lack of knowledge in defining the membership function. This provides a tool to deal the theory of uncertainty and has turned to be an important tool in modeling real situations [5, 6]. As in fuzzy environment, modeling real world situations via intuitionistic fuzzy sets also involves three main stages:

- Intuitionistic fuzzification (converting crisp to membership and non-membership values);
- Modification of membership and non-membership values (using IF rules, logic, IF sets, operators);
- Intuitionistic defuzzification (converting membership and non-membership values to crisp).

Fuzzification is the process of converting crisp to fuzzy. Parvathi et al., provides intuitionistic fuzzification functions to formulate membership and non-membership functions of any IFS [8]. Various standard types of intuitionistic fuzzification functions such as triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal, S-shaped, Z-shaped functions are useful in intuitionistic fuzzification process.

Defuzzification has attracted far less attention than other processes involved in fuzzy systems and technologies [2, 4, 9]. Defuzzification is sometimes not seen as a part of the core of the fuzzy system, fuzzy system "ends" where uncertainty and imprecision end. Defuzzification means the fuzzy to crisp conversions. The fuzzy results generated cannot be used to the real time applications and it is necessary to convert the fuzzy quantities into crisp quantities for further processing. In literature, various defuzzification methods are available for converting fuzzy into a crisp value [10, 11, 12]. Defuzzification of intuitionistic fuzzy set discussed in [7], gives a single defuzzified output. Adrian Ban et al., introduced de-i-fuzzification procedure for intutionistic fuzzy sets in [1]. De-i-fuzzification is the process of assigning a standard fuzzy value to the members of an intuitionistic fuzzy set. Vassia Atanassova et al., gave a new formula for de-i-fuzzification of intutionistic fuzzy sets [13].

As far as image processing is concerned, images are read as matrix and there arises the need of a defuzzified matrix instead of a single value. This happens in most of the cases where input is of matrix type and hence it is necessary to define intuitionistic defuzzification function to convert IF matrix into a crisp matrix taking into account both membership and non-membership values. In this paper, an attempt has been made to define several types of defuzzification functions characterizing IFSs such as triangular, trapezoidal, L-trapezoidal, R- trapezoidal, Gaussian, S-shaped, Z-shaped functions. Introduction of these defuzzification techniques leaves a way for development of intuitionistic fuzzy logic controller. If defuzzification is not the inverse of fuzzification, it is necessary to develop other techniques.

The remaining part of the paper is organized as follows: Section 2 gives basic definitions which are prerequisites for the study. In Section 3, seven types of IF-defuzzification functions such as triangular, trapezoidal, L-trapezoidal, R- trapezoidal, Gaussian, S-shaped, Z-shaped functions are discussed and Section 4 gives a numerical example. Section 5 concludes the paper.

2 Preliminaries

In this section, some basic definitions required for the study, are outlined. In [7] various standard types of intuitionistic fuzzification functions are defined. In this paper, iftrif is used for fuzzification process and similarly other intuitionistic fuzzification functions can also be used.

Definition 2.1 [5] Let the universal set X be fixed. An *intuitionistic fuzzy set* (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ define the degrees of membership and non-membership of the element $x \in X$ respectively, and for every $x \in X$ in A, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ holds.

Definition 2.2 [5] For every common intuitionistic fuzzy subset A on X, we have $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ called the *intuitionistic fuzzy index or hesitancy index* of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A. $\pi_A(x)$ expresses the degree of lack of knowledge of every $x \in X$ belongs to IFS or not. Obviously, for every $x \in X$ and $0 \leq \pi_A(x) \leq 1$.

Definition: 2.3 [12] *Membership function* for an intuitionistic fuzzy set A on the universe of discourse X is defined as $\mu_A : X \to [0,1]$, where each element X is mapped to a value between 0 and 1. The value $\mu_A(x), x \in X$ is called the membership value or degree of membership.

Definition: 2.4 [5] *Non-membership function* for an intuitionistic fuzzy set A on the universe of discourse X is defined as $\gamma_A : X \to [0,1]$, where each element X is mapped to a value between 0 and 1. The value $\gamma_A(x), x \in X$is called the non-membership value or degree of non-membership.

Definition: 2.5 [7] For a fixed universe E, the IFS A can be interpreted as a mapping $E \to [0,1] \times [0,1]$ and it can be defined by a pair $\langle \mu_A, \gamma_A \rangle$ where for $x \in E$, $\mu_A(x)$ denotes the degree of membership of x and $\gamma_A(x)$ denotes the degree of non -membership of x to the set A; and $\mu_A(x)$and $\gamma_A(x)$ satisfy the condition : $\mu_A(x)+\gamma_A(x) \leq 1$. The set B is a fuzzy set, when $\mu_B(x) + \gamma_B(x) = 1$. The *crispification* operation as a map $[0,1] \times [0,1] \to R$, where R is the set of real numbers is introduced. Here, $E = R$ for IFSs.

Definition: 2.6 [8] *Intuitionistic fuzzy triangular function* (iftrif) of A takes the form

$$\langle \mu_A(x), \gamma_A(x) \rangle = \begin{cases} \langle 0, 1-\epsilon \rangle & \text{if } \ x \leq a \\ \left\langle (\frac{x-a}{b-a}) - \epsilon, 1 - (\frac{x-a}{b-a}) \right\rangle & \text{if } \ a < x \leq b \\ \left\langle (\frac{c-x}{c-b}) - \epsilon, 1 - (\frac{c-x}{c-b}) \right\rangle & \text{if } \ b \leq x < c \\ \langle 0, 1-\epsilon \rangle & \text{if } \ x \geq c \end{cases}$$

ϵ is an arbitrary parameter chosen in such a way that $\mu_A(x) + \gamma_A(x) + \epsilon = 1$ and $0 \leq \epsilon < 1$.

Definition: 2.7 [5] Modal operator $\boxplus$. Let the universal set X be fixed and A be an IFS, then $\boxplus A = \left\{ \langle x, \frac{\mu_A(x)}{2}, \frac{\gamma_A(x)+1}{2} \rangle | x \in X \right\}$.

Definition: 2.8 [5] Modal operator $\boxtimes$. Let the universal set X be fixed and A be an IFS, then $\boxtimes A = \left\{ \langle x, \frac{\mu_A(x)+1}{2}, \frac{\gamma_A(x)}{2} \rangle | x \in X \right\}$.

3 IF-defuzzification functions

IF-defuzzification function is a function used to convert membership and non-membership values into precise quantity. The term *IF-defuzzification function (IFDF)* refers to formulation of defuzzification function of an IFS. This section discusses the formulation and the features of a few IF defuzzification functions. Suitable illustrations are also dealtwith. Throughout this paper, A represents an *intuitionistic fuzzy set*.

3.1 Intuitionistic fuzzy triangular (*iftridf*)

IF-triangular defuzzifiction function (iftridf) is given by,

$$C(y) = \begin{cases} \leq a & \text{if } \ y = 0 \\ a + (b-a)(y+\epsilon) - (\sqrt{\mu * (c_1 - \gamma)}) & \text{if } \ 0 < y \leq \frac{x-a}{b-a} - \epsilon \\ (b-a)(y+\epsilon) + c - \sqrt{\mu * (c_2 - \gamma)} & \text{if } \ \frac{x-a}{b-a} - \epsilon \leq y < \frac{c-x}{c-b} - \epsilon \\ \geq c & \text{if } \ y = 0 \end{cases}$$

where c_1 and c_2 are arbitrary constants and $y = \mu_A(x)$ is the fuzzified value which lies in $[0, 1]$ and ϵ is a small quantity such that $\mu_A(x) + \gamma_A(x) + \epsilon = 1$ and $0 \leq \epsilon < 1$.

Note. Hereafter, ϵ is a small quantity chosen in such a way that $\mu_A(x) + \gamma_A(x) + \epsilon = 1$ and $0 \leq \epsilon < 1$.

3.2 Intuitionistic fuzzy trapezoidal (*iftradf*)

IF-trapezoidal defuzzification function (iftradf) is given by

$$C(y) = \begin{cases} \leq a & \text{if } \ y = 0 \\ a + (b-a)(y+\epsilon) - (\sqrt{\mu * (c_1 - \gamma)}) & \text{if } \ 0 < y \leq \frac{x-a}{b-a} - \epsilon \\ b \leq x \leq c & \text{if } \ y = 1 - \epsilon \\ (c-d)(y+\epsilon) + d - \sqrt{\mu * (c_2 - \gamma)} & \text{if } \ 1 - \epsilon < y < \frac{d-x}{d-c} - \epsilon \\ \geq d & \text{if } \ y = 0 \end{cases}$$

where c_1 and c_2 are arbitrary constants.

3.2.1 Intuitionistic fuzzy R-trapezoidal (*ifrtdf*)

The IF-R-trapezoidal defuzzification function (ifrtdf) is given by

$$C(y) = \begin{cases} \leq c & \text{if } \ y = 1 - \epsilon \\ (c-d)(y+\epsilon) + d - \sqrt{\mu * (c_1 - \gamma)} & \text{if } \ 1 - \epsilon < y < \frac{d-x}{d-c} - \epsilon \\ \geq d & \text{if } \ y = 0 \end{cases}$$

where c_1 is an arbitrary constant.

3.2.2 Intuitionistic fuzzy L-trapezoidal (*ifltdf*)

IF-L-trapezoidal defuzzification function (ifltdf) takes the form

$$C(y) = \begin{cases} \leq a & \text{if } \; y = 0 \\ a + (b-a)(y+\epsilon) - \sqrt{\mu * (c_2 - \gamma)} & \text{if } \; 0 < y \leq \frac{x-a}{b-a} - \epsilon \\ \geq b & \text{if } \; y \geq 1 - \epsilon \end{cases}$$

where c_2 is an arbitrary constant.

3.3 Intuitionistic fuzzy Gaussian (*ifgaussdf*)

IF-Gaussian defuzzification functions (ifgaussdf) are defined as

$$C(y) = \begin{cases} m - k\sqrt{-2(log(y+\epsilon))} - \sqrt{\mu * c_1 * \gamma} & \text{if } \; x \leq m \\ m + k\sqrt{-2(log(y+\epsilon))} + \sqrt{\mu * c_2 * \gamma} & \text{if } \; x > m \end{cases}$$

where c_1 and c_2 are arbitrary constants.

3.4 Intuitionistic fuzzy S-shaped (*ifSdf*)

IF-S-shaped defuzzification function (ifSdf) takes the form

$$C(y) = \begin{cases} \leq a & \text{if } \; y = 0 \\ a + \frac{(b-a)\sqrt{y+\epsilon}}{\sqrt{2}} + (\mu * (c_1 - \gamma))^2 & \text{if } \; 0 < y \leq 2(\frac{x-a}{b-a})^2 - \epsilon \\ b - \frac{(b-a)\sqrt{1-(y+\epsilon)}}{\sqrt{2}} - (\mu * c_2 * \gamma)^2 & \text{if } \; 2(\frac{x-a}{b-a})^2 - \epsilon \leq y < 1 - 2(\frac{x-b}{b-a})^2 - \epsilon \\ \geq b & \text{if } \; y \geq 1 - \epsilon \end{cases}$$

where c_1 and c_2 are arbitrary constants.

3.5 Intuitionistic fuzzy Z-shaped (*ifZdf*)

IF-Z-shaped defuzzification function (ifZdf) is defined as

$$C(y) = \begin{cases} \leq a & \text{if } \; y = 1 - \epsilon \\ a + \frac{(b-a)\sqrt{1-(y+\epsilon)}}{\sqrt{2}} + (\mu * c_1 * \gamma)^2 & \text{if } \; 0 < y \leq 1 - 2(\frac{x-a}{b-a})^2 - \epsilon \\ b - \frac{(b-a)\sqrt{y+\epsilon}}{\sqrt{2}} - (\mu * (c_2 - \gamma))^2 & \text{if } \; 1 - 2(\frac{x-a}{b-a})^2 - \epsilon \leq y < 2(\frac{x-b}{b-a})^2 - \epsilon \\ \geq b & \text{if } \; y = 0 \end{cases}$$

where c_1 and c_2 are arbitrary constants.

4 Numerical examples

Example 4.1. In this example, IF-triangular fuzzification function is used for fuzzification and iftridf is used for defuzzification processes. The other defuzzification functions can also be verified in the similar way. No modification is done in the original input. The input considered here is the 3×3 gray matrix extracted from an image whose gray values vary from 0 to 255. The parameters are $a = 0$, $b = 128$, $c = 256$ and $\epsilon = 0.001$. This example is done only to check the validity of the proposed intuitionistic defuzzification functions. Consider a 3×3 gray matrix

$$A = \begin{bmatrix} 50 & 128 & 192 \\ 202 & 220 & 166 \\ 256 & 32 & 64 \end{bmatrix} \tag{1}$$

From Definition 2.6, IF-triangular fuzzified matrix is given by

$$[\langle \mu_A(x), \gamma_A(x) \rangle]_{3\times 3} = \begin{bmatrix} \langle 0.3896, 0.6094 \rangle & \langle 0.9990, 0 \rangle & \langle 0.4990, 0.5000 \rangle \\ \langle 0.4206, 0.5781 \rangle & \langle 0.2803, 0.7188 \rangle & \langle 0.7021, 0.2969 \rangle \\ \langle 0, 0, 9990 \rangle & \langle 0.2490, 0.7500 \rangle & \langle 0.4990, 0.5000 \rangle \end{bmatrix} \tag{2}$$

The corresponding IF-triangular defuzzified matrix is obtained as follows:

$$A = \begin{bmatrix} 49.6099 & 127.0005 & 191.1348 \\ 201.2231 & 219.7128 & 164.9096 \\ 256 & 31.7505 & 63.5005 \end{bmatrix} \tag{3}$$

Here, no modification is carried out in the input. After IF triangular fuzzification (iftrif) and IF triangular defuzzification process (iftridf) the output remains the same. Comparing (1) and (3), the loss of accuracy is mainly due to numerical approximation.

Example 4.2. Intuitionistic fuzzification, modification of membership and non-membership values and intuitionistic defuzzification are the three major steps involved in modeling real situations via IFSs. IF logic controller models human experience, human decision making behaviour and so on. In intuitionistic fuzzy inference system, modification is required so that result is more suitable than original for perception. In this example, intuitionistic fuzzy modal operators $\boxplus$ and $\boxtimes$ are used for modification of membership and non-membership values. Consider the same matrix A as in Example 4.1.

Based on the matrix (2), the modified matrix using modal operator $\boxplus$, is given by

$$\left[\left\langle \mu_A'(x), \gamma_A'(x) \right\rangle\right]_{3\times 3} = \begin{bmatrix} \langle 0.1948, 0.8047 \rangle & \langle 0.4995, 0.5 \rangle & \langle 0.2495, 0.75 \rangle \\ \langle 0.2103, 0.7890 \rangle & \langle 0.1401, 0.8594 \rangle & \langle 0.3510, 0.6484 \rangle \\ \langle 0, 0.9995 \rangle & \langle 0.1245, 0.875 \rangle & \langle 0.2495, 0.75 \rangle \end{bmatrix} \tag{4}$$

The corresponding defuzzified matrix for (4) is as follows:

$$C = \begin{bmatrix} 24.8673 & 63.5642 & 223.3775 \\ 228.4489 & 237.5394 & 210.2552 \\ 255.872 & 15.9392 & 31.8142 \end{bmatrix} \tag{5}$$

The modified matrix using modal operator $\boxtimes$, is given by

$$\left[\langle \mu_A^{''}(x), \gamma_A^{''}(x)\rangle\right]_{3\times 3} = \begin{bmatrix} \langle 0.6948, 0.3047\rangle & \langle 0.9995, 0\rangle & \langle 0.7495, 0.25\rangle \\ \langle 0.7103, 0.2890\rangle & \langle 0.6401, 0.3594\rangle & \langle 0.8510, 0.1484\rangle \\ \langle 0.5, 0.5\rangle & \langle 0.6245, 0.375\rangle & \langle 0.7495, 0.25\rangle \end{bmatrix} \tag{6}$$

The corresponding defuzzified matrix for (6) is as follows:

$$C = \begin{bmatrix} 88.3673 & 127.0642 & 159.1862 \\ 90.3357 & 172.9144 & 145.6887 \\ 191.005 & 79.2887 & 95.3142 \end{bmatrix} \tag{7}$$

Comparing (5) and (7), it is inferred after applying $\boxplus$ modal operator, that they can be used for enhancement. Low intensity gray levels are further reduced and high intensity gray levels are further increased. The modal operator $\boxtimes$ increases the low and decreases the high. These tools find applications in image processing.

5 Conclusion

Fuzzification and defuzzification functions of IFSs are necessary for designing the architecture of intuitionistic fuzzy logic controller. The authors formulated intuitionistic fuzzification functions [8]. In this paper, an attempt has been made to formulate defuzzification functions for IFSs. In literature, the existing defuzzification techniques [7] result in a single crisp value which is a representative for the given IFS as a whole. As far as image processing is concerned, there are situations to convert a modified IF matrix into a crisp gray matrix (not a single value) so that the output image can be used for elicitation of information. Hence, IF-defuzzification functions are designed to deal such situations. Several types of defuzzification functions such as triangular, trapezoidal, L-trapezoidal function, R-trapezoidal function, Gaussian, S-shaped, Z-shaped functions characterizing intuitionistic fuzzy sets are reviewed for designing intuitionistic fuzzy logic controller.

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