Connection between Generalized nets with characteristics of the places and Intuitionistic fuzzy generalized nets of type 1 and type 2

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Generalized nets with characteristics of the places

Generalized nets with characteristics of the places (GNCP) are defined in [1]. Again there it is proved that Σ_{CP} - the class of all GNCP - is a conservative extension of the class Σ. In a GNCP places can receive characteristics related to the number of tokens of different types that have entered them, the time moments when the tokens entered the places and other information about the flow of the tokens into the net. The formal definition of a GNCP is:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle$$

Ψ is a characteristic function that assigns new characteristics to the places when a token enters the place (this is the new component which cannot be find in the definition of the ordinary GN);
The algorithms for the functioning of transition and GNCP are the same as in the standard GNs (see [2]). The only difference is that now the characteristic function Ψ assigns characteristic to the output places for every token that has been transferred.

GNCP versus IFGN1 and IFGN2

Previously, the idea of assigning characteristics to the places has been known from the Intuitionistic fuzzy generalized nets of type 2 (IFGN2). An IFGN2 has the form:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi\rangle\rangle$$

where A is the set of transitions which have the ordinary GN form but the capacities of the arcs given by the index matrix M are real numbers. The function c which in the ordinary GNs gives the capacities of the places in non-negative integers now assigns a non-negative real value to each place. This real number corresponds to the volume of the place - that is the quantity of matter it can collect.

GNCP versus IFGN1 and IFGN2

The function f which calculates the truth values of the predicates has the form:

$$f(r_{i,j}) = \langle \mu(r_{i,j}), \nu(r_{i,j}) \rangle$$

where $\mu(r_{i,j})$ is the degree of truth of the predicate $r_{i,j}$ and $\nu(r_{i,j})$ is the degree of falsity. $\mu(r_{i,j}) \in [0,1]$ and $\nu(r_{i,j}) \in [0,1]$ satisfy the condition:

$$\mu(r_{i,j}) + \nu(r_{i,j}) \leq 1$$

Here the tokens are some kind of quantities that flow into the net and do not have initial or other characteristics. Instead the function Φ assigns to every place characteristics - the quantities of the tokens from each type in the place.

GNCP versus IFGN1 and IFGN2

In Intuitionistic fuzzy generalized nets of type 1 (IFGN1) the function f which evaluates the predicates of the transitions has the same form as in IFGN2. The tokens, however, are regarded in the classical GN sense and they obtain characteristics. Also, the capacities of the arcs, i.e. the elements of the index matrix M are non-negative integers, whereas in $IFGN2$ they are non-negative real numbers.

It is proved that

$\Sigma_{IFGN1} \equiv \Sigma$ and $\Sigma_{IFGN2} \equiv \Sigma$.

The same result is proved in [1] for the class Σ_{CP} :

$\Sigma_{CP} \equiv \Sigma$

Connection between GNCP and IFGN1

To see how the class Σ_{CP} is related to Σ_{IFGN1} we will first show how it is possible to represent an arbitrary IFGN1 in terms of GNCP.

Theorem

The functioning and the result of work of every IFGN1 can be represented by a GNCP.

Proof. Let a IFGN1 G be given.

$$G = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle \tag{1}$$

Connection between GNCP and IFGN1

We will construct a GNCP E which represents the functioning and the result of the work of G. The new net has the following components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f^*, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle \quad (2)$$

All components of E with the exception of f^* and the function Ψ are the same as in G. Before we define f^* we shall note that in the general algorithm for transition functioning in IFGN1 (see [4]) the transfer of tokens from input to output places is determined by one of the following conditions:

Connection between GNCP and IFGN1

C1 $\mu(r_{i,j}) = 1$, $\nu(r_{i,j}) = 0$ (the case of ordinary GN)
C2 $\mu(r_{i,j}) > \frac{1}{2}$ $(> \nu(r_{i,j}))$
C3 $\mu(r_{i,j}) \geq \frac{1}{2}$ $(\geq \nu(r_{i,j}))$
C4 $\mu(r_{i,j}) > \nu(r_{i,j})$
C5 $\mu(r_{i,j}) \geq \nu(r_{i,j})$
C6 $\mu(r_{i,j}) > 0$
C7 $\nu(r_{i,j}) < 1$, i.e. at least $\pi(r_{i,j}) > 0$, where $\pi(r_{i,j}) = 1 - \mu(r_{i,j}) - \nu(r_{i,j})$ is the degree of uncertainty(indeterminancy).

The condition for transfer of the tokens which will be used is determined for every transition before the firing of the net.

Connection between GNCP and IFGN1

In order to preserve this condition in E, where the function f^* assigns to the predicates values from the set $\{0,1\}$, for the condition $C1$ we define f^* in the following way:

C1* $f^*(r_{i,j}) = \lfloor pr_1 f(r_{i,j}) \rfloor$.

where $\lfloor x \rfloor$ is the floor function which maps a real number x to the largest integer smaller or equal to x.

Similarly, in the other cases we define $f^*(r_{i,j})$ as follows:

C2* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

C3* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

C4* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

Connection between GNCP and IFGN1

C5* $f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) \geq \nu(r_{i,j}) \\ 0\,, & \text{otherwise} \end{cases}$

C6* $f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) > 0 \\ 0\,, & \text{otherwise} \end{cases}$

C7* $f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \nu(r_{i,j}) < 1 \\ 0\,, & \text{otherwise} \end{cases}$

Since in IFGN1 the places do not obtain characteristics we can consider that the characteristic function Ψ of E does not assign any charcteristics to the places, i.e. $\Psi(l) = \emptyset$ for all places l.

Connection between GNCP and IFGN1

Let $\alpha \in pr_1 pr_2 G$ and $\alpha^* \in pr_1 pr_2 E$ be two tokens with equal characteristics that are in two corresponding input places $l_i \in pr_1 Z$ and $l_i^* \in pr_1 Z^*$. The transfer of α is determined by one of the conditions $C1, C2, \cdots, C7$. Let the conditions allow the transfer to output place l_j. At the same time α^* in place l_i^* will be transferred to the output place l_j^* because the function $f^*(r_{i,j}) = 1$ if the corresponding condition for the transfer from l_i to l_j is satisfied. Upon entering l_j^* the token α^* obtains the same characteristic as the token α because the characteristic function Φ is the same for the two nets. If α can not be transferred to any output places of Z, then the token α^* also can not be transferred because of the definition of f^*.

Connection between GNCP and IFGN1

Theorem

The functioning and the results of the work of every GNCP can be represented by an IFGN1.

Proof. Let E be a GNCP with components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle$$

We will construct an IFGN1 G which preserves the result of the work of E. Let $Z \in pr_1 pr_1 E$ be arbitrary transition with components $Z = \langle L', L'', t_1, t_2, r, M, \square\rangle$

For every such transition Z we construct a corresponding transition Z^* with components $Z^* = \langle L'^*, L''^*, t_1, t_2, r^*, M^*, \Box^* \rangle$(see Fig.2)

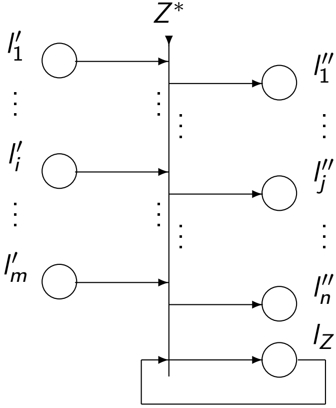

Fig. 2.

$$G = \langle\langle A^*, \pi_A^*, \pi_L^*, c^*, f, \theta_1, \theta_2\rangle, \langle K^*, \pi_K^*, \theta_K^*\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b^*\rangle\rangle$$

where

$$(\forall Z_i^* \in A^*)(\pi_A^*(Z_i^*) = \pi_A(Z_i))$$

$$\pi_L^* = \pi_L \cup \pi_{\{l_Z | Z \in A\}},$$

where function $\pi_{\{l_Z|Z\in A\}}$ determines the priorities of the new places that are elements of set $\{l_Z|Z \in A\}$ and the priorities of the places l_Z for every transition $Z \in A$ are the minimal among the priorities of all other places of the transition Z.

$$c^* = c \cup c_{\{l_Z|Z\in A\}},$$

where function $c_{\{l_Z|Z\in A\}}$ satisfies the equality

$$c_{\{l_Z|Z\in A\}}(l_Z) = 1,$$

$$K^* = (\bigcup_{l\in Q^l} K_l)\bigcup\{\alpha_Z|Z \in A\},$$

i.e. the set of the tokens of G consists of all tokens of E and all additional tokens in the l_Z-places.

$$\pi^*_K = \pi_K \cup \pi_{\{l_Z|Z\in A\}}$$

where the function $\pi_{\{l_Z|Z\in A\}}$ determines the priorities of the α_Z tokens. The α_Z tokens stay in the l_Z places during the entire period of functioning of the net and no other tokens can enter the l_Z places. The α_Z tokens should have the lowest priority among all tokens of the net.

$$\theta^*_K = \theta_K \cup \theta_{\{l_Z|Z \in A\}},$$

where $\theta_{\{l_Z|Z \in A\}}$ determines that each $_Z$-token stays in the initial time-moment T in the l_Z place.

$$X^* = X \cup \{x_0^Z | Z \in A\},$$

where x_0^Z is the initial characteristic of token α_Z and it is a list of all output places for the transition Z:

$$\text{"} < l_1^{''}, l_2^{''}, \cdots l_n^{''} > \text{"}$$

$$\Phi^* = \Phi' \cup \Psi^*_{\{l_Z|Z \in A\}},$$

where the function $\Psi^*_{\{l_Z|Z \in A\}}$ determines the characteristics of the $_Z$-tokens in the form

$$\Psi^*_{\{l_Z|Z \in A\}}(z) = \text{"}\{\langle l_j^{''}, \Psi(l_j^{''})\rangle | l_j^{''} \in L''\}\text{"},$$

The characteristic function Φ^* assigns to every token α_Z the characteristics of the places of the transition Z of E. The function Φ' which assigns the same characteristics to the tokens that have been transferred as the function Φ in E. If we strictly follow the definition of IFGN1, we should define $\Phi'(\alpha) = \langle \Phi(\alpha), \langle \mu(r_{i,j}), \nu(r_{i,j}) \rangle \rangle$. However, in this case the couple $\langle \mu(r_{i,j}), \nu(r_{i,j}) \rangle = \langle 1, 0 \rangle$ as this is the only way for the token to be transferred.

$$b^* = b \cup b_{\{\alpha_Z | Z \in A\}}$$

where the function $b_{\{\alpha_Z | Z \in A\}}(\alpha) = \infty$ determines the number of characteristics the α_Z tokens can keep.
To prove that the so constructed IFGN1 G represents the funtioning and the results of work of E let us take an arbitrary pair of corresponding transitions $Z_i \in pr_1 pr_1 E$ and $Z_i^* \in pr_1 pr_1 G$.

Let $\alpha \in K$ and $\beta \in K^*$ be two tokens of the same type with equal characteristics which are at two corresponding places of the transitions at some moment of time. Apparently neither of them is α_Z token. From the definition of the transition Z_i^* it is clear that the token β can be transferred to some output place $l_j''^*$ if and only if the token α can be transferred to the output place l_j'' (here $l_j''^*$ and l_j'' are two corresponding output places of the two transitions). The two tokens will receive the same characteristics because the characteristic functions Φ and Φ' coincide in all places except the l_Z places. The characteristic funtion Ψ assigns characteristic to the place l_j''. The same characteristic is assigned to the α_Z token. Since the transitions and the tokens were arbitrarily chosen, we can conclude that G represents the funtioning and the result of work of E. The case where splitting of tokens is allowed is analogous.

Connection between GNCP and IFGN2

Theorem

The functioning and the result of the work of every IFGN2 can be represented by a GNCP.

Proof. Let an IFGN2 G be given.

$$G = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi\underline{\rangle}\rangle$$

We will construct a GNCP E based on G which represents the functioning and the results of the work of G. Let $Z = \langle L', L'', t_1, t_2, r, M, \square\rangle$ be arbitrary transition of G. We will construct a corresponding transition $Z^* = \langle L', L'', t_1, t_2, r, M^*, \square\rangle$ which has the same graphic structure as Z, the same number of input and output places, the same time components, the same predicates and the same transition type .

Let A^* be the set of all transitions obtained from the transitions of G by the above procedure. Let the GNCP E have the following components:

$$E = \langle\langle A^*, \pi_{A^*}, \pi_L, c^*, f^*, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi^*, \Psi, b\rangle\rangle$$

The function π_{A^*} which determines the priorities of the transitions assigns to every transition of E the priority of its corresponding transition in G: $\pi_{A^*}(Z_i^*) = \pi_A(Z_i)$ for all $Z_i^* \in A^*$. The priorities of the places in E and the functions θ_1 and θ_2 are also the same. We use the same notation for them in E. Since the capacities of the places in E must be positive integers, we define $c^*(l_i) = \lceil c(l_i)\rceil$ for all places $l_i \in L$, where $\lceil x \rceil$ is the ceiling function which maps a real number x to the smallest integer greater or equal to it. The codomain of the function f^* which determines the degrees of truth and falsity of the predicates in the case of GNCP is the set $\{0, 1\}$. As it was in the case of IFGN1 the values of $f^*(r_{i,j})$ depend on the conditions for the transfer $C1, C2, \cdots, C7$.

C1* $f^*(r_{i,j}) = \lfloor pr_1 f(r_{i,j}) \rfloor$.
where $\lfloor x \rfloor$ is the floor function which maps a real number x to the largest integer smaller or equal to x.

C2* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

C3* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

C4* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

C5* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

C6* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > 0 \\ 0, & \text{otherwise} \end{cases}$

C7* $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \nu(r_{i,j}) < 1 \\ 0, & \text{otherwise} \end{cases}$

All other components of E are the same as in G except Φ^* and Ψ. The tokens in *IFGN*2 do not receive characteristics and therefore we define $\Phi^*(\alpha) = \emptyset$ for all $\alpha \in pr_1 pr_2 E$. The new characteristic function Ψ in E assigns the same characteristics to the places in E as the function Φ assigns to the places in G: $\Psi(l) = \Phi(l)$ for all places l.

To prove that the so constructed GNCP E represents the functioning and the result of the work of G we will use the theorem for the completeness of the GN transitions. Let $Z \in pr_1 pr_1 G$ and $Z^* \in pr_1 pr_1 E$ be two corresponding transitions, i.e.Z^* is obtained from Z by the procedure described above. We will trace the behavior of two corresponding tokens $\alpha \in pr_1 pr_2 G$ and $\alpha^* \in pr_1 pr_2 E$ which are in two corresponding input places l'_i and l'^*_i at the same moment of time. Here we denote the corresponding to l'_i input place l'^*_i to avoid any ambiguity. Without loss of generality we can consider that the two input places have the same characteristics. We can do this because at least for the transitions

Theorem

The functioning and the result of work of every GNCP can be represented by a IFGN2.

Proof. Let E be a GNCP with components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, \Psi, b\rangle\rangle$$

For every transition $Z = \langle L', L'', t_1, t_2, r, M, \Box\rangle$ of E we construct corresponding transition Z^* with the same graphic structure same number of input and output places, the same time components, index matrix of predicates, index matrix of the capacities of arcs and the same type of transition.

Let A^* be the set of all transitions obtained by the above procedure. Let G be IFGN2 with components:

$$G = \langle\langle A^*, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi^*\rangle$$

where the function Φ^* assigns to the places of G both the characteristics of the tokens that are transferred to the output places in E which are determined by function Φ and the characteristics of the output places determined by Ψ.

$$\Phi^*(l_j) = \langle\langle \alpha_1, \Phi_{l_j}(\alpha_1)\rangle, \langle \alpha_2, \Phi_{l_j}(\alpha_2)\rangle, \cdots, \langle \alpha_k, \Phi_{l_j}(\alpha_k)\rangle, \Psi(l_j)\rangle$$

where $\alpha_1, \cdots \alpha_k$ are the tokens that has entered the output place l_j and $\Psi(l_j)$ is the characteristic of the corresponding place of E.

Let Z_i and Z_i^* be two corresponding transitions of E and G. To compare their work let a token α be in the input place l_i' of Z_i and a corresponding token α^* be in the corresponding to l_i' input place $l_i'^*$ of G. Without loss of generality we can consider that the characteristics of α and l_i' coincide with characteristics of the place $l_i'^*$. If the splitting of tokens is prohibited, the truth value of the predicate $r_{i,j}$ is "*true*" and if the other conditions for the transfer allow it, the token α will be transferred to place $l_j'' \in pr_2 Z_i$. Since the components of the two transitions coincide, the token α^* will be transferred to the corresponding output place $l_j''^* \in pr_2 Z_i^*$. The token α and the place l_j'' in Z_i will receive the characteristics $\Phi_{l_j}(\alpha)$ and $\Psi(l_j)$. From the definition of the characteristic function Φ^* it is clear that this characteristics will be included in the characteristics of $l_j''^*$. The case when splitting of tokens is allowed is analogous. Therefore the two transitions function similarly and all information relevant to Z_i is also present in Z_i^*. From the theorem for completeness of the GN transitions it follows that G

Conclusion

The theorems in this paper state that given a GNCP we can construct IFGN1 and IFGN2(or vice versa) which preserve the functioning and the results of the work of the given net. Because the proofs are constructive they are important for the applications of GNs in the modelling of real processes. In future we intend to define intuitionistic fuzzy GNCP and study their relation to IFGN1 and IFGN2.

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Thank You For Your Attention!