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AN EXTENSION OF THE UNIVERSAL OPERATOR OVER INTUITIONISTIC FUZZY SETS AND SOME OF ITS PROPERTIES

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In 1991, during a lecture, the author was asked if an operator over Intuitionistic Fuzzy Sets (IFSs; see [1]) can be constructed such that it would include as partial cases all operators of modal type? On the next lecture the author gave a positive answer, preparing the text which was later published in [2]. The operator is described below.

Let E be a fixed universe and let A be an IFS over E. Let:

$$X_{a,b,c,d,e,f}(A) = \{\langle x, a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)), d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x))\rangle \\ |x \in E\}$$

where $a, b, c, d, e, f \in [0, 1]$ and:

$$a + e - e.f \leq 1,$$

$$b + d - b.c \leq 1.$$

Obviously, this operator is universal for all other operators defined over IFSs (see [3,4]).

Now we shall construct the following extension of it. Let $a, b, c, d, e, f : E \to [0, 1]$ be functions for which, for every $x \in E$:

$$a(x) + e(x) - e(x).f(x) \leq 1 \tag{1}$$

$$b(x) + d(x) - b(x).c(x) \leq 1 \tag{2}$$

Then we define:

$$X_{a,b,c,d,e,f}(A) = \{\langle x, a(x).\mu_A(x) + b(x).(1 - \mu_A(x) - c(x).\nu_A(x)), \\ d(x).\nu_A(x) + e(x).(1 - f(x).\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$

Theorem 1: For every IFS A and for every six functions $a, b, c, d, e, f : E \to [0, 1]$ satisfying (1) and (2), $X_{a,b,c,d,e,f}(A)$ is an IFS.

Proof: Let $a, b, c, d, e, f : E \to [0, 1]$ and let (1) and (2) be true. Let A be a fixed IFS. Then

$$a(x).\mu_A(x) + b(x).(1 - \mu_A(x) - c(x).\nu_A(x)) + d.\nu_A(x) + +e(x).(1 - f(x).\mu_A(x) - \nu_A(x)) \\ \geq a(x).\mu_A(x) + d(x).\nu_A(x) \geq 0$$

and from (1) and (2):

$$a(x).\mu_A(x) + b(x).(1 - \mu_A(x) - c(x).\nu_A(x)) + d(x).\nu_A(x) + +e(x).(1 - f(x).\mu_A(x) - \nu_A(x)) \\ = \mu_A(x).(a(x) - b(x) - e(x).f(x)) + \nu_A(x).(d(x) - b(x).c(x) - e(x)) + b(x) + e(x)$$

$\leq \mu_A(x).(1 - b(x) - e(x)) + \nu_A(x).(1 - b(x) - e(x)) + b(x) + e(x)$
$= (\mu_A(x) + \nu_A(x)).(1 - b(x) - e(x)) + b(x) + e(x)$
$\leq 1 - b(x) - e(x) + b(x) + e(x) = 1.\diamondsuit$

As it is noted in [2], all the IFS operators can be represented by the operator X at some appropriate values of its parameters. These representations are the following:

$$\begin{array}{ll} \Box A & = X_{1,0,r,1,1,1}(A), \\ \Diamond A & = X_{1,1,1,1,0,r}(A), \\ D_a(A) & = X_{1,a,1,1,1-a,1}(A), \\ F_{a,b}(A) & = X_{1,a,1,1,b,1}(A), \\ G_{a,b}(A) & = X_{a,0,r,b,0,r}(A), \\ H_{a,b}(A) & = X_{a,0,r,1,b,1}(A), \\ H^*_{a,b}(A) & = X_{a,0,r,b,0,a}(A), \\ J_{a,b}(A) & = X_{1,a,1,b,0,r}(A), \\ J^*_{a,b}(A) & = X_{1,a,b,b,0,r}(A), \end{array}$$

where r is an arbitrary real number and $r \in [0, 1]$.

Theorem 2: Let for $a(x), b(x), c(x), d(x), e(x), f(x), g(x), h(x), i(x), j(x), k(x), l(x) : E \to [0, 1]$ the functions $u, v, w, x, y, z : E \to [0, 1]$ be defined by:

$$\begin{array}{lllllll} u(x) & = & a(x).g(x) & +b(x) & -b(x).g(x) & -b(x).c(x).k + b(x).c(x).k.l & \geq 0, \\ v(x) & = & a(x).h(x) & +b(x) & -b(x).c(x).k & -b(x).h(x) & > 0, \\ w(x) & = & a(x).h(x).i & +b(x).c(x).j & -b(x).c(x).k & -b(x).h(x).i & \geq 0, \\ x(x) & = & d(x).j & +e(x) & -e(x).f(x).h(x) & +e(x).f(x).h(x).i - e(x).j & \geq 0, \\ y(x) & = & d(x).k & +e(x) & -e(x).f(x).h(x) & -e(x).k & > 0, \\ z(x) & = & d(x).k.l & +e(x).f(x).g(x) & -e(x).f(x).h(x) & -e(x).k.l & \geq 0. \end{array}$$

Then:

$$X_{a,b,c,d,e,f}(X_{g,h,i,j,k,l}(A)) = X_{u,v,w/v,x,y,z/y}.$$

Proof: $X_{a,b,c,d,e,f}(X_{g,h,i,j,k,l}(A)) =$

$$\begin{array}{ll} = & X_{a,b,c,d,e,f}(\{\langle x, g(x).\mu_A(x) + h(x).(1 - \mu_A(x) - i(x).\nu_A(x)), \\ & j(x).\nu_A(x) + k(x).(1 - l(x).\mu_A(x) - \nu_A(x))\rangle | x \in E\}) \\ = & \{\langle x, a(x).(g(x).\mu_A(x) + h(x).(1 - \mu_A(x) - i(x).\nu_A(x))) + b(x).(1 - g(x).\mu_A(x) - \\ & h(x).(1 - \mu_A(x) - i(x).\nu_A(x)) - c(x).(j(x).\nu_A(x) + k(x).(1 - l(x).\mu_A(x) - \nu_A(x)))), \\ & d(x).(j(x).\nu_A(x) + k(x).(1 - l(x).\mu_A(x) - \nu_A(x))) + e(x).(1 - f(x).(g(x).\mu_A(x) + h(x). \\ & (1 - \mu_A(x) - i(x).\nu_A(x))) - (j(x).\nu_A(x) + k(x).(1 - l(x).\mu_A(x) - \nu_A(x))))\rangle | x \in E\} \\ = & \{\langle x, \mu_A(x).(a(x).g(x) - b(x).g(x) + b(x).c(x).k(x).l(x) - a(x).h(x) + b(x).h(x)) + \\ & \nu_A(x).(b(x).c(x).k(x) - b(x).c(x).j(x) - a(x).h(x).i(x) + b(x).h(x).i(x)) + (a(x).h(x) + \\ & b(x) - b(x).c(x).k(x) - b(x).h(x)), \mu_A(x).(d(x).k(x).l(x) + e(x).f(x).g(x) - \\ & e(x).f(x).h(x) - e(x).k(x).l(x)) + \nu_A(x).(d(x).j(x) + e(x).f(x).h(x).i(x) - e(x).j(x) - \\ & d(x).k(x) + e(x).k(x)) + (d(x).k(x) + e(x) - e(x).f(x).h(x) - e(x).k(x))\rangle | x \in E\} \end{array}$$

$$
\begin{aligned}
= \; & \{\langle x, (a(x).g(x) + b(x) - b(x).g(x) - b(x).c(x).k(x) + b(x).c(x).k(x).l(x)).\mu_A(x) + \\
& (a(x).h(x) + b(x) - b(x).c(x).k(x) - b(x).h(x)).(1 - \mu_A(x) - (a(x).h(x).i(x) + \\
& b(x).c(x).j(x) - b(x).c(x).k(x) - b(x).h(x).i(x))/(a(x).h(x) + b(x) - b(x).c(x).k(x) - \\
& b(x).h(x)).\nu_A(x)), (d(x).j(x) + e(x) - e(x).f(x).h(x) + e(x).f(x).h(x).i(x) - \\
& e(x).j(x)).\nu_A(x) + (d(x).k(x) + e(x) - e(x).f(x).h(x) - e(x).k(x)).(1 - \\
& (d(x).k(x).l(x) + e(x).f(x).g(x) - e(x).f(x).h(x) - e(x).k(x).l(x))/ \\
& (d(x).k(x) + e(x) - e(x).f(x).h(x) - e(x).k(x)).\mu_A(x) - \nu_A(x))\rangle | x \in E\} \\
= \; & X_{u,v,w/v,x,y,z/y}.\Diamond
\end{aligned}
$$

For example, if $a(x) = c(x) = d(x) = f(x) = g(x) = i(x) = j(x) = l(x) = 1$, for every $x \in E$ and $b(x) = \alpha, e(x) = \beta, h(x) = \gamma, k(x) = \delta$ for every $x \in E$, where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\alpha + \beta \leq 1$, $\gamma + \delta \leq 1$, then

$$
\begin{aligned}
u(x) &= 1, \\
v(x) &= \gamma + \alpha - \alpha.\delta - \alpha.\gamma, \\
w(x) &= \alpha + \gamma - \alpha.\gamma - \alpha.\delta, \\
x(x) &= 1, \\
y(x) &= \beta + \delta - \beta.\gamma - \beta.\delta, \\
z(x) &= \beta + \delta - \beta.\gamma - \beta.\delta.
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
& F_{\alpha,\beta}(F_{\gamma,\delta}(A)) = X_{1,\alpha,1,1,\beta,1}(X_{1,\gamma,1,1,\delta,1}(A)) \\
= \; & X_{1,\alpha+\gamma-\alpha.\gamma-\alpha.\delta,1,1,\beta+\delta-\beta.\gamma-\beta.\delta,1}(A) \\
= \; & F\alpha + \gamma - \alpha.\gamma - \alpha.\delta, \beta + \delta - \beta.\gamma - \beta.\delta(A)
\end{aligned}
$$

Theorem 3: For every two IFSs A and B and for every six functions $a, b, c, d, e, fE :\to [0, 1]$ satisfying (1) and (2):
(a) $\overline{X_{a,b,c,d,e,f}(\overline{A})} = X_{d,e,f,a,b,c}(A)$,
(b) $X_{a,b,c,d,e,f}(A \cap B) \subset X_{a,b,c,d,e,f}(A) \cap X_{a,b,c,d,e,f}(B)$,
(c) $X_{a,b,c,d,e,f}(A \cup B) \supset X_{a,b,c,d,e,f}(A) \cup X_{a,b,c,d,e,f}(B)$,
(d) if $c = f = 1, a \geq b$ and $e \geq d$, then $X_{a,b,c,d,e,f}(A + B) \subset X_{a,b,c,d,e,f}(A) + X_{a,b,c,d,e,f}(B)$,
(e) if $c = f = 1, a \geq b$ and $e \geq d$, then $X_{a,b,c,d,e,f}(A.B) \supset X_{a,b,c,d,e,f}(A).X_{a,b,c,d,e,f}(B)$,
(f) $X_{a,b,c,d,e,f}(A@B) = X_{a,b,c,d,e,f}(A)@X_{a,b,c,d,e,f}(B)$.

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