

## **Intuitionistic fuzzy estimations of biological interactions**

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**Abstract:** All living organisms sustain their life, reproduce and evolve through interaction with the biotic and abiotic components of their ecosystems. Therefore, the mathematical formalism chosen to describe interactions generally predetermines the scope and properties of the ecological models it is part of. The goal of the present paper is to introduce intuitionistic fuzzy estimations of the interaction between two living organisms capable of aggregating the positive and negative aspects of the interaction, as well as the uncertainty with which they can be determined. Using the concept of intuitionistic fuzzy sets, new continuous formulation of the six basic types of biological interactions (neutralism, amensalism, commensalism, competition, mutualism, predation or parasitism) is presented. This formulation is capable of incorporating different sources of uncertainty, including the ambiguity in the discrimination of direct and environment mediated effects. In the special case when the uncertainty of interaction between two living organisms comes from unaccounted small changes in their relatively constant environment, a method for estimation the interaction components based on observations of interacting objects is provided. A simple averaging algorithm is presented for generalizing the results of multiple observations. Future extensions and possible applications of the intuitionistic fuzzy estimations of biological interactions are also discussed.

**Keywords:** Biological interactions, Intuitionistic fuzzy sets, Forward and inverse problems in ecology, Neutralism, Amensalism, Commensalism, Competition, Mutualism, Predation, Parasitism.

**AMS Classification:** 92D40, 03B52.

### **1. Introduction**

According to [18], more than 75% of the ice free Earth surface is altered as a result of human activities. The more extreme and faster these changes are, the more off equilibrium the systems become. First to adapt to the new conditions are the most abundant and simplest short lifecycle life forms (like bacteria and microorganisms) which affect the food web and along with initiating factors the whole ecosystem. The key to understanding how organisms adapt or maintain their existence in a pristine natural or anthropogenic ecosystem is to understand how they interact with the environment and with other species. Such understanding will help us reconnect to the intricate web of life and benefit from the evolutionary gained knowledge of how to live together, utilise and recycle resources more efficiently, and become active self-

aware stability factors in our own ecosystems. Brief review of some of the existing ecological theories directly involving the notion of interaction is provided below.

Replicator model is widely used for modelling the population dynamics and evolution of traits. Interactions in this model change the quantity of one species after encountering another. Populations are presumed to be well mixed and every species interacts with every other species with fixed in time interaction strength. The model uses predefined interaction strengths in the replicator differential equation to determine species concentrations change. Different approaches are employed for assigning interaction strengths from random [15] through presumption based i.e. Hebb rule [34] to directly measured [9, 19, 40]. In the classical replicator model, only trophic (food web related) interactions are considered. Increasing number of studies [12, 37] suggest a significant role of non-trophic interactions which led some researchers to introduce them into replicator model in the form of interaction modifications [20, 22]. Another extension of replicator model is the ability of the model to evolve interaction strength on a different time scale of the fast changes in the population dynamics [35]. Despite the recent developments, the replicator model still possesses some limitations emanated from the original assumptions that all species interact with fixed frequency in time and space. These two limitations are much better treated by another wide spread theory for modelling population dynamics cellular automata.

A cellular automaton [38] is a grid of identical cells each of which can be in one of finite number of states. States are synchronously updated in function of cell's state and the state of neighbouring cells. Cells can represent individuals or local populations [14] and functions can represent individual or species interactions [24], fertility, mortality, migration and other. Comprehensive review of cellular automata applications is presented in [30]. They separate the application of cellular automata in two broad categories: in illustrating theoretical consequences of different rules, and empirical applications utilizing parameterized rule systems to model experimental observations. Cellular automata have proved to be very flexible and computationally efficient tool for investigation of the various aspects of interspecies interactions, yet so far models have been built on a case by case basis with rules designed specifically for the considered set of species. Development of universal models is hampered by the definition of cellular automata in which each cell have to be identical and therefore the more species and possible interactions are considered, the more complex the definition of the cells becomes.

Another important property of natural interactions is their perceived uncertainty and variability. While both of these properties could be just the result of our limited knowledge of all contributing factors, they still need to be considered when building models of biological interactions. Variability is when different behaviour under seemingly preserved conditions is modelled by probabilities. In cellular automata models, this is expressed as probabilistic rules [39] and in replicator model – as random interactions. There are also entirely probabilistic models for predicting the population dynamics [25]. Uncertainty is generally modelled as fuzziness. In cellular automata models, uncertainty is introduced in terms of fuzzy rules and states [10, 36]. In niches theory, which models species interactions implicitly, fuzziness is also used to describe the degree of overlapping between niches [11, 42].

All of the methods above are designed to model forward problems in ecology, i.e. finding the state of an ecosystem, when species and their interactions are known and understood. The inverse problem [7], i.e. to quantify the interactions when observations of the ecosystem state are provided, proved to be even more challenging since discrete experimental observations have to be incorporated in often continuous theoretical models. A systematic overview of the origin of the major current methods is offered by [27]. The four basic theoretical models

described in the overview are: community matrix, Jacobian matrix, Inverted Jacobian matrix and removal matrix. Community matrix represents the direct effect an individual from one species has on individual of other species. It can be derived from generalized Lotka-Volterra equation [8, 28]. This approach has been extensively studied [13, 26, 29] and also applied to interacting populations instead of individuals. The elements of Jacobian matrix represent the direct effect of an individual from one species on the total population from another species at or near equilibrium [8, 32]. While community and Jacobian matrices account only for direct effects of one species on another, the inverted Jacobian matrix also accounts for indirect interactions [31]. This matrix describes the total direct and indirect effects of one species to a constant rate of removal or addition of another species [33, 41]. The removal matrix measures the interaction strength as a result of complete species removal. Each element in the matrix represents the difference in equilibrium abundance between a community with all species present, and the same community with one species removed. Various nontrivial extensions of those four major methods have been developed [16, 21] as well as some principally different approaches [17] have been introduced. A common assumption in all of these methods is that the structure of the interaction model is known and only the interaction coefficients have to be determined. While this could be the case in some well-studied simple ecosystems, in general it is a limiting factor for the complex natural and anthropogenic systems. This outlines the need of a general model which can define the individual interaction strengths with no previous assumptions based on observations only.

The goal of the present paper is to introduce a formal description of the interaction between two living organisms capable of aggregating the mixture of positive and negative effects rendered from the one organism to the other and the ambiguity in the discrimination of direct and environment dependent components of the interaction.

## 2. Intuitionistic fuzzy estimations of biological interaction

Let  $\Omega_{S+x}^x \in [0,1]$  be the health status of a living organism  $x$  in the presence of the set of objects,  $S = \{z|z \in \aleph\}$ , where 0 corresponds to dead organism and 1 to organism in perfect health, and let  $S^{+x}$  denote the set of objects  $S$  combined with the object  $x$ , or in general case

$$S^{+x_1, x_2, \dots, x_m} = \{z|z \in \aleph\} \cup \{x_1, x_2, \dots, x_m\}.$$

Then the *effect* of the object  $y \in \aleph$  over the living organism  $x \in \aleph$  in the presence of the objects in  $S$  could be described as the intuitionistic fuzzy number [4]

$$\alpha_{S(x,y)} = \langle \mu_S(\langle x, y \rangle), \vartheta_S(\langle x, y \rangle) \rangle,$$

where:  $\langle x, y \rangle$  is the ordered tuple of the two interacting objects,  $\mu_S: \aleph^2 \rightarrow [0,1]$  is the positive effect of  $y$  over  $x$  in the presence of  $S$ ,  $\vartheta_S: \aleph^2 \rightarrow [0,1]$  is the negative effect of  $y$  over  $x$  in the presence of  $S$ , and  $0 \leq \mu_S(\langle x, y \rangle) + \vartheta_S(\langle x, y \rangle) \leq 1$ . Level of uncertainty  $\pi: \aleph^2 \rightarrow [0,1]$  can be defined as  $\pi_S(\langle x, y \rangle) = 1 - \mu_S(\langle x, y \rangle) - \vartheta_S(\langle x, y \rangle)$ .

The *interaction* between two objects  $x, y \in \aleph$  in the presence of the objects in  $S$  is defined by the tuple

$$I_S(x, y) = (\alpha_{S(x,y)}, \alpha_{S(y,x)}).$$

The *pure* or *direct* effect of  $y$  over  $x$  can be defined as the effect observable in the constant environment  $\bar{S}$ :

$$\alpha_{\bar{S}(x,y)} = \alpha_{(x,y)} = \langle \mu(\langle x, y \rangle), \vartheta(\langle x, y \rangle) \rangle,$$

The **constant environment**  $\bar{S}$  is such an environment which is not affected by the presence of  $x$  and  $y$ , or formally:

$$\forall (z \in S), \Omega_{S+x,y}^z = \Omega_{S+y}^z = \Omega_{S+x}^z = \Omega_S^z, \text{ and}$$

$$\forall (z_1, z_2 \in S), I_S(z_1, z_2) = I_{S+x}(z_1, z_2) = I_{S+y}(z_1, z_2) = I_{S+x,y}(z_1, z_2).$$

The interaction  $I_S(x, y)$  is the cumulative effect of all direct interactions in time and space between  $x, y$  and  $S$  (see Figure 1).

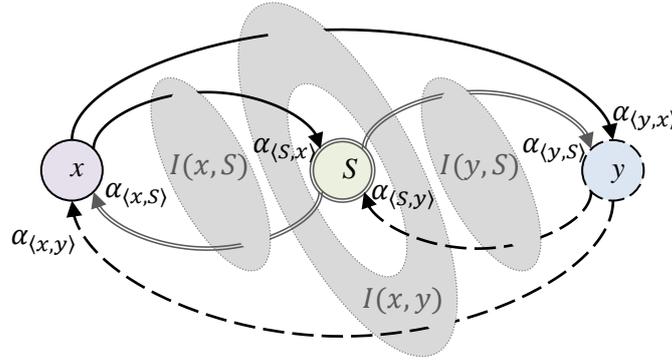


Figure 1. The cumulative effect of of all direct ineractions in time and space between  $x, y$  and  $S$

In the general case, this could be very complicated and mutually dependent relationship which cannot be easily divided into independent components. In the special case when  $S$  could be treated as relatively unchanged set of objects as a result of presence of  $x$  and  $y$ , then the interaction between  $x$  and  $y$  in the enviroment of  $S$  is close to direct interaction. In this case, we can assume that all uncertainty  $\pi_S$  for the direct interaction  $I(x, y)$  comes from the unaccounted small changes in  $S$ . This case provides a framework with a great practical significance where pure interactions can be studied by comparing the health indices  $\Omega_{S+x}^x$  and  $\Omega_{S+y}^y$  of  $x$  and  $y$  living by themselves in the enviroment  $S$  with their health indices  $\Omega_{S+x,y}^x$  and  $\Omega_{S+x,y}^y$  wnen living together. One possible way to estimate the direct effect  $\alpha_{\bar{S}(x,y)}$  is

$$\mu_{\bar{S}(x,y)} = \left( \frac{1}{2} + \frac{\Delta \Omega_{\bar{S}}^x}{2 * \max(\Omega_{\bar{S}+x}^x, 1 - \Omega_{\bar{S}+x}^x)} \right) (1 - \pi_{\bar{S}}(\langle x, y \rangle))$$

$$\vartheta_{\bar{S}(x,y)} = \left( \frac{1}{2} - \frac{\Delta \Omega_{\bar{S}}^x}{2 * \max(\Omega_{\bar{S}+x}^x, 1 - \Omega_{\bar{S}+x}^x)} \right) (1 - \pi_{\bar{S}}(\langle x, y \rangle))$$

where

$$\Delta \Omega_{\bar{S}}^x = \Omega_{\bar{S}+x,y}^x - \Omega_{\bar{S}+x}^x$$

and  $\pi_{\bar{S}}(\langle x, y \rangle) = \max(|\mu(\langle \bar{S}, x \rangle) - \vartheta(\langle \bar{S}, x \rangle)|, |\mu(\langle \bar{S}, y \rangle) - \vartheta(\langle \bar{S}, y \rangle)|)$  is one possible measure of the changes in  $\bar{S}$  as a result of presence of  $x$  and  $y$ . In the expressions above,  $\bar{S}$  denotes **relatively constant environment** which can be formally defined as following:

$$\forall (z \in S), \Omega_{S+x,y}^z \cong \Omega_{S+y}^z \cong \Omega_{S+x}^z \cong \Omega_S^z, \text{ and}$$

$$\forall (z_1, z_2 \in S), I_S(z_1, z_2) \cong I_{S+x}(z_1, z_2) \cong I_{S+y}(z_1, z_2) \cong I_{S+x,y}(z_1, z_2).$$

The estimation method above presumes maximum activity and represents neutral effects ( $\Delta\Omega_{\xi}^x$ ) with their equal maximal values  $\mu_{\xi(x,y)} = \vartheta_{\xi(x,y)} = 0.5$ , when  $\pi_{\xi}(\langle x, y \rangle) = 0$ .

The formal classification of biological interactions is given in Table 1.

<b>Interaction type</b>	<b>Description</b>	<b>Effect on x</b>	<b>Effect on y</b>	<b>Intuitionistic fuzzy definition</b>	<b>Extreme crisp case</b> $\pi(\langle x, y \rangle) = 0,$ $\pi(\langle y, x \rangle) = 0$
Neutralism	Neutralism describes the relationship between two objects which interact but do not affect each other.	0	0	$\begin{cases} \mu(\langle x, y \rangle) = \vartheta(\langle x, y \rangle) \\ \mu(\langle y, x \rangle) = \vartheta(\langle y, x \rangle) \end{cases}$	$\begin{cases} \mu(\langle x, y \rangle) = 0.5 \\ \vartheta(\langle x, y \rangle) = 0.5 \\ \mu(\langle y, x \rangle) = 0.5 \\ \vartheta(\langle y, x \rangle) = 0.5 \end{cases}$
Amensalism	Amensalism between two objects $x, y$ involves $y$ impeding the success of $x$ while the $x$ has no effect on $y$ .	-	0	$\begin{cases} \mu(\langle x, y \rangle) < \vartheta(\langle x, y \rangle) \\ \mu(\langle y, x \rangle) = \vartheta(\langle y, x \rangle) \end{cases}$	$\begin{cases} \mu(\langle x, y \rangle) = 0 \\ \vartheta(\langle x, y \rangle) = 1 \\ \mu(\langle y, x \rangle) = 0.5 \\ \vartheta(\langle y, x \rangle) = 0.5 \end{cases}$
Commensalism	Commensalism between two objects $x, y$ occurs when $x$ benefits from $y$ , while $x$ has no effect on $y$ .	+	0	$\begin{cases} \mu(\langle x, y \rangle) > \vartheta(\langle x, y \rangle) \\ \mu(\langle y, x \rangle) = \vartheta(\langle y, x \rangle) \end{cases}$	$\begin{cases} \mu(\langle x, y \rangle) = 1 \\ \vartheta(\langle x, y \rangle) = 0 \\ \mu(\langle y, x \rangle) = 0.5 \\ \vartheta(\langle y, x \rangle) = 0.5 \end{cases}$
Competition	Competition is an interaction between two objects that is mutually detrimental.	-	-	$\begin{cases} \mu(\langle x, y \rangle) < \vartheta(\langle x, y \rangle) \\ \mu(\langle y, x \rangle) < \vartheta(\langle y, x \rangle) \end{cases}$	$\begin{cases} \mu(\langle x, y \rangle) = 0 \\ \vartheta(\langle x, y \rangle) = 1 \\ \mu(\langle y, x \rangle) = 0 \\ \vartheta(\langle y, x \rangle) = 1 \end{cases}$
Mutualism	Mutualism is an interaction between two objects, which is mutually beneficial.	+	+	$\begin{cases} \mu(\langle x, y \rangle) > \vartheta(\langle x, y \rangle) \\ \mu(\langle y, x \rangle) > \vartheta(\langle y, x \rangle) \end{cases}$	$\begin{cases} \mu(\langle x, y \rangle) = 1 \\ \vartheta(\langle x, y \rangle) = 0 \\ \mu(\langle y, x \rangle) = 1 \\ \vartheta(\langle y, x \rangle) = 0 \end{cases}$
Predation or Parasitism	Predation between two $x, y$ organisms is when $x$ benefits at the expense of the $y$ .	+	-	$\begin{cases} \mu(\langle x, y \rangle) > \vartheta(\langle x, y \rangle) \\ \mu(\langle y, x \rangle) < \vartheta(\langle y, x \rangle) \end{cases}$	$\begin{cases} \mu(\langle x, y \rangle) = 1 \\ \vartheta(\langle x, y \rangle) = 0 \\ \mu(\langle y, x \rangle) = 0 \\ \vartheta(\langle y, x \rangle) = 1 \end{cases}$

Table 1. Formal classification of biological interactions using IFSs

Naturally occurring interactions often does not exclusively fall in a single category but are rather located in the continuum between any of the six main categories. This fact is well accommodated by their intuitionistic fuzzy definition which also provides additional parameter  $\pi$  quantifying the uncertainty of the described relationship. It must be also noted that these relationships are defined as pure interactions in a constant environment  $\bar{S}$  (the index in the definitions is omitted for better readability).

The more complex interacting organisms are and richer their constant environment is, the more individual differences and the accuracy of observational health index estimations can influence the measure of the pure interaction. Therefore, a simple algorithm can be applied to smooth those differences and combine them in an average estimate. This method can be used when all interactions take place in the same constant environment. For simplicity as in above definitions, the index  $\bar{S}$  will be omitted from the description below.

We can construct the following index matrix [1, 5, 6]:

	$O_1$	$O_2$	$\dots$	$O_m$
$O_1$	$\alpha_{\langle 1,1 \rangle}$	$\alpha_{\langle 2,1 \rangle}$	$\dots$	$\alpha_{\langle m,1 \rangle}$
$O_2$	$\alpha_{\langle 1,2 \rangle}$	$\alpha_{\langle 2,2 \rangle}$	$\dots$	$\alpha_{\langle m,2 \rangle}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$O_m$	$\alpha_{\langle 1,m \rangle}$	$\alpha_{\langle 2,m \rangle}$	$\dots$	$\alpha_{\langle m,m \rangle}$

where  $O_1, O_2, \dots, O_m$  are the interacting objects and  $\alpha_{\langle i,j \rangle}$  are the intuitionistic fuzzy estimations of the pure effect of object  $O_j$  over object  $O_i$  in the constant environment  $\bar{S}$ , where  $i, j = 1, 2, \dots, m$ . The direct interaction can be expressed as above:

$$I(O_i, O_j) = (\alpha_{\langle O_i, O_j \rangle}, \alpha_{\langle O_i, O_j \rangle}).$$

Initially, all effects are unknown and  $\pi_{\langle i,j \rangle} = \pi_{\langle j,i \rangle} = 1$ ,

	$O_1$	$O_2$	$\dots$	$O_m$
$O_1$	$\langle 0,0 \rangle$	$\langle 0,0 \rangle$	$\dots$	$\langle 0,0 \rangle$
$O_2$	$\langle 0,0 \rangle$	$\langle 0,0 \rangle$	$\dots$	$\langle 0,0 \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$O_m$	$\langle 0,0 \rangle$	$\langle 0,0 \rangle$	$\dots$	$\langle 0,0 \rangle$

Let  $E_{\langle i,j \rangle}^k$  be the  $k$ -th observation of the effect of object  $j$  on object  $i$

$$E_{\langle i,j \rangle}^k = \langle \mu_{E^k}(\langle i,j \rangle), \vartheta_{E^k}(\langle i,j \rangle) \rangle,$$

where  $i, j \in \aleph$  &  $0 \leq \mu_{E^k}(\langle x,y \rangle) + \vartheta_{E^k}(\langle x,y \rangle) \leq 1$ . Then, the  $k$ -th intuitionistic fuzzy interaction estimation for the objects  $i, j$  can be obtained as a weighted combination of the  $a_{\langle i,j \rangle}^{k-1}$  and the observation  $E_{\langle i,j \rangle}^k$

$$a_{\langle i,j \rangle}^k = \left\langle \frac{(k-1)\mu_{a^{k-1}}(\langle i,j \rangle) + \mu_{E^k}(\langle i,j \rangle)}{k}, \frac{(k-1)\vartheta_{a^{k-1}}(\langle i,j \rangle) + \vartheta_{E^k}(\langle i,j \rangle)}{k} \right\rangle.$$

Alternatively, to prevent error accumulation the above expression, it can be written in the following form:

$$a_{\langle i,j \rangle}^k = \left\langle \frac{1}{k} \sum_{l=1}^k \mu_{E^l}(\langle i,j \rangle), \frac{1}{k} \sum_{l=1}^k \vartheta_{E^l}(\langle i,j \rangle) \right\rangle$$

Using simple averaging does not lead to reduction of uncertainty with increased number of observations, which could be a desired property when the algorithm is applied to interaction

in relatively constant environment  $\tilde{S}$  since the uncertainty there comes from unaccounted cumulative effects. In case of constant environment  $\bar{S}$ , other operators reducing uncertainty can be used to aggregate the estimations from different observations [3].

### 3. Discussion

The formalism above focuses on the general case of interaction of two living organisms in a given environment. These results can be applied to the case when one of the interacting objects is not a living organism but rather a media or resource. In this case, the health status  $\Omega_{S+x}^x$  can be viewed as a measure of the availability of the resource within the range of: 0 – resource is not available, and 1 resource is available in infinite quantities. Another special case of general definition is the interaction without environment,  $S = \emptyset$ . In this case the interaction is direct and does not have any third party mediated components. This is also the case which is considered in the majority of the existing models, including those based on Lotka-Volterra equation and replicator dynamics.

Direct effect observed in one environment  $\tilde{S}_1$  does not necessary coincide with the direct effect observed in another environment  $\tilde{S}_2$ . Further investigation is needed to reveal what conditions need to be met so that direct effect observed in one environment can be adjusted to estimate the direct effect observed in another environment.

In the current definition, only the “quality” (positive, neutral, negative) of the interaction is considered but not its magnitude / strength. This, however, is an important factor when the quantitative estimation of the result of the interaction is needed.

In its present form, the intuitionistic fuzzy estimation of the biological interactions could not explicitly accommodate important properties of natural interactions. Some of these properties are:

- *Living organism state dependency.* A simple example of such a dependency is the interaction between the predator and prey – depending on the current need for food of the predator. Such dependencies can be modelled in the future by introduction of state machines models of the organisms.
- *Temporal aspect.* This aspect is tightly related to the state dependencies and introduces the time scale in which these state changes develop. A generalized measure of such temporal changes is the life stage of the living organism which clearly presents different needs and different interaction properties in the course of organism development, maturation and decline. Temporal aspect could be easily introduced utilizing the existing research in the field of Temporal Intuitionistic Fuzzy Sets [2, 3].
- *Spatial aspect.* Interaction strength and polarity (positive neutral negative) generally are significantly altered by the distance between the interacting objects. For example a nitrogen fixing plant at optimal distance could provide nitrogen for other plants without creating too much root or light competition, thus creating overall positive effect. The same plant if closer than optimal could have detrimental effect because of shading, and diminishing effect if it is too far.
- *Evolutionary aspect.* Crossbreeding and mutations – the driving force of evolution also change the way living organisms interact.
- *Multidimensionality.* Interactions can have multiple “dimensions” and can be beneficial in certain aspects while detrimental in others.

Considering all of these properties in a single model might look as an over-complication and unattractive for practical application but it could be the only plausible way to establish their relative importance while building a better model of arguably complex natural phenomena. Moreover, when considering state machine represented individual interacting objects in space, time and physical environment utilizing game theory, a simulation of such a virtual ecosystem does not look unattainable. Such virtual models could choose not to describe the abundant diversity of life forms but rather utilize the existing taxonomy classifications and build general models which simulate the relationships of kingdoms, genera, classes or families of organisms, investigating the evolutionary preserved properties of the respective taxonomy ranks.

## 4. Conclusions

The introduction of intuitionistic fuzzy estimations of biological interactions offers simple yet practically applicable framework for studying anthropogenic or natural ecosystems. It presents a unified formalism which can be used for modelling of both forward and inverse problems in ecology. A benefit over existing approaches is that intuitionistic fuzzy estimations provide explicit estimations of both positive and negative aspects of an interaction, as well as its uncertainty. It is also important to emphasize that interactions in this framework are conditionally defined and depend on the environment in which they are occurring. Intuitionistic fuzzy interactions are also defined on individual level and can be used for encoding or predicting of direct observations.

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