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Intuitionistic Fuzzy Delphi Method: More realistic and interactive forecasting tool

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Abstract: This paper presents a new and improved version of Fuzzy Delphi Method by using triangular intuitionistic fuzzy number (TIFN). In case of real life usage of Delphi Method, information communicated by experts is not used with complete potential. Only some of the information provided are actually accessed or used. And, hence we may not come to a highly accurate and realistic conclusion always. But, in case of Intuitionistic Fuzzy Delphi Method, communication with experts is the same as Fuzzy Delphi Method yet an improved and elaborative statistical tool is used to reach in better conclusions. Subjective information is more likely to be like a quasi-objective data in case of intuitionistic fuzzy number and hence use of intuitionistic fuzzy number is more justified. Also, the experts use their individual competency and subjectivity and are somehow uncertain to air their opinions. Thus, they prefer degree of non-membership over degree of membership and this is the very reason why use of intuitionistic fuzzy concepts is more relevant than fuzzy concepts. Moreover, by using TIFNs, it is easier for an expert to study the realization data which are nested within one another than triangular fuzzy numbers (TFNs). And, the concept of sheaf of intuitionistic fuzzy numbers is an aggregation process which appears to be very convenient for the objectification of (somehow hazy) subjective opinions.

Keywords: Fuzzy Delphi Method, Application of TIFN, Decision making technique.

AMS Classification: 03F55, 62C86.

1 Introduction

The Delphi Method [9, 10] is a structured communication technique, originally developed as a systematic, interactive forecasting method which relies on a panel of experts. The Delphi Method belongs to the subjective-intuitive methods of foresight. Delphi was developed in the 1940s by the Rand Corporation, Santa Monica, California, in operation research. In 1944, General Henry H. Arnold ordered the creation of the report for the U.S. Army Air Corps on the future technological capabilities that might be used by the military. Different approaches were tried, but the shortcomings of traditional forecasting methods, such as theoretical approach, quantitative models or trend extrapolation, in areas where precise scientific laws have not been established yet, quickly became apparent. To combat these shortcomings, the Delphi Method was developed by Project RAND during the 1950-1960s (1959) by Olaf Helmer, Norman Dalkey, and Nicholas Rescher, [9]. It has been used ever since, together with various modifications and reformulations, such as the Imen-Delphi procedure, [13].

The name can be traced back to the Delphic oracle, as Woudenberg reports that the name 'Delphi' was intentionally coined by Kaplan, an associate professor of philosophy at the UCLA working for the RAND corporation in a research effort directed at improving the use of expert predictions in policy making. Kaplan et al. referred to the 'principle of the oracle' as a 'non-falsifiable prediction', a statement that does not have the property of being 'true' or 'false'. Thus 'Delphi' for the modern foresight method seems to be more than a simple brand name.

The foundation of the temple at Delphi and its oracle took place before recorded history. For a thousand years of recorded history the Greeks and other peoples, sometimes as private individuals, sometimes as official ambassadors, came to Delphi to consult the prophetess, who was called Pythia. Her words were taken to reveal the rules of the Gods. These prophecies were not usually intended simply to be a prediction of the future as such. Pythia's function was to tell the divine purpose in a normative way in order to shape coming events. The information came in from the ambassadors through their queries and the answers were written down on metal or stone plates, several of them found by archeologists. The temple was the locus of knowledge, i.e. the Delphic oracle was probably the largest database of the ancient world.

2 Definition

Wechsler (1978) characterizes a 'Standard Delphi Method' in the following way, [13]:

> *'It is a survey which is steered by a monitor group, comprises several rounds of a group of experts, who are anonymous among each other and for whose subjective-intuitive prognoses a consensus is aimed at. After each survey round, a standard feedback about the statistical group judgment calculated from median and quartiles of single prognoses is given and if possible, the arguments and counterarguments of the extreme answers are fed back…'.*

Content of Delphi studies are always issues about which unsure incomplete knowledge exists. Otherwise there are more efficient methods for decision making. Delphi are judgment processes with unsure aspects. The persons involved in Delphi studies only give estimations. For the participation experts are to be involved who on the basis of their knowledge and experience are able to assess in a competent way. During the rounds, they have the opportunity to gather new information. Especially the psychological process in connection with communication and less in the sense of mathematical models have to be stressed. Delphi tries to make use of self-fulfilling and self-destroying prophecies in the sense of shaping or even 'creating' the future.

The Delphi Method is based on structural surveys and makes use of the intuitive and available information of the participants, who are mainly experts, [10]. Therefore, it delivers qualitative as well as quantitative results and has beneath its explorative, predictive even normative elements. There is not the one Delphi methodology but the applications are diverse. There is agreement that Delphi is an expert survey in two or more 'rounds' in which in the second and later rounds of the survey the results of the previous round are given as feedback. Therefore, the experts answer from the second round on under the influence of their colleagues' opinions. Thus, the Delphi Method is a 'relatively strongly structured group communication process, in which matters, on which naturally unsure and incomplete knowledge is available, are judged upon by experts', [10].

3 Acceptance and weakness

Overall, the track record of the Delphi Method is mixed. There have been many cases when the method produced poor results. One may attribute this to poor application of the method and not to the weaknesses of the method itself. It must also be realized that in areas such as science and technology forecasting, the degree of uncertainty is so great that exact and always correct predictions are impossible, so a high degree of error is to be expected, [13]!

Another particular weakness of the Delphi Method is that future developments are not always predicted correctly by consensus of experts. Firstly, the issue of ignorance is important. If panelists are misinformed about a topic, the use of Delphi may only add confidence to their ignorance. Secondly, sometimes unconventional thinking of amateur outsiders may be superior to expert thinking.

One of the initial problems of the method was its inability to make complex forecasts with multiple factors. Potential future outcomes were usually considered as if they had no effect on each other. Later on, several extensions to the Delphi Method were developed to address this problem, such as cross impact analysis that takes into consideration the possibility that the occurrence of one event may change probabilities of other events covered in the survey.

Despite these shortcomings, today the Delphi Method is a widely accepted forecasting tool and has been used successfully for thousands of studies in areas varying from technology forecasting to drug abuse, [13].

4 Online Delphi forecasting systems

A number of Delphi forecasts are conducted using web sites that allow the process to be conducted in Real-time Delphi. For instance, the Tech Cast Project uses a panel of 100 experts worldwide to forecast breakthroughs in all fields of science and technology. Further examples are several studies conducted by the Center for Futures Studies and Knowledge Management that use an online-based Delphi method, [13].

5 Fuzzy Delphi Method

Fuzzy Delphi Method was introduced by Kaufman and Gupta (1988) [8] and was also pro–posed by Ishikawa et al. (1993). Noorderhaben (1995) indicated that applying the Fuzzy Delphi Method to group decision [4] can solve the fuzziness [12] of common understanding of expert opinions. The expert prediction interval value was then used to derive the fuzzy numbers, resulting in the Fuzzy Delphi Method. Hsu and Chen (1996) [11] proposed the fuzzy similarity aggregation method. Hence, Fuzzy Delphi Method is a generalization of the classical method.

It consists of the following steps [8, 11]:

Step 1	Experts are asked to provide the possible realization dates of a certain event in science, technology, or business, namely: the earliest date, the most plausible date, and the latest date. The data given by the experts are presented to the moderator for fuzzy averaging for forecasting.
Step 2	First, the average (mean) is computed, [5]. Then for each expert the deviation between mean and respective data is computed. It is also a triangular fuzzy number, [7]. The deviation is sent back to each of the expert for reevaluation.
Step 3	Each expert again presents a new triangular number in second round. Next, the same process starting with Step 2 is repeated. The triangular averages are calculated once again and are substituted correspondingly. If necessary, new triangular numbers are generated and their averages are calculated. The process could be repeated again and again until two successive means become reasonably close.
Step 4	At a later time the forecasting may be reexamined by the same process if there is important information available due to new discoveries or any other misinterpretation

6 Intuitionistic Fuzzy Delphi Method

Here, an improved version of Fuzzy Delphi Method is being proposed by using triangular intuitionistic fuzzy numbers (TIFNs) [1–3]. The arguments that can be used in favour of using

TIFNs in place of TFNs are that we may not always get subjective information which may be transformed into objective values as required in Fuzzy Delphi Method. For example, let us consider the price of gold in retail market. If we ask an expert about the price range of gold in next one year, we may receive a reply as follows: '…we envisage (not predict) a (still wide) range in prices, of from $ 880 to $ 1280 with the inside channel of $ 970 to $ 1070 likely. For the time being, the average price is likely to stick fairly close to the 2009 number near $970 per ounce.'* If we try to obtain quasi-objective data from this subjective information to use in Fuzzy Delphi method, we are likely to accept a TFN (970, 970, 1070) or (880, 970, 1280) as our expert opinion. So, we are likely to leave some information; e.g. in first case, we are considering values from inside channel *only*; where as in second case, we are *not* considering values from inside channel. It means that information communicated by experts is not being used with full potential. We are using only some of the information provided. And, hence, we may not come to a highly accurate and realistic conclusion. So, we must have to improve the Fuzzy Delphi Method to produce better results. In Fuzzy Delphi Method, communication with experts is the same as Delphi Method yet the estimation procedure is much different. And in case of Intuitionistic Fuzzy Delphi Method, communication with experts is the same as Fuzzy Delphi Method yet an improved and elaborative statistical tool is used to reach in better conclusions. Subjective information is more likely to be like quasi-objective data in case of intuitionistic fuzzy number and hence the use of intuitionistic fuzzy number in our method is more justified. [9, 10].

Also, the experts use their individual competency and subjectivity and are somehow uncertain to air their opinions. Hence, they tend to secure their opinions. Thus, they prefer degree of non-membership over degree of membership and this is the very reason why use of intuitionistic fuzzy concepts [1–3] is more relevant than fuzzy concepts. Moreover, by using TIFNs, it is easier for an expert to study the realization data which are nested within one another than TFNs. And, the concept of sheaf of intuitionistic fuzzy numbers is an aggregation process which appears to be very convenient for the objectification of (somehow hazy) subjective opinions.

The steps of the proposed method are as follows.

Step 1	The expert E_i, $i = 1, 2, \ldots, n$, are requested to provide the possible realization dates of a certain event in science, business or technology, viz.: the earliest certain date $e_{c1}^{(i)}$, the earliest uncertain date $e_{u1}^{(i)}$, the most plausible date $m_{p1}^{(i)}$, the latest certain date $l_{c1}^{(i)}$ and the latest uncertain date $l_{u1}^{(i)}$. Here, '1' in the suffix indicates that this is the first phase of forecasting process.
Step 2	We create an objective data out of these subjective information by considering a triangular intuitionistic fuzzy number as follows: $$(E_i\,;\, e_{c1}^{(i)}, m_{p1}^{(i)}, l_{c1}^{(i)}; e_{u1}^{(i)}, l_{u1}^{(i)}).$$

* Source: http://www.commodityonline.com

Step 3	These responses from n experts form a sheaf $(E_i\,;\, e_{c1}^{(i)}, m_{p1}^{(i)}, l_{c1}^{(i)};\, e_{u1}^{(i)}, l_{u1}^{(i)})$, $i = 1, 2, \ldots, n$. The mean of TIFN sheaf is then computed $(e_{c1}^{m}, m_{p1}^{m}, l_{c1}^{m};\, e_{u1}^{m}, l_{u1}^{m})$ and for each expert the divergence is computed as follows: $(E_i\,;\, e_{c1}^{m} - e_{c1}^{(i)}, m_{p1}^{m} - m_{p1}^{(i)}, l_{c1}^{m} - l_{c1}^{(i)};\, e_{u1}^{m} - e_{u1}^{(i)}, l_{u1}^{m} - l_{u1}^{(i)})$, where these divergence numbers can be positive, negative or null. This information is then sent again to each individual expert.
Step 4	Each expert now gives a new TIFN $(E_i\,;\, e_{c2}^{(i)}, m_{p2}^{(i)}, l_{c2}^{(i)};\, e_{2}^{(i)}, l_{u2}^{(i)})$ and the process from Step 3 is repeated.
Step 5	The process is continued until two successive means become reasonably close, i.e. the Delphi manager is satisfied. The number of such iteration phases may also be limited *a priori*. There may be many variations of this procedure; e.g. the experts can be asked not to increase the divergence without disturbing his unbiasness. Now, since the word 'close' is fuzzy, we require some in depth study. It can be based on the concept of distance metric between intuitionistic fuzzy numbers, [6, 7], i.e. if necessary, a study of opinions from partial or full group of experts is realized by calculating the distance between TIFN and non-disjunctive group of experts are formed by finding maximum sub relations of similarity.
Step 6	At a later time, the forecasting may be reexamined and reevaluated by same process in case of discovery or availability of new or important information.

7 Case study: Time estimation for technical realization of an innovative product

We consider the data for the problem of the technological realization of a cognitive information processing computer (as used partially in literature for the purpose of easy understanding only), [8, 10]. It is requested a group of fifteen computer experts to give us a subjective estimation for the realization of this new computing technology in the format of intuitionistic fuzzy number, i.e. it will consist of the earliest certain date $e_{c1}^{(i)}$, the earliest uncertain date $e_{u1}^{(i)}$, the most plausible date $m_{p1}^{(i)}$, the latest certain date $l_{c1}^{(i)}$ and the latest uncertain date $l_{u1}^{(i)}$ for each expert E_i. It may be noted that the experts are ranked equally and hence their opinions carry same weight.

We assume that he sheaf formed from experts' opinions was as follows:

Expert no.	Earliest Uncertain Date $e_{u1}^{(i)}$	Earliest Certain Date $e_{c1}^{(i)}$	Most Plausible Date $m_{p1}^{(i)}$	Latest Certain Date $l_{c1}^{(i)}$	Latest Uncertain Date $l_{u1}^{(i)}$
1	1992	1995	2003	2020	2024
2	1995	1997	2004	2010	2013
3	1999	2000	2005	2010	2012
4	1997	1998	2003	2008	2010
5	1998	2000	2005	2015	2018
6	1992	1995	2010	2015	2019
7	2009	2010	2018	2020	2022
8	1993	1995	2007	2013	2016
9	1993	1995	2002	2007	2010
10	2006	2008	2009	2020	2023
11	2008	2010	2020	2024	2027
12	1995	1996	2002	2006	2008
13	1996	1998	2006	2010	2013
14	1995	1997	2005	2012	2015
15	2000	2002	2010	2020	2023

The computation from this sheaf gives the mean TIFN, [5]:

$$(e_{c1}{}^{m}, m_{p1}{}^{m}, l_{c1}{}^{m}; e_{u1}{}^{m}, l_{u1}{}^{m}) = (29996/15, 30109/15, 30210/15; 29968/15, 30253/15)$$
$$= (1999.733, 2007.267, 2014, 1997.867, 2016.867)$$
$$\approx (2000, 2007, 2014; 1998, 2017).$$

The deviations [5] for each expert are now calculated as in the following table:

Expert no.	$e_{u1}{}^{m(i)} - e_{u1}{}^{(i)}$	$e_{c1}{}^{m(i)} - e_{c1}{}^{(i)}$	$m_{p1}{}^{m(i)} - m_{p1}{}^{(i)}$	$l_{c1}{}^{m(i)} - l_{c1}{}^{(i)}$	$l_{u1}{}^{m(i)} - l_{u1}{}^{(i)}$
1	10	05	04	– 06	– 07
2	07	03	03	04	04
3	03	00	02	04	05
4	05	02	04	06	07
5	04	00	02	– 01	– 01
6	10	05	– 03	– 01	– 02
7	– 07	– 10	– 11	– 06	– 05
8	09	05	00	01	01
9	09	05	05	07	07
10	– 04	– 08	– 02	– 06	– 06

(table continues)

11	– 06	–10	– 13	– 10	– 10
12	07	04	05	08	09
13	06	02	01	04	04
14	07	03	02	02	02
15	02	– 02	– 03	– 06	– 06

Suppose that the manager is not satisfied with the mean (2000, 2007, 2014; 1998, 2017). The deviations for each expert are given to respective expert and are requested to review his previous forecast and a new sheaf of TIFNs is obtained as follows:

Expert no.	Earliest Uncertain Date $e_{u2}^{(i)}$	Earliest Certain Date $e_{c2}^{(i)}$	Most Plausible Date $m_{p2}^{(\mathrm{i})}$	Latest Certain Date $l_{c2}^{(i)}$	Latest Uncertain Date $l_{u2}^{(i)}$
1	1994	1997	2004	2018	2022
2	1996	1998	2004	2009	2014
3	1999	2000	2005	2010	2014
4	1997	1998	2004	2010	2012
5	1998	2000	2005	2015	2018
6	1994	1997	2009	2015	2019
7	2006	2008	2016	2018	2021
8	1995	1997	2007	2013	2016
9	1995	1996	2003	2009	2012
10	2005	2007	2009	2019	2022
11	2006	2008	2017	2022	2024
12	1996	1997	2003	2008	2010
13	1996	1998	2006	2011	2012
14	1996	1998	2005	2012	2015
15	2000	2002	2009	2018	2021

In a similar way, the computation from this sheaf gives the mean TIFN:

$$
\begin{aligned}
(e_{c2}^{m}, m_{p2}^{m}, l_{c2}^{m}; e_{u2}^{m}, l_{u2}^{m}) &= (30001/15, 30106/15, 30207/15; 29973/15, 30252/15) \\
&= (2000.067, 2007.067, 2013.8; 1998.2, 2016.8) \\
&\approx (2000, 2007, 2014; 1998, 2017).
\end{aligned}
$$

Now, the manager is satisfied that mean TIFN in both cases are almost the same. He stops the intuitionistic fuzzy Delphi process and accepts the TIFN as a combined conclusion of experts' opinion. This means that the realization of the invention will occur in time interval [1998, 2017] with the inside channel being [2000, 2014] and the most likely year for the realization is 2007.

Now, to find the non disjunctive group of experts, the distances [6] between experts' opinions are calculated. There is no standard procedure to calculate the distance between TIFNs [6]. Here, a technique described in [8] by Arnold Kaufmann, Madan M. Gupta, is further developed.

Arnold Kaufmann, Madan M. Gupta [8] used $d(N_i, N_j)$ to be the normalized distance between two TFNs N_i and N_j, with

$$d(N_i, N_j) = \frac{1}{2(\beta_2 - \beta_1)}\left[\Delta^l(N_i, N_j) + \Delta^r(N_i, N_j)\right],$$

where N_i and N_j are respective TFNs given by experts i and j, Δ^l is the left distance and Δ^r is the right distance, β_2 and β_1 are arbitrary values at the right and the left respectively chosen such that $0 \le d \le 1$.

It is proposed that the normalized distance [6] between two TIFNs E_i and E_j be

$$d(E_i, E_j) = \frac{1}{4(\beta_2 - \beta_1)}\left[| E_u^{(i)} - E_u^{(j)} | + | E_c^{(i)} - E_c^{(j)} | + | L_u^{(i)} - L_u^{(j)} | + | L_c^{(i)} - L_c^{(j)} |\right]$$

where β_2 and β_1 are proposed to be $E_{u2}^{\ m}$ and $L_{u2}^{\ m}$ respectively, provided $0 \le d \le 1$. Else, the values of β_2 and β_1 are suitably chosen so that the relation $0 \le d \le 1$ holds. The results of the computations are tabulated for $\beta_2 = E_{u2}^{\ m} = 1998$ and $\beta_1 = L_{u2}^{\ m} = 2017$.

Expert No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0.2632	0.3158	0.2895	0.1842	0.0789	0.3158	0.1579	0.2763	0.2895	0.3947	0.3158	0.2632	0.2105	0.1579
2		0	0.789	0.0526	0.1842	0.1842	0.4737	0.1053	0.0658	0.4737	0.5658	0.0789	0.0526	0.0526	0.3157
3			0	0.789	0.1316	0.2368	0.4079	0.1579	0.1447	0.3947	0.4868	0.1579	0.1053	0.1053	0.2368
4				0	0.1842	0.2105	0.4737	0.1316	0.0658	0.4737	0.5658	0.0789	***0.0263***	0.0789	0.2895
5					0	0.1053	0.2895	0.1316	0.25	0.2895	0.3816	0.2632	0.1842	0.1316	0.1316
6						0	0.3684	0.0789	0.1974	0.3684	0.4605	0.2368	0.1842	0.1316	0.2105
7							0	0.5395	0.5395	0.0526	0.0921	0.5526	0.4737	0.4211	0.1579
8								0	0.1184	0.4211	0.5132	0.1579	0.1053	0.0526	0.2632
9									0	0.5395	***0.6316***	0.0658	0.0658	0.1053	0.4079
10										0	0.0921	0.5789	0.4737	0.4211	0.1579
11											0	0.6447	0.5658	0.5132	0.25
12												0	0.0789	0.1316	0.3947
13													0	0.0526	0.3158
14														0	0.2631
15															0

It is to be noted that the minimum distance is $d(E_4, E_{13}) = 0.0263$ and the maximum distance is $d(E_9, E_{11}) = 0.6316$. Now, if we are interested to find pair of experts for whom the distance is less than 0.1, we obtain the table as below:

Expert No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	R					R									
2		R	R	R					R			R	R	R	
3			R	R											
4				R					R			R	***R***	R	
5					R										
6						R		R							
7							R			R	R				
8								R						R	
9									R			R	R		
10										R	R				
11											R				
12												R	R		
13													R	R	
14														R	
15															R

For instance, the experts (2, 12, 13) and (4, 12, 13) have given almost same estimation. Therefore, we may assume that the experts (2, 4, 12, 13) have almost same estimation. Similarly, the experts (9, 12) and (12, 13) have also almost the same estimation. And (9, 12, 13) have almost same estimation.

For different upper limit of the metric *d*, we have different class of experts. It may be noted that the maximal sub relations of similarity in a dissemblance relation play the same role as classes in a similarity relation (as described by Kauffman and Gupta), [8].

8 Another metric to calculate distance between a pair of TIFNs

The distance [7] between TIFNs E_i and E_j may also be defined as:

$$d(E_i, E_j) = \frac{1}{3}\left\{\max(|E_c^i - E_c^j|, |L_c^i - L_c^j|) + \min(|E_u^i - E_u^j|, |L_u^i - L_u^j|) + |M_p^i - M_p^j|\right\}.$$

In this case, another table is formed by measuring distances between every pair of experts' opinions as follows:

Expert No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	3.667	4.667	3.667	2.667	2.667	8	3	3.667	5	8.667	4.333	3.667	3	3.667
2		0	1	***0.667***	3	4.333	9.667	2.667	1.33	7.7	12	***0.667***	1.333	1.333	6
3			0	1.667	2	4.667	8.667	2.333	2.667	6.33	10.333	2.667	1.667	1	4.333
4				0	2.333	4.333	10.333	2.667	1	7.33	11.333	1.333	1	1.333	4.667
5					0	2.667	7.333	2.333	3.667	5	8.667	3.667	2.333	1.667	3
6						0	6.667	1.667	4.333	4.33	8	6	4.667	3	2.333
7							0	8.333	11.333	3	1.667	11.333	9.667	9	4.333
8								0	2.667	6	9.667	3.333	1.333	1.333	4
9									0	9	***12.667***	***0.667***	1.667	2	6.667
10										0	4	8.667	7	6.667	2
11											0	***12.667***	10.667	10.333	5.667
12												0	2	2	6.667
13													0	***0.667***	4.667
14														0	4.667
15															0

It is to be noted that the minimum distance is $d(2, 4) = d(2, 12) = d(9, 12) = d(13, 14) = 0.667$ and the maximum distance is $d(9, 11) = d(11, 12) = 12.667$.

Now, if we are interested to find pair of experts for whom the distance is less than 1.5, we obtain the table as below:

Expert No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	R														
2		R	R	***R***					R			***R***	R	R	
3			R											R	
4				R					R			R	R	R	
5					R										
6						R									
7							R								
8								R					R	R	
9									R			***R***			
10										R					
11											R				
12												R			
13													R	***R***	
14														R	
15															R

For instance, the experts (2, 12, 13) and (4, 12, 13) have given almost same estimation. Therefore, we may assume that the experts (2, 4, 12, 13) have almost same estimation. But, the experts (9, 12) and (12, 13) do not have the same estimation. And, hence (9, 12, 13) does not qualify for a subclass (if $d < 1.5$).

9 Observation

It may be noted that the two sets of distance measure do not result into same non-disjunctive group of experts, whatever way the restriction on values of metric d be chosen; i.e. we will not be able to get a table identical with the last one, whatever the values of d be taken in

$$d(E_i, E_j) = \frac{1}{4(\beta_2 - \beta_1)}\left[\,| E_u^{(i)} - E_u^{(j)} | + | E_c^{(i)} - E_c^{(j)} | + | L_u^{(i)} - L_u^{(j)} | + | L_c^{(i)} - L_c^{(j)} |\,\right]$$

It may also be noted that, $0 \le d \le 1$ in first case, where as d (≥ 0) has no restriction on upper bound in second case.

10 Comparison among traditional Delphi Method, Fuzzy Delphi Method and Intuitionistic Fuzzy Delphi Method

Method	Methodology	Weakness and strength
Traditional Delphi Method	Experts give independent opinions; Data are analyzed statistically and are communicated to experts; Experts' reviews are analyzed and this process is repeated until convergence.	• Takes more time to collate expert opinions as Survey must be repeated multiple times. So, the cost is high. • The survey recovery rate is low. • In pushing for a consensus it' easy to misinterpret expert opinion.
Fuzzy Delphi Method	Experts give independent opinions; Subjective information are converted into objective data using fuzzy number; A fuzzy statistical analysis is done and are communicated to experts; Experts' reviews are analyzed and this process is repeated until outcome converges to a reasonable solution	• Saves on survey time and hence saves cost by reducing number of surveys; increases questionnaire recovery rate. • Experts can better express their opinions, ensuring the completeness and consistency of the group opinions as it takes into account the fuzziness that cannot be avoided during the survey process.
Intuitionistic Fuzzy Delphi Method	Communication with experts is the same as fuzzy Delphi Method yet an improved and elaborative statistical tool is used to reach in better conclusions. Subjective information is more likely to be like a quasi-objective data in case of intuitionistic fuzzy number; Intuitionistic fuzzy statistical analysis is done and is communicated to experts; Experts' reviews are analyzed in detail and this process is repeated until outcome converges to a reasonable solution.	• Reduces number of surveys rapidly and increases questionnaire recovery rate. So, the cost is lower than Fuzzy Delphi Method. • Takes into account the degree of non-membership values that cannot be avoided during the survey process. Hence, it does not misinterpret experts' original opinions and provides a true reflection of their response.

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