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Convex temporal intuitionistic fuzzy sets

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Abstract

We introduce a concept of convexity for temporal intuitionistic fuzzy sets and a series of basic results are established.

1 Introduction

The purpose of this paper is the extension to temporal intuitionistic fuzzy sets of the concept of convexity (concavity) introduced for intuitionistic fuzzy sets in [2].

2 Preliminaries

In this paper, all the definitions, the symbols and the results in connection with t-norms (t-conorms), convexity (concavity) of fuzzy sets and intuitionistic fuzzy sets follows [2](also, see [3]).

The subject "Temporal Intuitionistic Fuzzy Sets" ($TIFS$) was introduced by A.K. Atanassov as a generalisation of a intuitionistic fuzzy sets (IFS) (see [1]).

Let X be an universe and T a non-empty set. A $TIFS$ is an object having the form

$$A(T) = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T\}$$

where

(1) $A \subset X$ is a fixed set

(2) $\mu_A(x,t) + \nu_A(x,t) \leq 1$ for every $(x,t) \in X \times T$

(3) $\mu_A(x,t)$ and $\nu_A(x,t)$ are the degrees of membership and non-membership of the element $x \in X$ at the time-moment $t \in T$, respectively.

For every $TIFS$

$$A(T) = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T\}$$

we define the operators (see [1])

$$\begin{aligned}
A^C(T) &= \{\langle x,\ \nu_A(x,t),\ \mu_A(x,t)\rangle \mid (x,t) \in X \times T\}\\
\Diamond A(T) &= \{\langle x,\ 1-\nu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T\}\\
\Box A(T) &= \{\langle x,\ \mu_A(x,t),\ 1-\mu_A(x,t)\rangle \mid (x,t) \in X \times T\}\\
CA(T) &= \left\{\langle x, \bigvee_{t\in T} \mu_A(x,t), \bigwedge_{t\in T} \nu_A(x,t)\rangle \mid (x,t) \in X \times T\right\}\\
IA(T) &= \left\{\langle x, \bigwedge_{t\in T} \mu_A(x,t), \bigvee_{t\in T} \nu_A(x,t)\rangle \mid (x,t) \in X \times T\right\}
\end{aligned}$$

Let T' be a set such that $T \subseteq T'$ and $\lambda \in FS(X \times T)$.
We define

$$\underline{\lambda}(x,t) = \begin{cases} \lambda(x,t) & \text{if } t \in T\\ 0 & \text{if } t \in T' \setminus T\end{cases}$$

$$\overline{\lambda}(x,t) = \begin{cases} \lambda(x,t) & \text{if } t \in T\\ 1 & \text{if } t \in T' \setminus T\end{cases}$$

Then

$$A(T') = \left\{\langle x,\ \underline{\mu}_A(x,t),\ \overline{\nu}_A(x,t)\rangle\ \mid (x,t) \in X \times T'\right\} \in TIFS(X)$$

is an extension of $A(T)$.

Now, let h be t-norm or t-conorm and $A(T')$, $B(T'') \in TIFS(X)$,

$$\begin{aligned}
A(T') &= \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T'\}\\
B(T'') &= \{\langle x,\ \mu_B(x,t),\ \nu_B(x,t)\rangle \mid (x,t) \in X \times T''\}.
\end{aligned}$$

We put

$$A(T')\widetilde{h}B(T'') = A(T' \cup T'')\widetilde{h}B(T' \cup T'') =$$

$$\left\{\langle x, h\left(\underline{\mu}_A(x,t), \underline{\mu}_B(x,t)\right), h^C\left(\overline{\nu}_A(x,t), \overline{\nu}_B(x,t)\right)\rangle \mid (x,t) \in X \times (T' \cup T'')\right\}$$

3 Convex temporal intuitionistic fuzzy sets

Obviously, every ordinary IFS can be regarded as a $TIFS$ for which T is one element set (see [1]), and if

$$A(T) = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T\} \in TIFS(X),$$

then

$$A_t = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid x \in X\} \in IFS(X) \text{ for each } t \in T.$$

This suggests the following

Definition 1 *Let $A(T) \in TIFS(X)$ and f be a t-norm or t-conorm. $A(T)$ is called to be f-convex (f-concave) in X iff $A_t \in IFS(X)$ is f-convex (f-concave) in X for each $t \in T$.*

Lemma 2 *Let f be a t-norm or t-conorm and $T \subseteq T'$. $A(T) \in TIFS(X)$ is f-convex (f-concave) in X iff $A(T')$ is f-convex (f-concave) in X.*

Proof. Firstly, we prove that the following statements are equivalent:

(1) $\lambda(\cdot,t)$ is f-convex (f-concave) for all $t \in T$
(2) $\underline{\lambda}(\cdot,t)$ is f-convex (f-concave) for all $t \in T'$
(3) $\overline{\lambda}(\cdot,t)$ is f-convex (f-concave) for all $t \in T'$,

where $\lambda \in FS(X \times T)$.

Obviously (2) $\Rightarrow$ (1) and (3) $\Rightarrow$ (1).

If $t \in T$, $\lambda(\cdot,t) \equiv \underline{\lambda}(\cdot,t) \equiv \overline{\lambda}(\cdot,t)$ hence (1) implies (2) and (3).
If $t \in T' \setminus T$, $\underline{\lambda}(\cdot,t) \equiv 0$ and $\overline{\lambda}(\cdot,t) \equiv 1$. Because $f(1,1) = 1$ and $f(0,0) = 0$ for every f (t-norm or t-conorm) we obtain (2) and (3).
Now,

$$A(T) = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T\} \text{ is } f\text{-convex} \Leftrightarrow$$
$$\Leftrightarrow A_t \text{ is } f\text{-convex } \forall t \in T \Leftrightarrow$$
$$\Leftrightarrow \mu_A(\cdot,t) \text{ is } f\text{-convex and } \nu_A(\cdot,t) \text{ is } f^C\text{-concave } \forall t \in T \Leftrightarrow$$
$$\Leftrightarrow \underline{\mu}_A(\cdot,t) \text{ is } f\text{-convex and } \overline{\nu}_A(\cdot,t) \text{ is } f^C\text{-concave } \forall t \in T' \Leftrightarrow$$
$$\Leftrightarrow A_t \text{ is } f\text{-convex } \forall t \in T' \Leftrightarrow$$
$$\Leftrightarrow A(T') \text{ is } f\text{-convex } \blacksquare$$

Using Definition 1, from [2] we obtain results for convexity of $TIFS$. For example, by Theorems 12 and 15 from [2] and Lemma 2 we have

Theorem 3 (1) *Let f be a t-norm or t-conorm and $A(T) \in TIFS(X)$. Then*

(i) *$A(T)$ is f-convex (f-concave) if and only if $A^C(T)$ is f^C-concave (f^C-convex).*

(ii) *If $A(T)$ is f-convex (f-concave) then $\Box A(T)$ and $\Diamond A(T)$ are f-convex (f-concave).*

(2) *Let f, g, h be three t-norms or three t-conorms and $A(T')$, $B(T'') \in TIFS(X)$.*

(i) *If $f \leq g$, $f \ll h$, $A(T')$ is f-convex and $B(T'')$ is g-convex, then $A(T')\widetilde{h}B(T'')$ is f-convex.*

(ii) *If $g \leq f$, $h \ll f$, $A(T')$ is f-concave and $B(T'')$ is g-convex, then $A(T')\widetilde{h}B(T'')$ is f-concave.*

We cannot obtain the results for convexity of $CA(T)$ and $IA(T)$ as above, but we have the next theorem

Theorem 4 *Let f be a continuous t-norm or t-conorm.*

(1) *If $A(T)$ is f-convex, then $CA(T)$ is f-convex.*

(2) *If $A(T)$ is f-concave, then $IA(T)$ is f-concave.*

Proof. Let $A(T) = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid (x,t) \in X \times T\} \in TIFS(X)$.

(1) If $t \in T$ then

$$\mu_A\left(kx + (1-k)y,\ t\right) \geq f\left(\mu_A(x,t),\ \mu_A(y,t)\right)$$

for all $x, y \in X$, $k \in [0,1]$. Using Lemma 5 on [2] we obtain

$$\begin{aligned}&\bigvee_{t\in T} \mu_A\left(kx + (1-k)y,\ t\right) \geq \bigvee_{t\in T} f\left(\mu_A(x,t),\ \mu_A(y,t)\right) \geq \\ &\geq f\left(\bigvee_{t\in T} \mu_A(x,t), \bigvee_{t\in T} \mu_A(y,t)\right)\end{aligned}$$

for all $x, y \in X$, $k \in [0,1]$.

Analogous we have

$$\nu_A\left(kx + (1-k)y,\ t\right) \leq f^C\left(\nu_A(x,t),\ \nu_A(y,t)\right)$$

for all $x, y \in X$, $k \in [0,1]$, hence

$$\begin{aligned}&\bigwedge_{t\in T} \nu_A\left(kx + (1-k)y,\ t\right) \leq \bigwedge_{t\in T} f^C\left(\nu_A(x,t),\ \nu_A(y,t)\right) \leq \\ &\leq f^C\left(\bigwedge_{t\in T} \nu_A(x,t), \bigwedge_{t\in T} \nu_A(y,t)\right)\end{aligned}$$

for all $x, y \in X, k \in [0,1]$,therefore $CA(T)$ is f-convex.

(2) Is similar with (1) ■

Remark. $A(T)$ f-concave does not imply $CA(T)$ f-concave. Also $A(T)$ f-convex does not imply $IA(T)$ f-convex.

References

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[3] Mashinchi, M., On convexity of fuzzy sets, *The Journal of Fuzzy Mathematics*, Vol.2, No.3(1994), 655-669.